불확실성하에서의 국가간의 통화정책 조정

김 훈 용*

A two-country overlapping generations model with fiat monies is used to study international coordination of monetary policies under the flexible exchange rate system. The optimal monetary policy and the welfare of individual countries are investigated for: coordination and non-coordination cases. It is shown that the coordination is Pareto superior to the non-coordination. The countries choose more inflationary policies in the non-coordination case; the world output decreases, which depends on the degree of risk aversion.

Since the mid-1980s, the policy markets are increasingly paying attention to international coordination of monetary policies. The pioneering papers on strategic policy making and international policy coordination are of Hamada (1976, 1979). Hamada(1976) uses n-country model with fixed exchange rates where each monetary authority chooses its credit expansion to maximize its objective function which depends on the common rate of inflation and the balance of payments. Recent contributions in monetary policy coordination involve numerical simulation to examine the potential inefficiencies of non-cooperative policy-game and the benefits of international policy coordination [Turnovsky, Basar and d'Orey (1988); Oudiz and Sachs (1985); Currie and Levine (1985); Miller and Salmon (1985); Carlozzi and Taylor (1985)]. Turnovsky, Basar, and d'Orey (1988) uses a two-country version of Dornbusch (1976) model in which the policy-makers in the two economies seek to optimize their respective objective functions — defined to be the squared deviations of output and inflation from the target level. Canzoneri and Gray (1985) uses the same objective functions to investigate the nature of inefficiencies associated with the non-cooperative Nash solutions to

* 동덕여자대학교 무역학과 조교수

policy-games. "When two economies adjust to a common external shock, whether the Nash solution is too expansionary or too contractionary depends on structural features of the world economy."[1]

Rogoff (1985) demonstrates that international monetary cooperation may be counterproductive —— counterintuitive results. "Cooperation between central banks may exacerbate the credibility problem of central banks vis-à-vis the private sector. One reason for such a credibility problem is that the central bank may be tempted to try to exploit the existence of nominal wage contracts to systematically raise employment. In a time-consistent equilibrium, wage inflation will be high enough so that the central bank's efforts will be futile. International monetary cooperation may raise the rate of wage inflation because wage setters recognize that a noncooperative regime contains a built-in check on each central bank's incentives to inflate. The reason is that when a central bank expends its money supply unilaterally, it causes its country's real exchange rate to depreciate thereby reducing the employment gains and increasing the CPI inflation costs. Cooperation may remove this disincentive to inflate, and thus raise time-consistent nominal wage growth."[2] van der Ploeg (1988) uses and optimizing equilibrium model, a two-country version of Calvo (1978), to reproduce the Rogoff results.

Compared to the existing literatures on international monetary policy coordination, the present paper differs in two respects. First, while the existing papers assume that monetary authorities attempts to minimize a loss function which depends on the squared deviations of output and inflation from their optimal values, the present paper assumes that monetary authorities maximize the utility of representative agents. Second, in the existing papers, the fundamental macroeconomic equations are exogenously specified. In the present paper, however, output, price, and labor supply functions are derived endogenously within the model.

The present paper studies monetary policy choices among countries under the

1) Canzoneri and Gray (1985), p.547.
2) Rogoff (1985), p.200.

flexible exchange rate system. The model of Bulow and Polemarchakis (1983) is extended to two-country version (Section 1). It is assumed that each country is a monetary production economy, and the representative agents live for two periods. The utility is derived from the leisure in their youth and the consumption in their old age. Fiat monies are issued by each government, and production function is linear in labor supply (overlapping generations model: Samuelson (1958) and Diamond (1965)).

In a Nash non-cooperative situation, policy makers of each country choose their monetary policies independently to maximize the welfare of its representative agents. On the other hand, policy makers can coordinate their monetary policies, and thereby, possibly improve the welfare of their representative agents. We find out the optimal policies for each country under Nash non-coordination situation (Situation 2), and coordination situation (Section 3). Major findings are (Section 4): countries choose more inflationary policies in Nash Non-coordination case. World output decreases, and utility level is lowered. Monetary policy coordination leads to Pareto improvement in welfare. Nash solution depends on the degree of risk-aversion (of representative agents). When the agents are highly risk-averse, governments choose more inflationary policies. But the degree of decreases in the world output (in %), from the level of coordination case, is smaller.

1. The Model

In this section, we derive the labor supply and price functions in terms of money growth rates and productivity shocks.

[Assumption 1] There are two countries, country 1 and country 2. They are indexed by the variable k, k=1 and 2. Representative agents of each country live for two periods. During each time period, two generations are alive — one young and one old. Young generations work but do not consume; old generations consume but do not work. Utility function of country k's representative agent has the form

$$U(L_k, C_k) = -L_k + (1/a)\cdot C_k^{\,a} \qquad a < 1, a \neq 0 \tag{1}$$

where L_k = labor supply when young

C_k = consumption when old

for k = 1, 2.

[Assumption 2] There is only one type of good. It is not storable. Labor is the only input for production of goods. Production functions are

$$Y_{ki} = \theta_{ki}\cdot L_{ki} \qquad \theta_{li} > 0 \tag{2}$$

where Y_{ki} = output of country k in state i

θ_{ki} = porductivity of labor of country k in state i

L_{ki} = labor supply of country k in state i

for k = 1, 2; i = 1, ..., s.

Productivity shocks $\{\theta_{ki}\}$ are distributed independently and identically over time. For the numerical examples that appear in the later sections; we assumed that productivity shocks are distributed as:

i	π_i	θ_{1i}	θ_{2i}
1	1/4	10	9
2	1/4	9	10
3	1/4	10	10
4	1/4	9	9

(3)

where π_i = probability belief of state i.

[Assumption 3] There are two kinds of fiat monies — of country 1 and country

2, issued by each government. There are no transactions costs of whatsoever. Goods and labor in both countries can be paid for with <u>any</u> kind of monies. There are no restrictions on goods and portfolio transactions.

