Convertible Bond Pricing Using Contingent Claims Analysis: An Empirical Investigation

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I. Introduction

The convertible bond is a hybrid security, which confers the potential pertaining to the underlying common stock while most of the properties of straight bond are persisting. In Korean capital market, convertible bond rose to the surface as a momentous financial instrument in the early 1980's. Tremendous changes have been made since then. Overwhelming amounts of convertible bonds are being issued by many listed firms lately, and the issuing conditions of convertible bonds recently issued differ from those early issued. While increasing quantity of convertible bonds in capital market, convertible bonds as a means of investment are spotlighted by many investors. The convertible bond market, however, is still thin in spite of development recently made. Especially, convertible bonds are rarely traded in the secondary market. The weakness of secondary market causes convertible bonds not to be priced accurately. Under these circumstances, the purpose of this study is to provide the investors and the issuers of convertible bonds with the rational guidance about the price of convertible bonds. This paper will deal with the valuation of domestic convertible bonds issued to the domestic investors.

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The brief sketch of what is in the several chapters is as follows: Next chapter details the properties of convertible bond. In chapter 3, we discuss the application of model which is appropriate for dealing with convertible bonds issued in Korean capital market. In chapter 4, we perform the empirical investigations. The data description, the empirical results and interpretations, and the sensitivity analysis will be detailed in this chapter. In last chapter, we conclude this paper, and discuss major contributions of the paper to a better understanding the valuation of convertible bonds.

II. Properties of Convertible Bond

1. General Properties of Convertible Bond

Convertible bonds are a common type of derivative security. The holder has the right to exchange a convertible bond for equity in the issuing company at certain times in the future according to a certain exchange ratio. The exercise price, however, equals the value of the bond and is therefore not constant. Very often and added complication is that the convertible bond has the call option, that is, it can be repurchased by the issuer at a certain price at certain times in the future. If the bonds have been called, the holder can always choose to convert prior to repurchase. Hence, the effect of the call provision is often to give the issuer the right to force conversion of the bonds into equity at an earlier time than the holders would otherwise choose. The call option embedded in convertible bonds assumes greater importance because of its potential use in forcing conversion. On the other hand, the number of shares of common stock that the bondholder will receive from exercising the option is called the conversion ratio. The conversion privilege may extend for all or only some portion of the bond's life like American or European Options. On the time of issuance of convertible bonds, the issuer has effectively granted the bondholder the right to purchase the common stock at a price equal to (Par value of convertible bond)/(Conversion ratio). This price is

referred to as the conversion price. The conversion value of a convertible bond is the value of the bond if it is converted immediately.[1] That is, the conversion value is the product of the conversion ratio and the market price of common stock.

2. Peculiarities of Convertible Bond in Korean Capital Market

(1) Market situations

Tremendous amounts of convertible bonds are being issued recently. Many listed firms raise the capital by means of convertible bonds, and 1,032.2 billion won convertible bonds were issued during the first five months in 1994. This amount is three times as large as the one in 1993, and closes in upon the largest quantity, 1,178.4 billion won, in 1989. The issuing portion of convertible bonds in raising the capital needed is increasing also: 2.5% in 1992, 8.3% in 1993, 9.1% in first quarter of 1994.

(2) Properties in Korean Capital Market

The convertible bond market of Korea has its own properties. On the time of issuance of convertible bond, issuing conditions include maturity, coupon rate, secured yield to maturity, conversion price, etc. Among these conditions, our focus is on the coupon and secured yield to maturity. Once investors don't exercise the conversion option, the profit induced by investment in the convertible bond will come from these factors. The coupon rate of convertible bond is very low compared with other straight bonds. Actually, the convertible bonds of some firms have 1% coupon rate. To compensate for low coupon rate, once investors (bondholders) do not exercise the option, secured yield to maturity (somewhat similar rate compared with other straight bonds) is given to the investors.

