The Impact on the Investment Signaling Equilibrium of the Capital Structure

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Abstract

We examine the existence of the investment signaling equilibrium without assuming a specific utility function for the managers of the corporations. We assume the managers have the initial holdings of their own corporations as a form of the executive compensation. Under the different financing schemes to finance the investment, the new equity financing and the risky debt financing, we derive the investment signaling equilibrium and compare the the investment signaling equilibrium under each financing scheme. We show that the investment signaling equilibrium with each financing will obtain with the underinvestment of the high quality firm and that the investment signaling equilibrium with the risky debt financing will dominate the investment signaling equilibrium with the new equity financing.

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I. Introduction

There have been some efforts to explain the investment activity of the firm as a signal for identifying the true quality of the firm. Trueman [1986]constructs a signaling model based on the internally financed investment decision, assuming a negative exponential utility function which is linear in terms of the mean and variance of the final wealth. He argues that an insider who holds some fraction of the firm equity will use an overinvestment above the optimal NPV maximizing investment level under symmetric information in order to signal a private information owned by the firm insider to the outside market investors who are assumed to be risk-neutral. Others use the investment decision as a signal along with other activities such as the dividends or the insider trading ,etc. Ambarish ,John and Williams[1987] employ the dividend decision and the investment decision as the signal activities. Firms issue new equity to finance the investment and the dividend and insiders are assumed to act in the interests of the current shareholders. The high quality firms can choose NPV maximizing investment by selecting high dividend level but they overinvest (underinvest) and pay out less (more) dividends if the marginal cost of increasing (decreasing) the investment for the high quality firm is less than the cost for the low quality firm. John and Mishra[1990] provide a signaling model based on the insider trading and the investment as the signal activities. They assume that there exists enough internal cash to finance the investment and the managers of the firms have the negative exponential utility function. They argue that an overinvestment (underinvestment) by the growing high quality firms (the declining high quality firms) can be used to signal the firm's true type to the market investors who don't have the perfect information about the firm quality. Williams [1988] use the investment , the stock repurchase(or new equity issue) and the dividend as the signal activities. In his

model, the true quality of the firm can be signaled by the stock repurchase ,the dividend and the investment level that is exactly the optimal investment under the symmetric information , and they are financed by the internal cash and the new equity issue.

In this study, we provide a signaling model based on the investment decision without assuming the objective function for the insider or assuming any specific type of utility function. Also, we take the financing decision into the consideration. Under the different financing schemes , the appropriate objective functions for the firm managers will be derived endogenously within the model and it will be examined what kind of investment decision the high quality firms will make to signal their true types to the market investors who cannot distinguish the high quality firms from the low quality firms. Then, it will be examined whether there is any difference between an investment signaling equilibrium with the new equity issue and an investment signaling equilibrium with the debt issue in terms of the welfare for the high quality firm manager ,i.e., whether either new equity issue or debt issue will yield an investment signaling equilibrium that will be more beneficial to the high quality firm manager.

II. The Asymmetric Information and Signaling with the Investment Decision

We will consider a competitive economy operating between now and then. All decisions including the investment decisions are made now and all payoffs on those decisions are received then. The corporations in the economy can be grouped by the industry and there are many risk- averse investors. In order to introduce uncertainty , let (Ω, B, Ψ) denote the probability space , where Ω is the set of

possible states of economy, B is the event space,and Ψ is the subjective probability measure for the occurrence of states of nature. Suppose that there are only a finite number of states of nature, i.e., $\Omega = \{\underline{\omega}, \ldots\ldots \overline{\omega}\}$; the event space B is the power set,i.e., the set of all subsets of Ω. Let Π_f denote the payoff of corporation f then and suppose that Π_f is a function mapping Ω into the real line ,i.e., $\Pi_f : R \times \Omega \Rightarrow R$. Then, $\Pi_f(I,\omega)$ is the payoff of corporation f in state ω in Ω when the firm f takes the investment I. Suppose that the financial markets are complete. There are N primitive assets while a primitive asset is a promise to pay one dollar in state ω and zero in all other states. Let $p(\omega)$ be the price of the state ω asset. Trading occurs in the primitive assets and the corporate assets. Finally, suppose that the information possessed by the market investors and the corporations is complete with respect to all the financial markets except those in which the assets of industry I firms are traded. By complete information we mean that all investors and corporations know the payoff functions Π_f of the firms and the payoff functions for all the assets issued by the firms.