[Assumption 4] Economy extends over discrete time, t. The tool at the disposal of government is monetary policy. Every period t, after observing the productivity shocks θ_{ki}, the government of country k Chooses (possibly negative) lump-sum payments, s_{kt}, to be distributed in the following period to what is the young generation of the current period. Country k's money transfer, s_{ki}, is given to current young generation of country k only; in the following period (k = 1,2; i = 1, ..., s).

The state of the economy is described as $(\theta_1, \theta_2, M_1, M_2)$. We define the monetary policy rules and the price systems as functions of the state of the economy.

Monetary Policy Rules:

$$S_k(\theta_1, \theta_2, M_1, M_2) + M_k > 0$$

for k = 1,2

Prices:

$$P_k(\theta_1, \theta_2, M_1, M_2)$$

for k = 1,2

Wages:

$$W_k(\theta_1, \theta_2, M_1, M_2)$$

for k = 1,2

where

$P_k =$ price of goods in currency of country k

$W_k =$ wage rate of country k in currency of country k

$M_k =$ money stock of country k.

$$P_1(\theta_{1i}, \theta_{2i}, M_1, M_2) = P_{1i} \qquad P_2(\theta_{1i}, \theta_{2i}, M_1, M_2) = P_{2i}$$

$$W_1(\theta_{1i}, \theta_{2i}, M_1, M_2) = W_{1i} \qquad W_2(\theta_{1i}, \theta_{2i}, M_1, M_2) = P_{2i}$$

$$S_1(\theta_{1i}, \theta_{2i}, M_1, M_2) = S_{1i} \qquad S_2(\theta_{1i}, \theta_{2i}, M_1, M_2) = P_{2i}$$

In state i where productivity shocks θ_{1i} and θ_{2i} are realized; and the money stocks are M_1 and M_2; each government announces S_{1i} and S_{2i} as money transfers for the next period, and then, the equilibrium prices and wages are P_{1i}, P_{2i}, W_{1i}, and W_{2i}.

Prices, wages and money transfers are linear in money stocks. When money stocks are M_1'. and M_2', the respective variables are multiplied by M_1'/M_1 and M_2'/M_2. That is,

$$P_1(\theta_{1i}, \theta_{2i}, M_1', M_2) = (M_1'/M_1)\cdot P_{1i}$$

$$P_2(\theta_{1i}, \theta_{2i}, M_1, M_2') = (M_2'/M_2)\cdot P_{2i}$$

$$W_1(\theta_{1i}, \theta_{2i}, M_1', M_2) = (M_1'/M_1)\cdot W_{1i}$$

$$W_2(\theta_{1i}, \theta_{2i}, M_1, M_2') = (M_2'/M_2)\cdot W_{2i}$$

$$S_1(\theta_{1i}, \theta_{2i}, M_1', M_2) = (M_1'/M_1)\cdot S_{1i}$$

$$S_2(\theta_{1i}, \theta_{2i}, M_1, M_2') = (M_2'/M_2)\cdot S_{2i}$$

In state i, given monetary policy rules and price systems, $S_1(\theta_{1i}, \theta_{2i}, M_1, M_2), S_2(\cdot\cdot), P_1(\cdot\cdot), P_2(\cdot\cdot), W_1(\cdot\cdot)$ and $W_2(\cdot\cdot)$, young generations of country k make labor supply and portfolio decisions (k = 1,2):

$$\underset{L_{ki},\ M_{1i}^{dk},\ M_{2i}^{dk}}{Max} \quad - L_{ki} + (1/a) \cdot \sum_{j=1,\cdot\cdot,S} \pi_j \{ C_{kj} \} \tag{4}$$

subject to

$$\frac{M_{1i}^{dk}}{P_{1i}} + \frac{M_{2i}^{dk}}{P_{2i}} = \frac{W_{ki} \cdot L_{ki}}{P_{ki}} \tag{5}$$

$$\frac{M_{1i}^{dk}}{P_{1j} \cdot \dfrac{M_1 + S_{1i}}{M_1}} + \frac{M_{2i}^{dk}}{P_{2j} \cdot \dfrac{M_2 + S_{2i}}{M_2}} + \frac{S_{ki}}{P_{kj} \cdot \dfrac{M_k + S_{ki}}{M_k}} = C_{kj} \tag{6}$$

$$\begin{aligned} &0 \le L_{ki} \\ &0 \le C_{kj} \end{aligned} \tag{7}$$

for i = 1, ··, S ; j = 1, ··, S ; k = 1,2

where

$\pi_j =$ subjective probability belief of state j

$C_j =$ consumption in the next period when state j is realized

$M_j^{dk} =$ country-k young generation's money demand for currency of country n ; for n = 1,2

Young generations, in state i, supply labor L_{ki} at the wage rate W_{ki}. They convert their wage income into portfolio of money holdings, $M_{1i}{}^{dk}$ and $M_{2i}{}^{dk}$, which are carried into the next period to buy the consumption goods when old. In the next period, they receive the money transfers S_{ki} which were announced when young. If state j is realized, $(\theta_{1j}, \theta_{2j})$; the price of consumption goods will be $P_{kj} * (M_{ki} + S_{ki}) / M_{ki}$ (k=1,2). The factors multiplying P_{kj} arise out of the degree one homogeneity of prices with respect to the money stock.

The necessary and sufficient conditions for the optimality are

$$\frac{1}{W_{ki}/P_{ki}} = \frac{\sum_{j=1,\cdots,S} \pi_j \left\{ C_{kj(a-1)} \cdot \frac{1}{P_{1j} \cdot \left(\frac{M_1 + S_{1i}}{M_1} \right)} \right\}}{(1/P_{1i})} = \frac{\sum_{j=1,\cdots,S} \pi_j \left\{ C_{kj(a-1)} \cdot \frac{1}{P_{2j} \cdot \left(\frac{M_2 + S_{2i}}{M_2} \right)} \right\}}{(1/P_{2i})} \tag{8}$$

for k = 1,2

where C_{kj}'s are as expressed in the budget constraints (6). (8) is the marginal condition between leisure and consumption; when wage incomes can be transformed into consumption, via either country 1 or country 2 currencies.

