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STABILITY OF THE TWO-TEMPERATURE ACCRETION DISK *

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ABSTRACT

The stability of the geometrically thin, two-temperature hot accretion disk is studied. The general criterion for thermal instability is derived from the linear local analyses, allowing for advective cooling and dynamics in the vertical direction. Specifically, classic unsaturated Comptonization disk is analysed in detail. We find five eigen-modes: (1) Heating mode grows in thermal time scale, $(5/3)(\alpha\omega)^{-1}$, where *alpha* is the viscosity parameter and ω the Keplerian frequency. (2) Cooling mode decays in time scale, $(2/5)(T_e/T_i)(\alpha\omega)^{-1}$, where T_e and T_i are the electron and ion temperatures, respectively. (3) Lightman-Eardley viscous mode decays in time scale, $(4/3)(\Lambda/H)^2(\alpha\omega)^{-1}$, where Λ is the wavelength of the perturbation and H the unperturbed disk height. (4) Two vertically oscillating modes oscillate in Keplerian time scale, $(3/8)^{1/2}\omega^{-1}$ with growth rate $\propto (H/\Lambda)^2$. The inclusion of dynamics in the vertical direction does not affect the thermal instability, adding only the oscillatory modes which gradually grow for short wavelength modes. Also, the advective cooling is not strong enough to suppress the growth of heating modes, at least for geometrically thin disk. Non-linear development of the perturbation is followed for simple unsaturated Compton disk: depending on the initial proton temperature perturbation, the disk can evolve to decoupled state with hot protons and cool electrons, or to one-temperature state with very cool protons and electrons.

Key Words : Accretion, Neutron Stars, Black Holes

I. INTRODUCTION

The first quantatative models of accretion disks around black holes were calculated by Shakura and Sunyaev (1973, hereafter SS73), using 'α-viscosity' description for the angular momentum transport. The particular disk model they constructed is effectively optically thick and geometrically thin. But the disk temperature is too low to produce the high energy X-rays observed from many binary X-ray sources including Cyg X-1. Moreover, the inner part of the disk has been found unstable thermally and secularly. Shapiro, Lightman, and Eardley (1976, hereafter SLE) subsequently found a second type of solution for the accretion disk around black holes: the main cooling mechanism is unsaturated Comptonization and the plasma is in two-temperature (2T) state. The proton is quite hot, $T_p \sim 10^{11}$K, and the electron mildly hot, $T_e \sim 10^9$K. The disk is effectively optically thin and geometrically thin. They believed that unstable part of the optically thick disk would change to this high temperature solution, thereby explaining the hard X-ray spectrum.

As soon as these steady disk solutions were found, their stabilities have been studied through linear anlysis: Pringle, Rees, and Pacholczyk (1973) discussed the thermal stability of the optically thin disk radiating by thermal bremsstrahlung. Pringle (1976) derived the condition for thermal stability through simple stability analysis. Shakura and Sunyaev (1976, hereafter SS76) did thorough study of the SS73 disk, allowing the time dependence in the surface mass density, finding thermal and viscosly driven secular instabilities wherever the radiation pressure dominates over the gas pressure. Piran (1978) extended the analysis for more general form of viscosity. He confirmed that most cooling mechanism with α-viscosity is thermally or secularly unstable.

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Pringle (1976) and Piran (1978) applied these stability anaysis of one-temperature (1T) disk to SLE's 2T disk under certain assumptions: the cooling and heating of the electron is assumed to be balanced (Pringle 1976) or the Compton parameter Y is fixed to be 1 (Piran 1978). They both found SLE's 2T unsaturated Compton disk thermally unstable. White and Lightman (1990) also studied the time-dependent behaviour of the 2T disk to find elctron-positron pair production instability.

However, these studies neglect several potentially important factors: the cooling due to the radial advection, the expansion and contraction of the disk in the vertical directions, and proton velocity effects in the Coulomb coupling between protons and electrons.

So in this paper, we study through linear anayses the stability of the gas pressure dominated hot accretion disk incorporating all the factors. The result is general enough to include any kind of electron cooling function. As an important example, we apply the result to the classic unsaturated Comptonization disk of SLE.

II. STEADY-STATE SOLUTIONS

We recalculate the classic two-temperature disk solution of Shapiro, Lightman, and Eardley (1975) as the unperturbed solution. This disk is vertically averaged, geometrically thin, cooled by unsaturated Comptonization of soft photons. Proton velocity and relativistic effects are included in the Coulomb coupling formula, which has been neglected in SLE's calculation (Stepney 1983).