Next, suppose that the financial markets for the assets of corporations in industry H are characterized by the incomplete information. Suppose the information is only incomplete with respect to the earnings,equivalently, the payoffs, of the firms in industry H. Suppose there are two types of firms in this industry H. Type one firm managers know that the random payoff then for their firms is $\Pi_1(I,\omega)$. Similarly, type two firm managers know that the random payoff then for their firms is $\Pi_2(I_2,\omega) > \Pi_1(I_1, \omega)$, for $I_2 = I_1$ and all ω in Ω. Although the firms have the complete information , ex ante, concerning their payoff functions, individual investors in the market do not have such a complete information about the payoff functions of the firms in the industry H. The investor who purchases an asset from a firm f in industry H only knows that if state of nature ω is realized then, the corporate payoff will be $\Pi_1(I_1, \omega)$ with probability θ or $\Pi_2(I_2, \omega)$ with probability $1-\theta$ when θ is the proportion of firms in the industry H that are type one

, or equivalently, the probability of randomly selecting a type one firm. We assume that the information about the proportion θ is a public information. It follows that the investors know the average industry payoff will be a pooled value of $\Pi_1(I,\omega)$ and $\Pi_2(I,\omega)$,i.e., $\Pi_0(I,\omega)=\theta\Pi_1(I,\omega)+(1-\theta)\Pi_2(I,\omega)$. Since the investors in the market don't have the information about the random payoffs of the firms, the type one firms and the type two firms will be valued at $V_0=\sum_\Omega p(\omega)\Pi_0(I,\omega)$ unless they can be identified properly. It is obvious that the type two firms will try to be identified as the high quality firms since V_0 is less than $V_2=\sum\Omega p(\omega)\Pi_2(I,\omega)$ while the type one firms prefer being valued at V_0 to being valued at their true value $V_1=\sum\Omega p(\omega)\Pi_1(I,\omega)$ since V_0 is greater than V_1. Therefore, the type two firms try to convey their information about the true payoff to the market investors through being engaged in some signaling activity . Here, we interpret the investment decision of the firms as such a signaling equilibrium.

III. Investment Signaling Equilibrium with the Equity Issue

First, we explore the existene f the investment signaling equilibrium when both the type one firms and the type two firms issue the new equities to finance the investments of their firms. We assume that all firms have N stocks of the equities before issuing new equities. In a signaling equilibrium, the type two firm manager should have higher welfare given signaling than given no signaling and the type one manager must be better off by not signaling ,i.e., revealing his firm type, than by signaling. In other words, a signaling must provide the type two firm manager higher welfare than no signaling and being identified as the type one firm manager while the signaling should keep the type one manager from mimicking such a signaling activity by the type two firm manager.

1. The Investment Levels That deter the Type One Firm Manager from Mimicking

The type one firm managers are assumed to have X_{11}^0 stocks of initial holdings of their own firms. Suppose that there exists a signaling equilibrium where the investment level is a signal. If the type one manager does not signal and the type two manager does signal, he will make an investment decision to maximize the function of $\alpha_1\{S_1(I) - I\}$ where α_1 represents $\frac{X_{11}^0}{N}$ and $S_1(I)$ denotes $\int_\Omega p(\omega)\Pi_1(I,\omega)d\omega$ which is the true market value of the type one firm.[1] Since the type one firm manager optimally selects I_1^S if he does not signal where I_1^S denotes the optimal investment for the type one firm under the symmetric information [2], the payoff to the type one firm manager will be $\alpha_1 S_1^0(I_1^S)$ if he does not signal.

However, the type one manager can try to mimic the type two manager's investment selection and signal by taking the same amount of the investment selected by the type two firm manager. The following theorm identifies the objective function for the type one managers given mimicking.

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[1] In the equity issue case, the risk adjusted present value of the firm's payoff, V_f, is equivalent to the market value of the stock issue, i.e., $V_f = S_f$. We assume that $D_1\Pi_f > 0$ and $D_{11}\Pi_f < 0$ where D_1 denotes the first partial derivative with respect to the first variable and D_{11} denotes the second partial derivative with respect to the first variable.

[2] I_1^S is an investment level such that $\int_\Omega p(\omega)D_1\Pi_1(I_1^S,\omega) - 1 = 0 \Leftrightarrow S_1(I_1^S) - 1 = 0$

Theorem 1.[3] Suppose both the type one firm and the type two firm issue the new equities to finance the investments. If the type one firm manager decides to mimic the type two firm manager's investment decision in the signaling equilibrium , the objective function for the type one firm manager will be

$$\alpha_1\{S_2(I) - I\} - \alpha_1\{\frac{S_2(I) - I}{S_2(I)}\}\{S_2(I) - S_1(I)\}.$$

The first term of the objective function is the old shareholders' true value of the type two firm and the second term is the capital loss to the type one firm manager by mimicking.We will denote such an objective function as $\alpha_1 S^0_{1m}(I)$.