Market clearing conditions are

$$\text{Labor: } \frac{W_{ki}}{P_{ki}} = \theta_{ki} \tag{9}$$

$$\text{Money: } M_{ki1}^{k1} + M_{ki}{}^{d2} = M_k \tag{10}$$

$$\text{Goods: } \theta_{1i} \cdot L_{1i} + \theta_{2i} \cdot L_{21} = C_{1i} + C_{2i} \tag{11}$$

for i = 1, ⋯, S ; k = 1,2

Portfolio composition of representative agents are such that their post-transfer compositions are identical across countries; and are equal to the post-transfer money stocks. That is,

$$\frac{M_{1i}^{d1} + S_{1i}}{M_{2i}^{d1}} = \frac{M_{1i}^{d2}}{M_{2i}^{d2} + S_{2i}} = \frac{M_1 + S_{1i}}{M_2 + S_{2i}} \tag{12}$$

for i = 1, ⋯, S

By holding portfolio as expressed in (12); agent k's consumption share - $\text{consumption}_{kj}/\text{world output}_j$ - is invariant to the realizations of (P_{1j}/P_{2j}).

Proof

Define V_{1i} and V_{2i} such that

$$V_{1i} = \frac{(M_{1i}^{d1} + S_{1i})/P_{1i}}{(M_{1i}^{d1} + S_{1i})/P_{1i} + M_{2i}^{d1}/P_{2i}}$$

$$V_{2i} = \frac{M_{1i}^{d2}/P_{1i}}{M_{1i}^{d2}/P_{1i} + (M_{1i}^{d2} + S_{2i})/P_{2i}} \tag{13}$$

which are the portfolio weights (%) for currency of country 1; when weights are measured in terms of goods. From the budget constraint (5), we obtain

$$(M_{1i}^{d1} + S_{1i})/P_{1i} + M_{2i}^{d1}/P_{2i}) = (W_{1i} \cdot L_{1i} + S_{1i})/P_{1i} \tag{14}$$

From (13) and (14), we have

$$M_{1i}^{d1} + S_{1i} = [\ (W_{1i} \cdot L_{1i} + S_{1i}) / P_{1i}] \cdot V_{1i} \cdot P_{1i}$$
$$M_{2i}^{d1} = [\ (W_{1i} \cdot L_{1i} + S_{1i}) / P_{1i}] \cdot (1 - V_{1i}) \cdot P_{2i} \tag{15}$$

Substituting (15) for the money balances in the budget constraint (7); the objective function (4) can be rewritten as

$$\underset{L_{1i},\ M_{1i}^{d1},\ M_{2i}^{d1}}{Max} \quad - \ L_{1i} + (1/a) \cdot \sum_{j=1,\cdots,S} \pi_j \cdot C_{1j}^{\ a} \tag{16}$$

where

$$C_{1j} = \frac{(W_{1i} \cdot L_{1i} + S_{1i})}{P_{1i}} \cdot \frac{V_{1i} \cdot P_{1i}}{P_{1j} \cdot \dfrac{M_1 + S_{1i}}{M_1}} + \frac{(W_{1i} \cdot L_{1i} + S_{1i})}{P_{1i}} \cdot \frac{(1 - V_{1i}) \cdot P_{2i}}{P_{2j} \cdot \dfrac{M_2 + S_{2i}}{M_2}} \tag{17}$$

the second term of the objective function (16) can be rewritten as

$$(1/a) \cdot \frac{(W_{1i} \cdot L_{1i} + S_{1i})^{a}}{P_{1i}^{\ a}}$$

$$\cdot \sum_{j=1\cdots s} \pi_j \left\{ V_{1i} \cdot \frac{P_{1i}}{P_{1j} \cdot \dfrac{M_1 + S_{1i}}{M_1}} + (1 - V_{1i}) \cdot \frac{P_{2i}}{P_{2j} \cdot \dfrac{M_2 + S_{2i}}{M_2}} \right\}^{a} \tag{18}$$

Following the similar steps, we have for agent of country 2,

$$(1/a) \cdot \frac{(W_{2i} \cdot L_{2i} + S_{2i})^{a}}{P_{i}^{\ a}}$$

$$\cdot \sum_{j=1 \cdots s} \pi_j \left\{ V_{2i} \cdot \frac{P_{1i}}{P_{1j} \cdot \frac{M_1 + S_{1i}}{M_1}} + (1 - V_{2i}) \cdot \frac{P_{2i}}{P_{2j} \cdot \frac{M_2 + S_{2i}}{M_2}} \right\}^a \tag{19}$$

Given P_{ki}, P_{kj}, S_{ki} and W_{ki} ($k = 1,2$; $i = 1, \ldots, s$), the portfolio choice problem is equivalent to choosing V_{ki} in (17) and (18). Since the power a of the utility functions are the same for agents of both countries; the choice V_{1i} will be equal to V_{2i}. Therefore,

$$\frac{(M_{1i}^{d1} + S_{1i})}{M_{2i}^{d1}} = \frac{M_{1i}^{d2}}{(M_{2i}^{d2} + S_{2i})} \tag{20}$$

By the money market clearing conditions; from (20), we get (12). Q.E.D.

Money demands are

$$\begin{aligned} M_{ki}^{dk} &= (M_k + S_{ki}) \cdot \delta_{ki} - S_{ki} \\ M_{qi}^{dk} &= (M_q + S_{qi}) \cdot \delta_{ki} \end{aligned}$$

for $k = 1, 2$; $q = 1, 2$; $q \neq k$; $i = 1, \cdot \cdot, s$ (21)

where

$$\delta_{ki} = \frac{(w_{ki} \cdot L_{ki} + S_{ki}) / P_{ki}}{(M_1 + S_{1i}) / P_{1i} + (M_2 + S_{2i}) / P_{2i}} \tag{22}$$

We show late, that δ_{ki} is

$$\delta_{ki} = (\theta_{ki} / \theta_i^*)$$

where $\theta_i^* = (\theta_{1i}^{[1/(1-a)]} + \theta_{2i}^{[1/(1-a)]})^{(1-a)}$ (23)

Substituting the money demands (21) into the budget constraints (6), we get

$$C_{kj} \;=\; (\frac{M_1}{P_{1j}} + \frac{M_2}{P_{2j}}) \cdot \delta_{ki} \tag{24}$$

for j = 1, ..., s

where δ_{ki} is as expressed in (22).