Another peculiarity is that the conversion privilege may be prohibited for some portion of bond's life in Korean convertible bond market. Actually, the conversion

1) The theoretically correct definition for the conversion value is that it is the product of the conversion ratio and the stock price after conversion.

cannot take place during the 7 months after the issuance. The difference between the conversion request and the actual conversion has to be understood. In the view of conversion request, we cannot make a request to convert the bonds into the stocks during the 6 months after issuance and the 1 month before the maturity. Conventionally, it takes the investors from 20 to 50 days to obtain the stocks which are issued as a result of conversion. That is, there exists lead time on which the conversion request is in the process. For instance, if the request is made on this month, the actual conversion will occur on the 20th next month in Korean convertible bond market. This clause of conversion prohibition could shrink the willingness of investment in convertible bonds.

In addition to the properties above, the last one is that only block exercise on conversion option can be made. In early 1980's, the partial exercise on conversion option was a unique property in Korean capital market. That peculiarity, however, is forgone nowadays. On the other hand, the domestic convertible bonds issued to demestic investors are not callable and putable.

III. Valuations of Convertible Bond

The security pricing is the most important factor in capital market. Accurate prices help the securities trading to be promoted in the market. We'll perform an empirical investigation using the modified model and boundary conditions of Brennan & Schwartx [1980] in this chapter.

1. Model for Convertible Bond

To calibrate the PDE and boundary conditions, first of all, we have to set up the several assumptions. The assumptions are enumerated as follows:

(a) Perfect market

(b) Trading takes place continuously in time and there are no restrictions against borrowing or short sales.
(d) Continuous and uniform cash payments are made: The payouts by the firm are made per unit time continuously and uniformly to either its shareholders or liabilities holders.
(e) Modigliani-Miller theorem I obtains.
(f) No advance conversion notice is required.
(g) The term structure of interest rate is flat and known with certainty.
(h) The dynamics for the value of the firm through time can be described by lognormal diffusion process.
(i) It is possible for firm to default on the bond.
(j) Capital structure consists of common stock, senior debt, and convertible bond.
(k) The management of firm acts at all times to maximize shareholder's wealth subject to the restrictions placed on it.
(l) No securities including stocks, senior bonds, and convertible bonds are issued during the life of convertible bond.

Under these assumptions, we can elicit the appropriate model. The stochastic process of firm value is

$$dV=[\ V\mu_V-Q_1]\ dt+V\sigma_V dz \tag{1}$$

where V is firm value, μ_V is the instantaneous expected rate of return on the firm value per unit time, Q_1 is the total payouts paid continuously and uniformly by firm per unit time to securities holders, σ_V^2 is the instantaneous variance of return of firm value, and dz is a Wiener process. As we postulate that the capital structure consists of common stock, senior debt, and convertible issues, total payout, Q_1, is

$$Q_1 = I_B + I_{CB} + D \tag{2}$$

where I_B is the coupon payment to the senior bondholders, I_{CB} is the coupon payment to the convertible bondholders, and D is the dividend payment to the stockholders. We can derive the PDE governing the value of convertible bond.

$$\frac{1}{2}\sigma_V^2 V^2 C_{VV} + (rV - Q_1)C_V - rC - C_\tau + I_{CB} = 0 \tag{3}$$

where C denotes the value of the convertible bond, r is risk free rate, τ is the time to maturity, and subscripts denote partial differentiation.

Now, we can consider several boundary conditions to price the convertible bond. The convertible value corresponding to a zero firm value is zero. The expression of lower boundary condition is

$$\lim_{V \to 0} C(V, t) = 0$$

The conversion condition is

$$\lim_{V \to \Phi(t)} C(V, t) = z(\Phi(t) - B)$$

where $\Phi(t)$ is optimal exercise boundary, z and B are the dilution factor, the market value of senior debt respectively. The conversion can take place every time except the period of prohibition. Hence the condition has to be taken into account every time except the period in an empirical investigation. On the other hand, the optimality condition for exercise boundary is

$$\lim_{V \to \Phi(t)} \frac{\partial C}{\partial V} = z$$

The maturity condition assumes great importance because of its post on which the computation algorithm set out. The convertible bond is assumed to mature at T, prior to the maturity of the senior debt. The value of convertible bond on maturity T is,