The type one firm manager will not signal,i.e., not mimic the investment decision of the type two firm manager if the payoff with the signal is less than the payoff without the signal. Therefore, the following condition under which the type one firm manager will not mimic the type two firm manager's investment decision can be established as follows;

$$\alpha_1 S^{o}_{1m}(I) \leq \alpha_1 S^0_1(I^s_1) \qquad (1)$$

(1) will be satisfied for an investment level inside $(0, I_m]$ or $[I_M, \infty)$, where I_m and I_M are the investments that satisfy the following;

$$S^0_{1m}(I_m) = S^0_1(I^s_1) \qquad S^0_{1m}(I_M) = S^0_1(I^s_1) \ , \qquad I_m < I_M$$[4]

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[3] The proofs for the theorems appearing in this work can be obtained from the author.

[4] The existence of I_m and I_M can be easily proved by showing that $S^0_{1m}(I^s_1)$ is greater than $S^0_1(I^s_1)$.

2. The Investment Levels That Let the Type Two Firm Manager Prefer Signaling to No Signaling

The type two firm managers have an initial equity holdings of his own firms. [5] We denote such an initial holding as x^0_{h2}. If the type two manager signals, it can be easily shown that $\alpha_2 S^0_2(I) = \int_\Omega p(\omega) \frac{x^0_{h2}}{N + n_2(I)} \Pi_2(I, \omega) d\omega = \alpha_2 \{S_2(I) - I\}$ will be the objective function for the type two firm manager .

If the type two manager does not signal and others do , his objective functions will be one identified in the following theorem.

Theorem 2. Suppose both the type one firm and the type two firm issue the new equities to finance the investments. If the type two firm manager decides not to signal , the objective function for the type two firm manager will be $\alpha_2 \{S_1(I) - I\} + \alpha_2 \{S_1(I) - I\} \frac{S_2(I) - S_1(I)}{S_1(I)}$.

We will denote such an objective function by $\alpha_2 S^0_{2n}(I)$. Since he will take the optimal investment level for the type one firm under the symmetric information , his payoff will be $\alpha_2 S^0_{2n}(I^S_1)$ if he does not signal. The type two firm manager will signal only if the payoff from signaling is greater than the payoff from no signaling. Therefore, he will signal only if the following condition is satisfied. In (2) , the LHS

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[5] See Leland & Pyle[1977], and John & Mishra[1990] for signaling by the insider trading.

represents the payoff given signaling and the RHS does the payoff given no signaling ;

$$\alpha_2 S_2^0(I) > \alpha_2 S_{2n}^0(I_1^s) \qquad (2)$$

The investment levels to satisfy (2) are such investments as inside (I_l, I_L) ,where I_l, and I_L are defined as follows;

$$S_2^0(I_l) > S_{2n}^0(I_1^s), \quad S_2^0(I_L) > S_{2n}^0(I_1^s) \qquad \text{where } I_l < I_L \,^6$$

3. The Existence of the Investment Levels That Can Result in the Signaling Equilibrium with the New Equity Financing

The investment levels that may result in the existence of the signaling equilibrium ought to satisfy both (1) and (2). Therefore, identifying the investment levels to satisfy (1) and (2) is equivalent to identifying the existence of the signaling equilibrium with the investment decision. As was stated before, the investments to satisfy (1) will be the investments inside $(0, I_m\}$ or $\{I_M, \infty)$. Similarly, the investments inside (I_l, I_L) will satisfy (2). It depends on the relative size of I_m, I_M , I_l ,and I_L whether there exist the investment levels to satisfy (1) and (2).

Proposition 1. Suppose firms issue the new equity to finance the investments. Signaling equilibria will exist with underinvestments by type two firm manager. Any investment inside $(I_l, I_m]$ obtains such an underinvestment signaling equilibrium.

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[6] The existence of I_l and I_L can be proved by showing that $S_2^0(I_1^s)$ is greater than $S_{2n}^0(I_1^s)$.

proof. Note that the signaling equilibria with the unverinvestments are feasible if I_l is less than I_m. Before showing I_l is less than I_m, note that $S_2^O(I)$ is greater than $S_{Im}^0(I)$ at all I and $S_{Im}^0(I)$ will be maximized at the investment level greater than I_2^S. Then the condition that I_l is less than I_m is equivalent to the following condition since $S_2^O(I)$ is increasing inside the interval $[0, I_2^S)$ which contains both I_l and I_m ;

$$S_2^O(I_l) < S_2(I_m) - I_m = S_2^0(I_m)$$

Recall that I_l and I_m will satisfy the following ,respectively;

$$S_2^0(I_l) = S_{2n}^0(I_1^s) = \frac{S_1(I_1^s) - I_1^s}{S_1(I_1^s)} S_2(I_1^s) \quad , \; S_2^0(I_m) = S_2(I_m) - I_m = \frac{S_2(I_m)}{S_1(I_m)} \{S_1(I_1^s) - I_1^s\}$$

Then , $S_2^0(I_l) - S_2^0(I_m) = \{\frac{S_2(I_1^s)}{S_1(I_1^s)} - \frac{S_2(I_m)}{S_1(I_m)}\}\{S_1(I_1^s) - I_1^s\} < 0$ since I_m is less than I_1^S.