Substituting (24) and (22) for consumption in the optimality condition (8); we derive the labor supply function L_k (S_1, S_2 , P_1 , P_2 , W_1 , M_1 , M_2) for k = 1,2

$$L_{1i} = \frac{1}{(w_{1i}/P_{li})} \cdot \left\{ [\; Q_1 \cdot \frac{W_{1i}}{(M_1 + S_{1i})/M_1}]^{[1/(1-a)]} \right.$$
$$\left. [\; (M_1 + S_{1i}) / P_{1i} + (M_2 + S_{2i})/P_{2i}] \; - \; \frac{S_{1i}}{P_{1i}} \right\} \tag{25}$$

$$L_{2i} = \frac{1}{(w_{2i}/P_{2i})} \cdot \left\{ [\; Q_1 \cdot \frac{(P_{li}/P_{2i}) \cdot W_{2i}}{(M_1 + S_{1i})/M_1}]^{[1/(1-a)]} \right.$$
$$\left. [\; (M_1 + S_{1i}) / P_{1i} + (M_2 + S_{2i})/P_{2i}] \; - \; \frac{S_{2i}}{P_{2i}} \right\} \tag{26}$$

where

$$Q_1 \;\equiv\; \sum_{j=1 \cdot \cdot S} \pi_j \;\{\; (M_1/P_{1i} \;+\; M_2/P_{2i})^{(a-1)} \;\cdot\; 1 \,/\, P_{1i} \;\} \tag{27}$$

From the second equality of the optimality condition (8); which is the marginal condition for holding currencies of country 2; we obtain the labor supply functions, expressed in currency of country 2. They have the same forms as (25) and (26) except that the first terms in the bracket are replaced with:

For (25), replace with

$$Q_2 \cdot \frac{(P_{2i} / P_{1i}) \cdot W_{1i}}{(M_2 + S_{2i}) / M_2} \tag{28}$$

For (26), replace with

$$Q_2 \cdot \frac{W_{2i}}{(M_2 + S_{2i}) / M_2} \tag{29}$$

where

$$Q_2 \equiv \sum_{j=1 \cdot \cdot S} \pi_j \{ (M_1/P_{1i} + M_2/P_{2i})^{(a-1)} \cdot 1 / P_{2i} \} \tag{30}$$

From the budget constraints, and the goods and money market clearing conditions; we obtain

$$\theta_{1i} \cdot L_{li} + \theta_{2i} \cdot L_{2i} = M_1 / P_{1i} + M_2 / P_{2i}$$

$$\text{for i} = 1, \cdot \cdot , \text{s.} \tag{31}$$

If we substitute the labor supply functions (25) and (26), which are expressed in terms of currency of country 1, for L_{1i} and L_{2i} in (31), we obtain

$$\sum_{j=1 \cdot \cdot S} \pi_j \{ (M_1/P_{1i} + M_2/P_{2i})^{(a-1)} \cdot 1 / P_{1i} \} \frac{P_{1i}}{(M_1 + S_{1i}) / M_1} \cdot \theta_i^* = 1$$

$$\text{for i} = 1, ..., 2. \tag{32}$$

where

$$\theta_i^* \equiv (\theta_{1i}{}^{[1/(1-a)]} + \theta_{2i}{}^{[1/(1-a)]})^{(1-a)}$$

Similarly, by substituting the labor supply functions expressed in currency of country 2; we obtain

$$\sum_{j=1\cdot\cdot S} \pi_j \{ (M_1/P_{1j} + M_2/P_{2j})^{(a-1)} \cdot 1/P_{2j} \} \frac{P_{2j}}{(M_2 + S_{2i})/M_2} \cdot \theta_i^* = 1$$

for $i = 1, \cdot \cdot, s.$ (33)

We defined the terms, $\Sigma \pi_j\{\cdot\}$, appearing in (32) and (33) ; as Q_k, k=1, 2. (27) and (30).

Q_k is the expected marginal utility of, one of currency of country k; for the "aggregater" who has utility function $u(c) = (1-a)\cdot c^a$, and consumes the world outputs, $(M_1/P_{1j} + M_2/P_{2j})$, for all states. After normalizing Q_1 and Q_2 as $Q_1 = Q_2$, We compute the price and labor supply functions in terms of the exogenous variables (θ_1, θ_2, S_1, S_2, M_1, M_2).

Proposition : Given monetary policy $S_k(\theta_1, \theta_2, S_1, S_2, M_1, M_2)$, k = 1, 2 ; there exists a unique price [$P_k(\cdot\cdot)$, $W_k(\cdot\cdot)$], k = 1, 2 ; such that [$S_k(\cdot\cdot)$, $P_k(\cdot\cdot)$, $W_k(\cdot\cdot)$] is an equilibrium for the economy.

proof : Define Z_k, k = 1 , 2 and θ_i^* such that

$$Z^1{}_i \equiv m_1/(M_1 + S_{1i})$$
$$Z^2{}_i \equiv m_2/(M_2 + S_{2i}) \qquad (34)$$
$$\theta_i^* \equiv (\theta_{1i}{}^{[1/(1-a)]} + \theta_{2i}{}^{[1/(1-a)]})^{(1-a)}$$

Then for

$$Q_1 = [\sum_{j=1 \cdot \cdot S} \pi_j \{ \theta_j^{*\,a} (M_1 Z_{1j} + M_2 Z_{2j})^{(a-1)} \cdot Z_{1j}\}]^{[1/(1-a)]}$$

$$Q_2 = [\sum \pi_j \{ \theta_j^{*\,a} (M_1 Z_{1j} + M_2 Z_{2j})^{(a-1)} \cdot Z_{2j}\}]^{[1/(1-a)]}$$