$$C(V, T) = \begin{cases} z(V - B(V, T)) & \text{if} \quad CB_P + \alpha \le z(V - B(V, T)) \\ CB_P + \alpha & \text{if} \quad z(V - B(V, T)) \le CB_P + \alpha \le V - B_P \\ V - B_P & \text{if} \quad 0 \le V - B_P \le CB_P + \alpha \\ 0 & \text{if} \quad V - B_P < 0 \end{cases}$$

where CB_P is par value of convertible bond and α, B_P are the premium which makes the secured yield to maturity kept and the par value of senior debt, respectively. Secured yield to maturity is guaranteed for the convertible bondholders at maturity who didn't exercise the conversion option. In Korean convertible bond market, the lump sum premium is given to keep the secured yield to maturity. The third part of maturity condition needs to be investigated thoroughly. That shows the situation of bankruptcy. The convertible bondholders are paid after the senior bondholders. The bankruptcy condition has a complicate and moot question. Actually, we cannot define the bankruptcy explicitly, and there has been a controversy over the definition itself. The capacity to pay the interest to bondholders and the firm value are also considered in defining the bankruptcy. In our boundary conditions, we'll quote the bankruptcy condition in Brennan & Schwartz [1980]. It says that the convertible bond indenture is written so that the bondholders will receive a fraction k of the par value when the firm goes bankrupt. The premium which keeps the secured yield to maturity is considered in the condition. If the value of firm falls to the sum of the par value of the senior debt and k times the par value of the convertible issue and the premium, the firm will go bankrupt. Besides it is assumed that convertible bondholders are rarely paid provided the firm goes bankrupt. The expression of bankruptcy condition is,

$$C(V, t) = k(CB_P + \alpha) \qquad \text{if} \quad V = B_P + k(CB_P + \alpha)$$

There exists conversion prohibition period in Korean convertible bond market.

The conversion request is prohibited during the 6 months after the issuance and the 1 month before the maturity in Korean capital market. For this reason, the conversion condition needs not to be checked during the 7 months after issuance considering the lead time.

2. Model for Senior Debt

For computing the price of convertible issue, the market value of senior debt has to be taken into consideration. The conversion and maturity conditions of convertible bond involve the value of senior debt. The stochastic process of firm value is,

$$dV=[\ V\mu_V-Q_2]\ dt+V\sigma_V dz \tag{4}$$

where $Q_2=I_B+D^*$,and D^* is the dividend payout which reflects the changed capital structure. The convertible issues are no longer in the firm's capital structure by *conversion or maturity*. The following PDE governs the value of senior debt.

$$\frac{1}{2}\sigma_V^2V^2B_{VV}+(rV-Q_2)B_V-rB-B_\tau+I_B=0 \tag{5}$$

The boundary conditions are also put up. The maturity condition is,

$$B(V,T)=\text{Min}[\ V,B_P]$$

where T' is the maturity date of senior debt. The bankruptcy condition is similar to one for convertible bond. The notation of h follows the bond indenture. The expression of condition is,

$$B(V,t)=hB_P \qquad \text{if} \quad V=hB_P$$

If the firm value falls to the h times the par value of the senior debt, the firm will go bankrupt.

3. Algorithm for Pricing Convertible Bond

To employ the finite difference procedure, the PDE has to be converted into a set of difference equations and the difference equations are solved iteratively. The PDE in equation (3) can be approximated by

$$a_i C_{i-1,j} + b_i C_{i,j} + c_i C_{i+1,j} = C_{i,j+1} + I_{CB} \cdot \Delta t \tag{6}$$

$$\text{where} \begin{cases} a_i = -0.5\sigma_V^2 i^2 \Delta t + 0.5ri\Delta t - 0.5Q_1(\Delta t/\Delta V) \\ b_i = \sigma_V^2 i^2 \Delta t + r\Delta t + 1 \\ c_i = -0.5\sigma_V^2 i^2 \Delta t - 0.5ri\Delta t + 0.5Q_1(\Delta t/\Delta V) \end{cases}$$
$$i = 1, 2, \cdots, M-1 \qquad j = 0, 1, 2, \cdots, N-1$$