Q.E.D.

{ Proposition 1} shows that signaling equilibria can exist only with the underinvestments inside the interval $(I_l, I_m]$ by the type two firm manager. However, it is obvious that an underinvestment signaling equilibrium with I_m dominates other underinvestment signaling equilibria since the investment I_m yields the highest payoff to the type two manager while any choice of an underinvestment inside $(I_l, I_m]$ by the type two manager does not provide an incentive for the type one manager to

mimic. Therefore, the signaling equilibrium obtains with the underinvestment I_m by the high quality firm managers. In this signaling equilibrium, the high quality firm managers select I_m and the type one firm managers select I_1^s,respectively and they reveal their true identities by doing so.

IV. Investment Signaling with the Risky Debt Issue

The next financing scheme we will consider is the debt financing. Suppose both type one and type two firm issue the zero coupon bond to finance the investment. Also, we assume that the Principle of Increasing Uncertainty (PIU) holds. The PIU implies that the riskiness of the price distribution increases with the investment , after correcting for the mean of the price distribution.[7]

1. The Investment Levels That Prevent the Type One Firm Manager from Mimicking

As in the new equity issue case , we start our analysis by supposing that the type two manager takes some investment level and such an investment financed by the issue the risky debt signals the firm type. Suppose that the type one manager does not signal and the type two manager does signal. It is well known that the type two firm manager will maximize the function of $\alpha_1\{\int_\Omega p(\omega)\Pi_1(I,\omega)d\omega - I\}$,i.e., $\alpha_1\{V_1(I)-I\}$[8] . we denote $V_1(I)-I$, the true shareholder value for the type

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[7] The PIU was first introduced by Hayne Leland [1972] . Such an interpretation of the PIU in terms of the Rothschild-stiglitz notion of risk is demonstrated in MacMinn and Holtmann[1983] .

[8]

one firms, as $S_1^L(I)$. Then $\alpha_1 S_1^L(I)$ represents the objective function for the type one firm manager given no mimicking. The type one firm manager selects I_1^S as his optimal investment given no mimicking. Therefore, the payoff to the type one firm manager will be $\alpha_1\{V_1(I_1^S)-I_1^S\}$ if he does not signal.

The type one firm manager can try to mimic the type two manager's investment selection and signal by taking the same amount of the investment selected by the type two firm manager. The type one firm manager will compare the payoff given signaling and the payoff given no signaling ,i.e., $\alpha_1\{V_1(I_1^S)-I_1^S\}$ and make a decision whether he will mimic the type two firm manager's action. The following Theorem 3 identifies the objective function for type one managers given mimicking.

Theorem 3. Suppose both the type one firm and the type two firm issue the zero coupon bonds to finance the investments. If the type one firm manager decides to mimic the type two firm manager's investment decision in the signaling equilibrium , the objective function for the type one firm manager will be $\alpha_1[V_1(I)-D_1\{I,B_2(I)\}]$,where $D_1\{I,B_2(I)\}$ denotes $\int_\Omega p(\omega)\min\{\Pi_1(I,\omega),B_2(I)\}d\omega$. [9]

In the equity issue case, $V_f = S_f = \int_\Omega p(\omega)\Pi_f(I,\omega)d\omega$. In the debt issue case, we denote the firm value by V_f, the shareholder value by S_f^L and the bondholder value by D_f.

[9] $B_f(I)$ is a promised payment on the risky debt issue of the firm f(f=1,2) that is selected so that the market value of the risky debt is equal to the investment amount;

$$D_f\{I,B_f(I)\} = \int_\Omega p(\omega)\min\{\Pi_f(I,\omega),B_f(I)\}d\omega = I$$

Now, let $S^L_{1m}(I)$ denote $[V_1(I)-D_1\{I,B_2(I)\}]$. $S^L_{1m}(I)$ can be expressed in terms of the capital loss to the type one firm manager as follows;

$$S^L_{1m}(I) = \{V_2(I) - I) - [\int_{S_1^2} p(\omega)\{\Pi_2(I,\omega) - \Pi_1(I,\omega)\}d\omega + \int_{I_1^2 \setminus B_2} p(\omega)\{\Pi_2(I,\omega) - B_2(I)\}d\omega]$$

$$I_1^2 = \{\omega \in \Omega; \Pi_1(I,\omega) < B_2(I)\}$$
$$S_1^2 = \{\omega \in \Omega; \Pi_1(I,\omega) > B_2(I)\}$$
$$B_f = \{\omega \in \Omega; \Pi_f(I,\omega) < B_f(I)\} \qquad , f = 1,2$$

The first term is the value to the type two firm shareholders and the second term in the bracket is the capital loss to the type one firm manager with mimicking.