(35)

Let

$$P_{1i} = 1 / (Z_{1i} Q_1 \theta_i^*)$$

$$P_{2i} = 1 / (Z_{2i} Q_2 \theta_i^*)$$

$$W_{1i} = \theta_{1i} / (Z_{1i} Q_1 \theta_i^*)$$

$$W_{2i} = \theta_{2i} / (Z_{2i} Q_2 \theta_i^*)$$

(36)

$$L_{1i} = (\theta_i^* / \theta_{1i}) [(\theta_{1i} / \theta_i^*)^{[1/(1-a)]} \cdot (M_1 Q_1 + M_2 Q_2) - M_1 Q_1 \cdot (1 - Z_{1i})]$$

$$L_{2i} = (\theta_i^* / \theta_2^*) [(\theta_{1i} / \theta_i^*)^{[1/(1-a)]} \cdot (M_1 Q_1 + M_2 Q_2) - M_2 Q_2 \cdot (1 - Z_{2i})]$$

for i = 1, · ·, s (37)

They system of equations (9) and (11) is then satisfied.

Q.E.D.

δ_{ki} of (22) is the consumption of country-k agent, divided by the world output in the next period —— "consumption share". Using (31),

$$\delta_{ki} = \frac{(w_{ki} \cdot L_{ki} + S_{ki}) / P_{ki}}{(M_1 + S_{1i}) / P_{1i} + (M_2 + S_{2i}) / P_{2i}}$$
$$= \frac{\theta_{1i} \cdot L_{ki} + S_{ki} / P_{ki}}{\theta_{1i} \cdot L_{1i} + S_{1i} / P_{1i} + \theta_{2i} \cdot L_{2i} + S_{2i} / P_{2i}}$$

(38)

Substituting the labor supply function (37) for L_{1i} and L_{2i}, we obtain

$$\delta_{ki} = (\theta_{ki} / \theta_i^{*})^{[1/(1-a)]}$$

where

$$\theta_i^{*} = (\theta_{1i}^{[1/(1-a)]} + \theta_{2i}^{[1/(1-a)]})^{(1-a)}$$

for i = 1, · ·, s (39)

"Consumption share" is function of productivities only. There are two reasons. First, by holding portfolios as expressed in (12), consumption share does not depend on the realizations of (P_{1j}, P_{2j}). Second, utility function is linear in leisure; and production function is linear in labor supply, too.

In the optimality conditions of (8), marginal utility of leisure and real wage are independent of labor supply. In the right-hand side of the equality,

$$C_{kj} = \delta_{ki} Y_j^{*}.$$

Comparing the optimality condition of each agent, we obtain $(\theta_{1i}/\theta_{2i}) = \delta_{1i} / \delta_{2i})(1-a)$: which is (39).

Equations (36) and (37) above are the equilibrium price, wage, and labor supply functions which are expressed in terms of the exogenous variables (θ_1, θ_2 , S_1, S_2, M_1, M_2).

2. Nash Non-Coordination

Government of country k chooses monetary policy {Z_{kh}}, maximizing the utility of its representative agents; taking as given the policy {Z_{gh}} of country q (h = 1, ..., s; k = 1,2; q = 1,2; q ≠ k).

$$\underset{\cdot\cdot, Z_{kh}, \cdot\cdot}{Max} \sum_{i=1,\cdot\cdot,S} \pi_i [-L_{ki} + \frac{1}{a} \{ \sum_{j=1,\cdot\cdot,S} (C_{kji})^a \}] \qquad (40)$$

subject to

$0 < Z_h$, $0 < L^{hi}$

for h = 1, · · ·, S   i = 1, · · ·, S   j = 1, · · ·, S

where C_{kji} is consumption of agent k in state j, when state i is realized in the previous period. From (39) above, C_{kji} is

$$C_{kji} = (\theta_{1j} \cdot L_{1j} + \theta_{2j} \cdot L_{2j}) \cdot \delta_{ki}$$
$$= (\theta_{1j} \cdot L_{1j} + \theta_{2j} \cdot L_{2j}^{*}) \cdot (\theta_{ki} / \theta_i^{*})^{\{1/(1-a)\}}$$

where

$$\theta_i^{*} \equiv (\theta_{1i}^{[1/(1-a)]} + \theta_{2i}^{[1/(1-a)]})^{(1-a)} \tag{41}$$

The necessary and sufficiemt conditions for the optimality are

$\frac{dU_k}{dZ_{kh}}$:

$$0 = \sum_{i=1,\cdots,s} \pi_i [-\frac{dU_{ki}}{dZ_{kh}}] + \sum_{i=1,\cdots,S} \pi_i [(\theta_{ki} / \theta_i^{*})^{\{a/(1-a)\}}]$$
$$\cdot \sum_{j=1,\cdots,S} \pi_j [(\theta_{1i} \cdot L_{1j} + \theta_{2j} \cdot L_{2j})^{(a-1)} \cdot (\theta_{kj} \cdot \frac{dL_{kj}}{dZ_{kh}} + \theta_{qj} \cdot \frac{dL_{qj}}{dZ_{kh}})]$$

for h = 1, ···, S ; k = 1, 2 ; q = 1, 2 ; q ≠ k. (42)

Solving two equations of (42) simultaneously for k=1 and k=2, we obtain

[Nash Non-Coordination]