The Δt, ΔV are equally spaced time and firm value, i, j are the orders for firm value and time and the finite numbers of equally spaced time and firm value are N and M ,respectively. The boundary conditions can be converted into the finite difference form also. The upper and lower bounds of gird diagram of finite difference procedure and the maturity condition have to be determined as known constants. The value of convertible bond is zero when the firm value is zero. Hence, $C_{0,j} = 0 \qquad j = 0, 1, 2, \cdots, N$. The limiting firm value condition can be approximated by

$$\frac{C_{M,j} - C_{M-1,j}}{\Delta V} = z \qquad j = 0, 1, 2, \cdots, N$$

The maturity condition can be converted into the finite difference terms in a similar way. The upper three conditions define the value of convertible bond along the three edges of the grid diagram. For a given j=N-1, equation (6) generates

(M-1) simultaneous equations. We can perform the computation which is for the (M-1) unknowns: $C_{i,N-1}$ $i=1,2,\cdots,M-1$. In a same way, we can obtain the (M-1) unknowns on the date of valuation : $C_{i,0}$ $i=1,2,\cdots,M-1$. The one value corresponding to the firm value of interest will be selected. That's the price of convertible bond. In a case of senior debt, the same algorithm can be applied to solve the PDE governing the value of senior debt basically. However, the alteration is in the condition of the limiting firm value. As the firm value tends to infinity, no bankruptcy will take place. Hence, the bond is free from default risk. The upper edge of the grid diagram is defined like this:

$$\int_0^{\tau} e^{-rs} I_B ds + B_P e^{-r\tau}$$

Using the same logical flow which is utilized in pricing the convertible bond, we can capture the value of senior debt, B.

IV. Empirical Analysis

1. Data Description

The data include the issuing conditions of convertible bonds, the estimation of firm value(V) and variance (σ_V^2) of firm value's return, the interest rate, and the issuing conditions of senior debt.

(1) Firm Value Estimations

To obtain a sample of empirical study, the 62 convertible bonds of 45 listed firms are selected. They are issued on the interval between 18 Jan 1994 and 30 May

1994. We define the firm value as the sum of common stocks, senior debt, and convertible bonds in the modificatory model. The value of common stocks is estimated as the product of the total number of stocks which have been issued until the valuation date and the closing stock price of the valuation date. In a case of convertible bonds, the issuing price is used for the component of firm value. Much efforts have to be made to estimate the value of senior debt. We collect the senior debt of each company, which doesn't mature on the valuation date. Henceforth, we obtain the present value of senior debt on the valuation date. The present value of senior debt is employed for the total firm value.

(2) Variance of Firm Value's Return

(a) Estimation method

The trading of convertible bonds is also few and far between in the market. For this reason, we are in a trouble to observe the fluctuation of firm value. Usually, the volatility of stock return is employed for the variance of firm value's return. However, the levered firm has the lower volatility than the volatility of stock return. This means that the volatility of stock return has to be adjusted for the levered firm. In our study, we employ the following equation for adjusting the standard deviation of return to the equity.

$$\sigma_E = E_V(\sigma_V)\ V/E\ \sigma_V$$

where $E_V(\sigma_V)$ is the partial derivative of the value of equity with respect to the value of the firm, σ_E is the standard deviation of return to the equity, and V, E are firm value and equity respectively. In the adjustment of the stock return's volatility, added assumptions are that capital structure consists of common stocks and senior debt, and no cash payments are made by the firm. The computation of $E_V(\sigma_V)$ rests on this assumption. Bisection numerical procedure is employed for searching σ_V. Once we know that an interval contains a root, the bisection

method is available to refine it. The new adjusted σ_V is formed using σ_E, E, V, and E_V. On the other hand, σ_E should be obtained preceding the bisection procedure. The standard deviation of return to equity is calculated, using the daily stock return data, for the two months period, immediately preceding the valuation date. It is assumed that stock prices are lognormally distributed. The lognormal stock price distribution implies that the variance over any given time period is proportional to its length. Thus, the computing the constant future instantaneous variance is to simply rely on past stock return data; that is, to assume that this past variability of the stock's returns is indeed invariant across time. We employ the variance calculating method of Parkinson. Extreme values variance measurement procedure by Parkinson is used to obtain variance estimates based on recent past market data.