It is obvious that the type one firm manager will not signal if the payoff given signaling is less than the payoff given no signaling ,i.e., if the following is true;

$$\alpha_1 S^L_{1m}(I) = \alpha_1[V_1(I) - D_1\{I,B_2(I)\}] \le \alpha_1 S^L_1(I^S_1) = \alpha_1\{V_1(I^S_1) - I^S_1\} \quad (3)$$

Such an inequality will be satisfied for any investment level inside $(0, I_d)$ (I_D, ∞) where I_d and I_D are the investment levels to satisfy the following;

$$S^L_{Im}(I_d) = S^L_1(I^S_1) \qquad S^L_{Im}(I_D) = S^L_1(I^S_1) \text{ where } I_d < I_D$$ [10]

2. The Investment Levels That Let the Type Two Firm Manager Prefer Signaling to No Signaling

Next, we have to find out when the signaling yields the higher payoff to the type two firm manager than no signaling with the risky debt issue. To achieve such a goal, we ought to identify and compare the objective functions and the payoffs given signaling and given no signaling for the type two firm manager. If he signals, the objective function for the type two firm manager will be $\frac{x^0_{h2}}{N}\int_\Omega p(\omega)\{\Pi_2(I,\omega) - I\}d\omega$, i.e., $\alpha_2\{V_2(I)-I\}$. The type two firm manager will maximize the value of the equity holders, $\{V_2(I)-I\}$, when he signals. By denoting $\{V_2(I)-I\}$ as $S^L_2(I)$, we let $\alpha_2 S^L_2(I)$ represent the objective function for the type two firm manager when he signals.

If the type two manager does not signal and others do, what would be his objective function and payoff ? Theorem 4 provides the objective functions for such managers.

Theorem 4. Suppose both the type one firm and the type two firm issue the zero coupon bonds to finance the investments. If the type two firm manager decides not to signal, the objective function for the type two firm manager will be $\alpha_2\{V_2(I)-D_2\{I,B_1(I)\}\}$, where $D_2\{I,B_1(I)\}$ denotes $\int_\Omega p(\omega)\min[\Pi_2(I,\omega), B_1(I)]d\omega$.

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[10] The existence of I_d and I_D can be proved by showing that $S^L_{Im}(I^S_1) > S^L_1(I^S_1)$.

Let $S^L_{2n}(I)$ denote $[V_2(I)-D_2\{I,B_1(I)\}]$. $S^L_{2n}(I)$ can be expressed in terms of the capital gain as follows;

$$S^L_{2n} = \{V_1(I) - I) + [\int_{\Omega \setminus B_1} p(\omega)\{\Pi_2(I,\omega) - \Pi_1(I,\omega)\}d\omega + \int_{B_1 \setminus I_2^1} p(\omega)\{\Pi_2(I,\omega) - B_1(I)\}d\omega]$$

$$I_2^1 = \{\omega \in \Omega; \Pi_2(I,\omega) < B_1(I)\}$$

The first term is the true value to the type one firm shareholders and the second term in the bracket is the capital gain to the type two firm manager. Since he will take the optimal investment level for the type one firm under the symmetric information in order not to signal, his payoff will be $\alpha_2 S^L_{2n}(I_1^s)$ if he does not signal.

The type two firm manager will signal only if the payoff given signaling is greater than the payoff given no signaling ,i.e., if the following is true;

$$\alpha_2 S^L_2(I) = \alpha_2\{V_2(I)-I\} > \alpha_2 S^L_{2n}(I_1^S)) = \alpha_2 [V_2(I_1^S)) - D_2\{I_1^S, B_1(I_1^S))\] \quad (4)$$

Since the RHS is a constant , Such an inequality will be satisfied for the investment inside the interval $[\ I_e, I_E]$ where I_e and I_E are to be defined as follows;

$$S^L_2(I_e) = S^L_{2n}I(^S_1) \quad S^L_2(I_E) = ^L_{2n}I(^S_1) \quad \text{where } I_e < I_E \qquad 11$$

3. The Existence of the Investment Levels That Can Result in the Signaling Equilibrium with the Risky Debt Issue

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11 The existence of I_e and I_E can be proved by showing that $S^L_1(I_1^s) > S^L_{2n}(I_1^s)$.