Monetary Policy:

$$Z_{kh} = A_2 \cdot [(A_1 + 1) \cdot \theta_{qh} / (\theta_{kh} + \theta_{qh}) - 1] \cdot$$
$$[\theta_{kh} \cdot \theta_{qh} / (\theta_{kh} + \theta_{qh})]^{\{1/(1-a)\}} \cdot (1/\theta_h^{*}) \tag{43}$$

where

$$\theta_h^* \equiv [\ \theta_{kh}^{\ (1/(1-a))} + \theta_{qh}^{\ (1/(1-a))}]^{\ (1-a)}$$

for k = 1, 2 ; q = 1, 2 ; q ≠ k ; h = 1, ··, S

Utility level :

$$U_k^N = [\ -\{sa_4 - a_1(A_1 + 1)\cdot A_2\cdot a_2 - A_2\cdot a_3\}] \cdot$$

$$a_3^{\ (1/(1-a))}\cdot\{(A_1 - 1)\}\cdot^{\ (1/(1-a))} \qquad (44)$$

World Outputs:

$$Y_h^* = [\ (A_1 - 1)\cdot A_2\cdot a_3\cdot\{\theta_{kh}\cdot\theta_{qh}/(\theta_{kh} + \theta_{qh})\}]^{\ (a/(1-a))} \qquad (45)$$

for k = 1,2 ; q = 1,2 ; q ≠ k ; h = 1,··,S

where

$$A_1 = \frac{b_1 - 1\cdot(1+a) + (1+2b_2)(1-a) - \sqrt{B}}{2\{b_1 + (1-b_2)\cdot(1-a)\}}$$

where

$$B = \{b_1(1+a) + (1+2b_2)(1-a)\}^2$$

$$-4\{b_1 + (1-b_2)(1-a)\}\{b_1 a + (2-b_2)(1-a)\}$$

$$A_2 = b_3\,\frac{(1-a)(A_1 + 1)}{(A_1 - a)(A_1 - 1)}$$

and

$$a_1 = \sum_{i=1,\cdots S} \pi_i(\theta_i^* / \theta_{ki})$$

for k = 1, 2

where

$$\theta_1^* \equiv [\ \theta_{ki}^{\{1/(1-a)\}} + \theta_{qi}^{\{1/(1-a)\}}\ ]^{(1-a)}$$

$$a_2 \equiv \sum_{i=1,\cdots,s} [\ \{\theta_{ki} \cdot \theta_{qi} / (\theta_{ki} + \theta_{qi})\}^{[1/(1-a)]} (1/\theta_{ki}) \{\theta_{qi} / (\theta_{ki} + \theta_{qi})\}]$$

$$a_3 \equiv \sum_{i=1,\cdots,s} [\ \{\theta_{ki} \cdot \theta_{qi} / (\theta_{ki} + \theta_{qi})\}^{[1/(1-a)]} (1/\theta_{ki})]$$

$$a_4 \equiv \sum_{i=1,\cdots,s} [\ \{\theta_{ki} \cdot \theta_i^*\}^{[1/(1-a)]}\ ]$$

for k = 1, 2 ; q = 1, 2 ; q $\neq$ k

$$b_1 \equiv a_1 / a_4$$

$$b_2 \equiv a_2 / a_3$$

$$b_3 \equiv a_4 / a_3$$

3. Coordination

Countries choose monetary policies cooperatively, maximizing the sum of the utilities of the representative agents.

$$\underset{\cdots z_{1h}, \cdots, z_{2h},}{Max} \quad \sum_{i=1,\cdots,S} \pi_i \cdot [\ -L_{1i} + 1/a \cdot [\ \sum_{j=1,\cdots,S} \pi_j \cdot C_{1ji}^{\ a}] \quad -L_{2i} + 1/a \cdot [\ \sum_{j=1,\cdots,S} \pi_j \cdot C_{2ji}^{\ a}]\ ] \qquad (46)$$

subject to $0 < Z_{1h}$, $0 < Z_{2h}$, $0 < L_{kh}$

for h = 1, ⋯, S i = 1, ⋯, S j = 1, ⋯, S

where C_{kji} is consumption of agent k in state j, when the state i was realized in the previous period. From (39) above, C_{kji} is

$$C_{kji} = (\theta_{1j} \cdot L_{1j} + \theta_{2j} \cdot L_{2j}) \cdot (\theta_{ki} / \theta_{i}^{i})^{(1/(1-a))} \tag{47}$$

for k = 1,2 ; h = 1, …, S ; i = 1, …, S ; j = 1, …, S

where

$$\frac{dU_{1+r}}{dZ_{kh}} \quad :$$

$$0 = \sum_i \pi_i [-\frac{dL_{ki}}{dZ_{kh}}] + \sum_i \pi_i [(\theta_{ki} / \theta_i^*)^{a/(1-a)}]$$

$$\cdot \sum_j \pi_j [(\theta_{kj} \cdot L_{kj} + \theta_{qj} \cdot L_{qj})^{(a-1)} \cdot (\theta_{kj} \cdot \frac{dL_{kj}}{dZ_{kh}} + \theta_{qj} \cdot \frac{dL_{qj}}{dZ_{kh}})]$$

$$+ \sum_i \pi_i [-\frac{dL_{qi}}{dZ_{kh}}] + \sum_i \pi_i [(\theta_{qi} / \theta_i^*)^{(a/(1-a))}]$$

$$\cdot \sum_j \pi_j [(\theta_{kj} \cdot L_{kj} + \theta_{qj} \cdot L_{qj})^{(a-1)} \cdot (\theta_{kj} \cdot \frac{dL_{kj}}{dZ_{kh}} + \theta_{qj} \cdot \frac{dL_{qj}}{dZ_{kh}})] \tag{48}$$

for k = 1, 2 ; q = 1, 2 ; q ≠ k ; i ≠ 1, …, S ; j ≠ 1, …, S
In order to present the results, we divide the states into three subsets according to the relative magnitudes of θ_{1i} and θ_{2i}.

$$S_1 = \{ i \mid \theta_{1i} > \theta_{2i} \}$$

$$S_2 = \{ i \mid \theta_{1i} < \theta_{2i} \}$$

$$S_3 = \{ i \mid \theta_{1i} = \theta_{2i} \}$$

Solving two equations of (48) simultaneously, we obtain:

[Coordination]