(b) Adjusted volatility in an empirical investigation

We deal with 45 listed firms, and the volatility of firm value's return varies from firm to firm. It is worth analyzing the relation between the firm's debt ratio and the reduction ratio in adjusting the volatility. Average debt ratio of 45 listed firms on the valuation date (issuing date of convertible bonds) is 0.335472, and average reduction ratio $(\sigma_E-\sigma_V)/\sigma_E$ is 0.360261. The linear relationship can be inferred from these average ratios. Actually, when we perform the simple linear regression where the independent variable is the debt ratio and the dependent variable is the reduction ratio, the following relationship is appeared.

$$\text{reduction ratio} = 0.016927 + 1.023436 \times \text{debt ratio}$$

This regression line has the high explanatory power(R^2=0.9661). From this result, we can guess the volatility of levered firm, and this can be utilized for pricing the securities.

(3) Riskless Rate of Interest

The last factor which needs to be estimated is the risk free rate of interest. The yield of government bond is hired for the proxy for riskless rate of interest.

(4) Issuing Conditions of Convertible Bonds and Senior Debt

We'll enumerate the various conditions pertaining to the securities issues of convertible bonds and senior debt in appendix. The data are collected from Korea Stock Exchange, Samsung Securities, Coryo Research Institute, Korea Investors Service, Inc., and so on.

2. Results and Interpretations

Utilizing the algorithm of implicit finite difference method, we can elicit the theoretical convertible bond price using the data described above.[2] The emphasis is directed toward comparing the price estimated from the model with actual price observed in the market. In order to make a comparison, several statistical tests are required. They include the test where null hypothesis is that the correlation coefficient between estimated and actual is zero, the test of the regression coefficients determined from the relationship between estimated and actual where the intercept and slope are expected to be zero and one, respectively, and the comparison of mean paired differences between actual and estimated.

(1) Empirical Results

The actual and estimated prices are presented in appendix. At a glance, the observed prices (actual price) are undervalued for all firms provided it is assumed that model yields the accurate price. The number attached to the name of firm is the order of bond issuance.

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2) It is assumed that the convertible bondholders recover 3/5 of the par value of their investment in the event of bankruptcy, and N(the number of time interval), M(the number of firm value interval) are 80 and 190 respectively.

(2) Several Tests

The first test of the model is a simple test of correlation between the actual and estimated prices. The correlation coefficients is 0.9938. According to the result, the null hypothesis of a zero correlation coefficient between actual and estimated prices is clearly rejectable. The high correlation coefficient says that there exists the (linear) relationship between two prices.

Now, we perform the simple linear regression analysis where the independent variable is the estimated price and the dependent variable is the actual price. The result to this point suggest that the estimated price is associated with the actual price (correlation coefficient greater than zero). If the model contains no systematic bias, the linear relationship between actual and estimated should produce an intercept of zero and slope coefficient of one. The following expression shows the linear relationship between two prices.

$$\text{Market Price} = -6.169141 + 0.818137 \times \text{Model Price}$$

In the case of intercept term, we cannot reject the null hypothesis (parameter=0) at the significant level 0.05. Thus, the intercept term has no meaning, and we can regard the intercept as zero. On the other hand, the null hypothesis of a zero slope coefficient is clearly rejectable. This finding can be conjectured from the correlation test also. The null hypothesis of one slope coefficient is also rejectable provided t-value is captured (T for H0: parameter=1 is -15.3905). Thus, we cannot say that the slope coefficient is one. That is to say, the model price (estimated price) is not equal to the market price (actual price), but we can adopt that it has high explanatory power.

If the data errors are unsystematic, then the average estimated price should not be significantly different from the average actual price. The data errors should average out. Thus, next test is a test of differences in the mean paired observations. The null hypothesis of zero difference between mean paired observations can be clearly rejectable (significance level 0.0001).