In the previous two sections, the investment levels that will prevent the type one firm manager from mimicking the signaling activity of the type two firm manager and the investment levels that will induce the type two firm manager prefer signaling to no signaling were identified. Recall that those investment levels were identified by the conditions (3) and (4) . The investment levels that may result in the existence of the signaling equilibrium ought to satisfy both (3) and (4). Therefore, identifying the investment levels to satisfy (3) and (4) is equivalent to identifying the existence of the signaling equilibrium with the investment decision. As was stated before, the investments to satisfy (3) will be the investments inside (0, I_d} or {I_D,∞). Similarly, the investments inside (I_e,I_E) will satisfy (4). It depends on the relative size of I_d,I_D , I_e ,and I_E whether there exist the investment levels to satisfy (3) and (4).

Proposition 2. Signaling equilibria with the underinvestments by the type two firm manager will exist with the risky debt financing. Such an underinvestment signaling equilibrium obtains with the investment inside [I_e,I_d].

proof. The signaling equilibrium with the underinvestment will exist if I_e is less than I_d. First note that $S_2^L(I)$ is greater than $S_{1m}^L(I)$ at all I and $S_{1m}^L(I)$ will be maximized at the investment level greater than I_2^s . ThenI_e will be less than I_d if $S_2^L(I_e) < S_2^L(I_d)$ since $S_2^L(I)$ is increasing inside the interval (0, I_2^s) which contains both I_e and I_d . Since I_e is the investment level such that $S_2^L(I_e) = S_{2n}^L(I_1^s)$, the signaling equilibrium with underinvestment will exist if $S_{2n}^L(I_1^s) < S_2^L(I_d)$. Note that

$$S_{2n}^{L}(I_1^s) - S_2^{L}(I_d) = [\int_{\varepsilon_1^s}^{\varpi} P(\omega)\{\Pi_2(I_1^s,\omega) - \Pi_1(I_1^s,\omega)\}d\omega + \int_{\varepsilon_2^s}^{\varepsilon_1^s} P(\omega)\{\Pi_2(I_1^s,\omega) - B_1(I_1^s)\}d\omega]$$

$$-[\int_{\zeta_1^d}^{\varpi} P(\omega)\{\Pi_2(I_d) - \Pi_1(I_d)\}d\omega + \int_{\zeta_2^d}^{\zeta_1^d} P(\omega)\{\Pi_2(I_d) - B_2(I_d)\}d\omega]$$

where $\Pi_1(I_1^s, \varepsilon_1^s) - B_1(I_1^s) = 0 \quad \Pi_2(I_1^s, \varepsilon_2^s) - B_1(I_1^s) = 0$

$\Pi_1(I_d, \zeta_1^d) - B_2(I_d) = 0 \quad \Pi_2(I_d, \zeta_2^d) - B_2(I_d) = 0$

Note that

$$[\int_{\zeta_1^d}^{\varpi} p(\omega)\{\Pi_2(I_d,\omega) - \Pi_1(I_d,\omega)\}d\omega + \int_{\zeta_2^d}^{\zeta_1^d} p(\omega)\{\Pi_2(I_d,\omega) - B_2(I_d)\}d\omega]$$

$$> [\int_{\varepsilon_1^d}^{\varpi} p(\omega)\{\Pi_2(I_d,\omega) - \Pi_1(I_d,\omega)\}d\omega + \int_{\varepsilon_2^d}^{\varepsilon_1^d} p(\omega)\{\Pi_2(I_d,\omega) - B_1(I_d)\}d\omega]$$

Therefore , the following is true;

$$S_{2n}^{L}(I_1^s) - S_2^{L}(I_d) = [\int_{\varepsilon_1^s}^{\varpi} p(\omega)\{\Pi_2(I_1^s,\omega) - \Pi_1(I_1^s,\omega)\}d\omega + \int_{\varepsilon_2^s}^{\varepsilon_1^s} p(\omega)\{\Pi_2(I_1^s,\omega) - B_1(I_1^s)\}d\omega]$$

$$-[\int_{\zeta_1^d}^{\varpi} p(\omega)\{\Pi_2(I_d,\omega) - \Pi_1(I_d,\omega)\}d\omega + \int_{\zeta_2^d}^{\zeta_1^d} p(\omega)\{\Pi_2(I_d,\omega) - B_2(I_d)\}d\omega]$$

$$< \left[\int_{\varepsilon_1^s}^{\varpi} p(\omega)\{\Pi_2(I_1^s,\omega)-\Pi_1(I_1^s,\omega)\}d\omega + \int_{\varepsilon_2^s}^{\varepsilon_1^s} p(\omega)\{\Pi_2(I_1^s,\omega)-B_1(I_1^s)\}d\omega\right]$$

$$-\left[\int_{\varepsilon_1^d}^{\varpi} p(\omega)\{\Pi_2(I_d,\omega)-\Pi_1(I_d,\omega)\}d\omega + \int_{\varepsilon_2^d}^{\varepsilon_1^d} p(\omega)\{\Pi_2(I_d,\omega)-B_1(I_d)\}d\omega\right]$$