Monetary Policy:

for h $\in$ S_1

$$Z_{1h}{}^{c} = X\cdot\theta_{1h}{}^{\{1/(1-a)\}}\cdot(1/\theta_{h}{}^{*}) + 2\cdot(\theta_{2h}/\theta_{h}{}^{*})^{\{1/(1-a)\}} - 1 \qquad (49)$$

$$Z_{2h}{}^{c} = 1 - 2\cdot(\theta_{2h}/\theta_{h}{}^{*})^{\{1/(1-a)\}}$$

for h $\in$ S_2

$$Z_{1h}{}^{c} = 1 - 2\cdot(\theta_{1h}/\theta_{h}{}^{*})^{\{1/(1-a)\}}$$

$$Z_{2h}{}^{c} = X\cdot\theta_{2h}{}^{\{1/(1-a)\}}\cdot(1/\theta_{h}{}^{*}) + 2\cdot(\theta_{1h}/\theta_{h}{}^{*})^{\{1/(1-a)\}} - 1 \qquad (50)$$

for h $\in$ S_1

$$Z_{1h}{}^{c} = 1/2\cdot X\cdot(\theta_{1h})^{\{1/(1-a)\}}\cdot(1/\theta_{h}{}^{*})$$

$$Z_{2h}{}^{c} = 1/2\cdot X\cdot(\theta_{2h})^{\{1/(1-a)\}}\cdot(1/\theta_{h}{}^{*}) \qquad (51)$$

where $x = 4\cdot\sum_{i=1,\cdot\cdot,S}\pi_{1}\{(\theta_{ki}/\theta_{i}{}^{*})^{\{a/(1-a)\}}\}/\sum_{i=1,\cdot\cdot,s}\pi_{i}[M]$

where $M = \{\max(\theta_{1i},\theta_{2i})\}^{\{a/(1-a)\}}$

for h = 1, ‥, S ; j = 1, ‥, S ; k = 1, 2.

Utility level :

$$U^c = \{-1+(1/a)\}\cdot 2^{\{a/(1-a)\}}$$

$$\cdot \sum_{i=1,\cdots s} \pi_1\{ [\max(\theta_{1i}, \theta_{2i})]^{[a/(1-a)]} \} \tag{52}$$

$$\cdot \sum_{i=1,\cdots s} \pi_i\{ (\theta_{ki} / \theta_i)^{\{a/(1-a)\}} \}$$

$$Y_h^* = [\sum_{i=1,\cdots,s} \pi_i\{ (\theta_{ki} / \theta_i)^{\{a/(1-a)\}} \}]^{[1/(1-a)]} \tag{53}$$

$$\cdot [\max(\theta_{1i}, \theta_{2i})]^{\{a/(1-a)\}}$$

for k = 1,2. ; h =1, ··, S.

4. Comparison

Compared to the coordination case, Nash non-coordination equilibrium has the following properties: Countries choose more inflationary policies; world output decreases; and utility level goes down. Coordination is Pareto superior to Nash non-coordination.

Due to the simplicity of our model, when a country increases money supply, its representative agent's share of consumption in the world output remains unchanged (Section 1). However, all motives to choose inflationary policies in a Nash non-coordination situation are preserved. Suppose both governments are maintaining the money growth rates at the level which is the solution of coordination case. Then it is suboptimal for a government playing a Nash policy-game. Taking the policy-choice of the other government as given, a government can improve the utility level of its representative agents, by raising money transfers that are given to its agents. It will provide more leisure but less consumption. Nash solution is inefficient. By coordinating monetary policies, world output increases, and utility level goes up. Formal proof of the results is presented for the case where productivity shocks are perfectly corrected across countries ($\theta_{1i} = \theta_{2i}$ for $i = 1, \cdots, S$). For the case where productivity shocks are less then perfectly correlated, we show the results with numerical examples: Tables 1 to 3.

The degree of risk-aversion (of representative agents) has the following effects on the Nash solution: Compared to the case where the degree of risk-aversion is low; when the agents are highly risk-averse, governments choose more inflationary policy. But the degree of decreases in the world output (in percentage) from the level of coordination is smaller. We assumed that the utility function has the form $u(L, C) = -L + (1/a)C^a$, $a < 1$. For a given increase in the money growth rates, labor supply of low-risk-averse (large α) agent is more elastic: Due to the differences in the concavity of u(c), leisure and consumption are closer substitutes for low-risk-averse agents. Raising money growth rates has negative effects on consumption, low-risk-averse agent switches to more leisure at a faster rate. On the other hand, labor supply of highly risk-averse agent is less elastic to changes in the money growth rates. Marginal negative effects of increasing money growth rates, on the utility of consumption is large. Therefore, Nash solution of highly risk-averse agents is: the degree of decreases in the world output, from the level of coordination solution, is smaller. More inflationary policies are chose, because labor supplies are less elastic.

[Perfectly-correlated Productivity Shocks]

	Nash Non-Coordination	Coordination
Monetary Policy:		
	$Z_h^N = \dfrac{2 \cdot \theta_h^{1/(1-a)}}{(3-a) \cdot \sum \pi_i \theta_i^{a/(1-a)}}$	$Z_h^C = \dfrac{\theta_h^{a/(1-a)}}{\sum \pi_i \theta_i^{a/(1-a)}}$

Utility level:

$$U^N =$$
$$-[\ 2/(3-a)]^{\ 1/(1-a)}$$
$$+\frac{1}{a}\cdot[\ 2/(3-a)]^{\ a/(1-a)}$$
$$\cdot\sum \pi_i \cdot \theta_i^{\ a/(1-a)}$$

$$U^C =$$
$$\frac{(1/a-1)^{a/(1-a)}}{\sum \pi_i \cdot \theta_i^{\ a/(1-a)}}$$

Output level:

$$Y_h^{*N} =$$
$$2\cdot[\ 2/(3-a)]^{\ 1/(1-a)}$$
$$\cdot \theta_h^{\ a/(1-a)}$$

$$Y_h^{*C} =$$
$$2\cdot \theta_h^{\ a/(1-a)}$$

for h = 1, ··, S ; i = 1, ··, S.