How much larger is the model price than the market price? To calibrate the gap between two prices, we perform the simple linear regression analysis, where the independent variable is the market price and the dependent variable is the (Model price - Market price)/Model price. The null hypothesis of zero intercept and zero slope coefficient is clearly rejectable. However, the slope of fitted regression line is nearly zero, and the intercept is 0.2562. This finding says that the model price exceeds the actual price by 25% of model price.

(3) Results Analysis

The result to this point suggest that the model price is related with the market price. However, on average, the two prices are not equal, and the model price always exceeds the market price by the consistent amount. Thus, eliciting the approximate price, the model has the somewhat high explanatory power. Why does the model price exceed the market price systematically? We can conjecture several grounds. These are enumerated as follows:

(a) Inefficiencies in market for convertible bond: As stated previously, the Korean market for convertible bond has the structural drawbacks. For instance, only a few convertible bonds are traded in the secondary market. This fact causes the price of convertible bond to be distorted.

(b) Bias in proxy estimation: Most data need to be estimated. Especially, we have a difficulty in estimating the firm value and the volatility. The bias in estimating these information can make the result distorted.

(c) Advance conversion notice in the market situations: It is assumed that no advance conversion notice is required in the model. This fact means that the conversion request and the actual conversion take place simultaneously. Investors make a decision about the conversion every time, observing the fluctuation of firm value. As soon as the option is exercised, the stock which are issued as a result

of conversion will be given to the investors. However, there exists the lead time on which the conversion request is in the process in the market. The discord between the option exercise and the actual conversion causes the convertible bond price to be distorted.

3. Sensitivity Analysis

The model can be utilized to observe the sensitivity of the convertible bond value to changes in pertinent parameters. To observe the sensitivity, the only one firm can be selected. We choose the convertible bond of YuHan Corporation. The results of the sensitivity analysis are shown in appendix. Each parameter is increased by 10% of original value except z value. If the variance of firm value return is increased, two effects are involved simultaneously: increasing the probability of bankruptcy and the increasing the value of the conversion option. The former reduces the value of convertible bond, and the latter raises the bond value. In this case, the effect of latter outweighs the former. On the other hand, zero z fraction case deserves comments. That case means the removal of conversion privilege, which would make the bond a straight bond. Thus, in this case, 27.637% of the basic value represents the premium of conversion privilege. The 10% increase in the stock price raises the bond value by 5.622%, and the increase in interest rate induces the decreasing bond value. The change in conversion price causes the z value to become different. In turn, this affects the value of convertible bond. A 10% of increase in the coupon rate of convertible bond raises the bond value by 0.998%. Finally, an increase in the firm's dividend payout also reduces the bond value since this increases the probability of bankruptcy. This is induced by the fact that the convertible bonds are not protected against dividend payments by the firm.

V. Conclusion

We dealt with the theorectical price of convertible bond, and observed the difference between model and market price. What is more, we inferred the causes that make the price distorted. The structural drawbacks of convertible bond market are the critical factors of the price distortion. First of all, the activated trading of convertible bonds is needed in the secondary market. Actually the only partial amounts (i.e. 4% on May in 1994.) of issuing convertible bonds are traded in the market. The systematic maintenances have to be encouraged to incite the participation of the investors in the market. First, the period of conversion prohibition has to be reduced. Second, the lead time between the conversion request and the actual conversion has to be reduced also. It takes the investors from 20 to 50 days to capture the stocks which are issued as a result of conversion after their conversion request. Third, the public announcement of trading information has to be reinforced. In addition to the remedy of the structural problems in Korean convertible bond market, the model for theoretical price of convertible bond has to be developed to guide the decision of investors. We want this paper to contribute to this part. We tried to consider the actual market circumstances in the model. The capital structure, bankruptcy situation, and conversion patterns in the present market were handled also. In an empirical analysis, the theoretical prices are always overvalued systematically. As stated-above, this fact seems to be induced by the market situations. This paper has such problems waiting solutions as the considerations of preferred stocks, advance notice of option exercise and so on. We defer those to the further research.