It suffices to show that

$$\left[\int_{\varepsilon_1^s}^{\varpi} p(\omega)\{\Pi_2(I_1^s,\omega)-\Pi_1(I_1^s,\omega)\}d\omega + \int_{\varepsilon_2^s}^{\varepsilon_1^s} p(\omega)\{\Pi_2(I_1^s,\omega)-B_1(I_1^s)\}d\omega\right]$$

$$-\left[\int_{\varepsilon_1^d}^{\varpi} p(\omega)\{\Pi_2(I_d,\omega)-\Pi_1(I_d,\omega)\}d\omega + \int_{\varepsilon_2^d}^{\varepsilon_1^d} p(\omega)\{\Pi_2(I_d,\omega)-B_1(I_d)\}d\omega\right] < 0 \qquad (5)$$

Now let the function f(I) denote the following ;

$$f(I) = \int_{\varepsilon_1}^{\varpi} p(\omega)\{\Pi_2(I,\omega)-\Pi_1(I,\omega)\}d\omega + \int_{\varepsilon_2}^{\varepsilon_1} p(\omega)\{\Pi_2(I,\omega)-B_1(I)\}d\omega$$

Then, (5) is equivalent $f(I_1^s) - f(I_d) < 0$. This is true since f(I) is an decreasing function of I and I_1^S is greater than I_d as was shown before, so $S_{2n}^L(I_1^s) < S_2^L(I_d)$. Q.E.D.

{ Proposition 2} shows that signaling equilibria can exist with the underinvestments inside the interval $(I_e, I_d]$ by the type two firm manager when both the type one firm

and the type two firm issue the risky bond. [12] However, it is obvious that an underinvestment signaling equilibrium with the I_d dominates other underinvestment signaling equilibria since the underinvestment I_d yields the highest payoff to the type two manager while any choice of an underinvestment inside $(I_e, I_d]$ by the type two manager does not provide an incentive for the type one manager to mimic. As a result, the underinvestment signaling equilibrium obtains by the investment decision of the type two firm managers to take I_d. At the investment level I_d, the capital loss to the type one firm managers after their true firm quality is revealed will be greater than the overvaluation by mimicking the signaling by the type two firm managers. The type one firm managers opt to choose I_1^S and reveal their true firm quality in such a signaling equilibrium.

V. A Comparison between the Investment Signaling Equilibrium with The New Equity Issue and the Investment Signaling Equilibrium with the Risky Debt Issue

Now, we are prepared to examine if the signaling equilibrium with the new equity issue would dominate the other signaling equilibrium with the risky debt issue, or vice versa. The type two firm manager will prefer a signaling equilibrium that will make him underinvest less since such a signaling equilibrium with the less underinvestment provides the higher payoff to the type two firm manager.

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[12] Proposition 2 examines the feasibility of the underinvestment signaling equilibrium only. We won't examine the feasibility of the overinvestment signaling equilibrium since it will be dominated by the underinvestment signaling equilibrium even if it is feasible.

1. The Objective Functions for the Type One Firm Manager with the New Equity Issue and the Risky Debt Issue When He Signals

In the underinvestment signaling equilibrium with the equity issue, the investment level that would result in the signaling equilibrium is I_m as was shown in the section III.1. and I_m is defined as follows;

$$S^0_{Im}(I_m) = S^0_1(I^s_1)$$

On the other hand, the investment level that would result in the signaling equilibrium with the risky debt issue is I_d as was shown in the section IV.1. and I_d is defined as follows;

$$S^L_{Im}(I_d) = S^L_1(I^s_1)$$

The underinvestment levels in the signaling equilibrium with the equity issue and signaling equilibrium with the risky debt issues will depends on the objective functions of the type one firm manager with the new equity issue and with the risky debt issue given mimicking. If these objective functions are identical, the underinvestment levels implied in each signaling equilibrium will not be different and there is no possibility of dominance between the underinvestment signaling equilibrium with the equity issue and the underinvestment signaling equilibrium with the risky debt issue. However, if these objective functions are different, we need to closely look at the investment levels that will provide the signaling equilibrium with each financing scheme. The following theorem shows that the objective functions for the type one firm manager with the equity issue and with the risky debt issue are different.

Theorem 5. The objective function for the type one firm managers given mimicking when they issue the new equity is different from the objective function when they issue the risky debt.

Theorem 5 shows that the type one firm manager would have the different objective functions given mimicking when they issue the new equity and when they issue the risky debt issue. These different objective functions for the type one firm manager will result in the different signaling equilibrium depending on whether the firms issue the new equity or the risky debt to finance the investment.