Proof: $U^N < U^C$

Define $U(X) \equiv [\ (1/a)\cdot X^a - X]\ \cdot \sum \pi_{i\cdot\theta_i}{}^{a/(1-a)}$

Then $U(1) = U^C$; and $U([\ 2/(3-a)]^{\ 1/(1-a)}) = U^N$

$$\frac{dU(X)}{dx} = (x^{a-1} - 1)\cdot \sum \pi_{1\cdot\theta_i}{}^{a/(1-a)} = 0 \quad \leftrightarrow \quad x = 0$$

$U(X)$ is maximum when $x = 1$ (for a<1). Q.E.D.

[TABLE 1]

[α = .5]

i	Nash Non-Coordination		Coordination	
	$\frac{1}{Z_{1i}^{N}}$	$\frac{1}{Z_{2i}^{N}}$	$\frac{1}{Z_{1i}^{C}}$	$\frac{1}{Z_{2i}^{C}}$
1	.88	2.13	.49	9.52
2	2.13	.88	9.52	.49
3	1.18	1.18	.98	.98
4	1.30	1.31	1.08	1.08

i	L_{1i}^{N}	L_{2i}^{N}	L_{1i}^{C}	L_{2i}^{C}
1	9.0	2.9	20.	0.
2	2.9	9.0	0.	20.
3	6.5	6.5	10.	10.
4	5.9	5.9	9.	9.

i	Y_i^{*N}	Y_i^{*C}
1	117	200
2	117	200
3	130	200
4	105	162

$U^N = 9.2$ $\qquad$ $U^C = 9.7$

$1/Z_{ki}$ = (gross) growth rate of money supply, of country k in state i; k=1,2; i= 1,2,3,4.

[Productivity Shocks]

i	π_i	θ_{1i}	θ_{2i}
1	1/4	10	9
2	1/4	9	10
3	1/4	10	10
4	1/4	9	9

[TABLE 2]

[α = .2]

i	Nash Non-Coordination		Coordination	
	$\frac{1}{Z_{1i}^N}$	$\frac{1}{Z_{2i}^N}$	$\frac{1}{Z_{1i}^C}$	$\frac{1}{Z_{2i}^C}$
1	1.11	1.90	.49	15.20
2	1.90	1.11	15.20	.49
3	1.38	1.38	.99	.99
4	1.41	1.41	1.02	1.02

i	L_{1i}^N	L_{2i}^N	L_{1i}^C	L_{2i}^C
1	1.5	.8	3.6	0.
2	.8	1.5	0.	3.6
3	1.2	1.2	1.8	1.8
4	1.1	1.1	1.7	9.

i	Y_i^{*N}	Y_i^{*C}
1	22	36
2	22	36
3	23	36
4	215	31
	U^N = 6.9	U^C = 7.1

[TABLE 3]

[α = -1]

i	Nash Non-Coordination		Coordination	
	$\frac{1}{Z_{1i}}^{N}$	$\frac{1}{Z_{2i}}^{N}$	$\frac{1}{Z_{1i}}^{C}$	$\frac{1}{Z_{2i}}^{C}$
1	1.81	2.24	.49	37.96
2	2.24	1.81	37.98	.49
3	2.06	2.06	1.01	1.01
4	1.95	1.95	.96	.96

i	$L_{1i}{}^{N}$	$L_{2i}{}^{N}$	$L_{1i}{}^{C}$	$L_{2i}{}^{C}$
1	.25	.20	.63	0.
2	.20	.25	0.	.63
3	.22	.22	.32	.32
4	.24	.24	.33	.33

i	$Y_i{}^{*N}$	$Y_i{}^{*C}$
1	4.4	6.3
2	4.4	6.3
3	4.5	6.3
4	4.2	6.0

U^N = -6.9 $\qquad$ U^C = -.64

BIBLIOGRAPHY

Bulow, J. and H.M. Polemarchakis, "Retroactive money," *Economica,* V.50, (1983), 301-310.

Calvo, G.A., "On the time consistency of optimal policy in a monetary economy," *Econometrica* V.46, (1978), 1411-1428.

Canzoneri, M.B. and J.A. Gray, "Monetary policy games and the consequences of non-cooperative behavior," *International Economic Review* V.26 No.3, (1985), 547-564.

Carlozzi, N. and J.B. Taylor, "International capital mobility and the coordination of monetary rules," in J.S. Bhandari, ed., *Exchange rate management under uncertainty,* MIT press, (1985).

Currie, D.A. and P. Levine, "Credibility and time inconsistency in a stochastic world," Discussion paper No.94 (Centre for Economic Policy Research, London), (1986).

Diamond, P., "National debt in a Neoclassical growth model, " *American Economic Review,* (1965), 1126-1150.

Dornbusch, R., "Expectations and exchange rate dynamics," *Journal of Political Economy,* V.84, (1976), 1161-1176.

Geanakoplos, J.D. and H.M. Polemarchakis, "Walrasian indcterminacy and Keynesian Macroeconomics," *Review of Economic Studies* V.LIII, (1983), 755-779.

Hamada, K., "A strategic analysis of monetary independence," *Journal of Political Economy* V.84,(1976), 677-700.

Hamada, K., "Macroeconomic strategy and coordination under flexible exchange rates," in R. Dornbusch and J. A. Frenkel, eds, *International economic policy: Theory and evidence* Johns Hopkins, Baltimore, (1979).

Oudiz, G. and J. Sachs, "international policy coordination in dynamic macroeconomic models," in W. H. Buiter and R. C. Marston, eds., *International economic policy coordination,* Cambridge University Press, (1985).

van der Ploeg, F., "international policy coordination in interdependent monetary economies,: *Journal of International Economics* V.25, (1988), 1-23.

Rogoff, K., "can international monetary policy cooperation be counterproductive?" *Journal of International Economic* V.18, (1985), 199-217.

Turnovsky, S.J.,T. Basar, and V. d'Orey, "Dynamic strategic monetary policies and coordination in interdependent economics," *American Economic Review* V.78 No.3, (1988), 341-361.