2. The Dominance of the Signaling Equilibrium with the Risky Debt over the Signaling Equilibrium with the New Equity Issue

It is now obvious that signaling equilibrium with the new equity issue will be different from the signaling equilibrium with the risky debt issue. In order to determine whether there exists a dominance between these two signaling equilibria , we need to compare the investment levels which can yield the signaling equilibria,i.e., I_d and I_m since both imply an underinvestment for the high quality firms.

Proposition 3. The signaling equilibrium with the risky debt issue will dominate the signaling equilibrium with the new equity issue.

proof. Recall that

$$S^0_{Im}(I_d) = S^L_2(I_d) - \frac{S^L_2(I_d)}{S_2(I_d)}\{S_2(I_d) - S_1(I_d)\} \qquad S^L_{Im}(I_d) = S^L_1(I^s_1)$$

Then,

$$S^0_{Im}(I_d) - S^L_{Im}(I_d) = [S^L_2(I_d) - \frac{S^L_2(I_d)}{S_2(I_d)}\{S_2(I_d) - S_1(I_d)\}] - S^L_1(I^s_1)$$

$$= \frac{1}{S_2(I_d)}[S_2(I_d)g(I_d) - S^L_2(I_d)g(I_d)] \quad > \quad 0$$

Therefore, $S^0_{Im}(I_d) > S^L_{Im}(I_d)$. It follows that

$$S^0_{Im}(I_d) > S^L_{Im}(I_d) = S^L_1(I^s_1) = S^0_{Im}(I_m)$$

Therefore, I_d is greater than I_m since $S^0_{Im}(I)$ is an increasing function inside the interval $[0, I^s_1)$ and I_d is less than I^s_1. Now, recall that both I_d and I_m are smaller than I^s_1, so $S^0_2(I)$ which is the same function as $S^L_2(I)$ is still increasing at I_d and I_m. As a result, $S^0_2(I_d)$ is greater than $S^0_2(I_m)$. Q.E.D.

The investment signaling equilibrium with the new equity financing and the investment signaling equilibrium with the risky debt financing are the underinvestment signaling equilibrium. As shown in the [Proposition 3] , the investment level leading to the signaling equilibrium with the risky debt financing is greater than the investment level leading to the signaling equilibrium with the new equity financing. Since these investments are below the optimal investment for the high quality firms under the symmetric information, the high quality firm managers will prefer the signaling equilibrium which entails less severe degree of the underinvestment. Therefore, the underinvestment signaling equilibrium with the risky debt financing will be preferred and it will dominate the underinvestment signaling equilibrium with the new equity financing.

VI. Conclusion

Most of the studies available in the literature about the investment signaling start with the assumption that the managers are to maximize the current shareholders' value or have some specific form of utility function to make the derivation of the managers' objective function easier. It is obvious that these assumptions about the managers' objective functions have their own limitations. In this study, such assumptions are not employed. Rather, the managers' objective functions are endogenously derived within the model for the managers who are risk-averse and hold an initial holding in his own firm. Such derivations of the objective functions also enable us to endogenously derive the signaling cost function. The analysis in section III shows that a signaling equilibrium with the investment decision can exist if the high quality firm managers take an investment below the optimal investment level for the high quality firms under the symmetric information. Within such a signaling equilibrium, the high quality firms signal their true firm type by taking an underinvestment below the optimal investment level for the high quality firms under the symmetric information and the low quality firms reveal their true firm type by taking the optimal investment level under the symmetric information. In section IV., we show that as in the new equity financing case, a signaling equilibrium can exist with the underinvestment by the high quality firm managers below the optimal investment level for the high quality firms under the symmetric information. The low quality firm managers reveal their true type by taking the optimal investment under the symmetric information. We compare these signaling equilibria in order to see if the financing choice can affect such an investment signaling equilibrium. We already observed the prevailing signaling equilibria with the new equity financing and the risky debt financing are the underinvestment signaling equilibria. If the investment levels to

obtain the signaling equilibria with the new equity financing and the risky debt financing are equal, there would be no difference between the signaling equilibrium with the new equity financing and the signaling equilibrium with the risky debt financing. However, if the investment levels are different, we can try to see if the investment level to obtain the underinvestment signaling equilibrium with the new equity financing is greater than the investment level to obtain the underinvestment signaling equilibrium with the risky debt financing ,or vice versa. It is rather obvious that the high quality firm managers will prefer a financing choice which entails the underinvestment signaling equilibrium with the less severe degree of the underinvestment. In section V., it is shown that the investment level to obtain the underinvestment signaling equilibrium with the risky debt financing is greater than the investment level to obtain the underinvestment signaling equilibrium with the new equity financing ,so the former implies less severe degree of the underinvestment than the latter. Therefore, the underinvestment signaling equilibrium with the risky debt financing will dominate the underinvestment signaling equilibrium with the new equity financing .

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