Steady-state solution satisfies the proton and electron energy equations:

$$\frac{n_p k(T_p + T_e)}{t_h} = \frac{n_p k(T_p - T_e)}{t_{ep}}, \tag{1}$$

$$\frac{(3/2)n_p k(T_p + T_e)}{t_h} = \frac{n_e k(T_e - T_C)}{t_c}. \tag{2}$$

The three important time scales in equations (1) and (2) are ion heating, Coulomb coupling, and Compton cooling times given as

$$t_h = \frac{1}{\alpha\omega}\frac{T_p}{T_p + T_e}, \tag{3}$$

$$t_{ep} = \left(\frac{\pi}{2}\right)^{1/2}\frac{m_p}{m_e}\frac{1}{\ln\Lambda}\frac{1}{n_e\sigma_T c}(\theta_p^* + \theta_e)^{3/2}, \tag{4}$$

$$t_c = \frac{m_e c}{4\sigma_T U_s}, \tag{5}$$

respectively, where $\omega \equiv (c/R_g)(r^*)^{-3/2}$ is the Keplerian frequency, $r_* \equiv R/R_g$, $R_g = R_{sch}/2 \equiv GM/c^2$, α the viscosity parameter, $\theta_p^* \equiv kT_p/m_e c^2$, $\theta_e \equiv kT_e/m_e c^2$, T_C the Compton temperature of the soft-photon radiation field, i.e., the energy-weighted mean of photon energy, U_s the radiation energy density of soft photons.

Since the source of the soft photons is far from trivial in the generic models of high-energy sources, the electron energy equation (2) is usually replaced by the condition for the unsaturated Comptonization (SLE):

$$Y \equiv \left(\frac{4kT_e}{m_e c^2}\right)\tau_{es}(1 + \tau_{es}) = 1, \tag{6}$$

where τ_{es} is the Thomson optical depth in the vertical direction.

Equation (1) and (6) with the angular momentum conservation with zero-stress boundary condition and hydrostatic equilibrium in the vertical direction are numerically solved for the steady state solutions: one of them for $L/L_E = 0.1$ and $\alpha = 0.1$ (L_E is the Eddington luminosity.) is shown in Figure 1. Improved Coulomb coupling and slightly different definition of Y result in higher T_p and lower T_e than those of SLE's calculation.

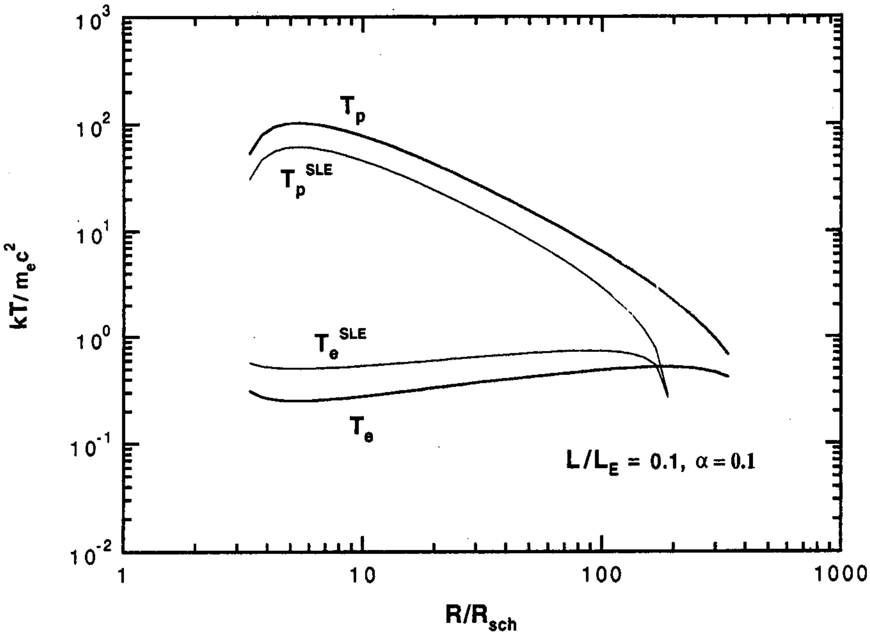

Fig. 1. The proton and elctron temprature profiles of the steady-state solution of $L/L_E = 0.1$ and $\alpha = 0.1$. The thin line show the original calculation of SLE.

III. LINEAR ANALYSES

Time dependent behavior of the disk is studied using linear approaches following SS76. But we do not assume $Y = 1$ or time-independent electron energy equation for the perturbation. Coulomb coupling is treated more accurately and the PdV work term, neglected in White and Lightman (1990), as well as the vertical expansion/contraction of the disk and advective cooling due to the radial motion is explicitly included. In the following subsections, linearized equations are derived and solutions are studied for several limiting cases to gain physical insights. General case is discussed in the end.

(a) Linearization

The equations to be linearized are the mass diffusion equation and the energy equations for protons and electrons.

$$\frac{\partial \Sigma}{\partial t} = \frac{\alpha}{2R}\frac{\partial}{\partial R}\frac{1}{R\omega}\frac{\partial}{\partial R}\Sigma H^2\omega^2 R^2, \tag{7}$$

$$\frac{\partial}{\partial t}(\epsilon_p H) + P_p\frac{\partial H}{\partial t} = -\frac{1}{R}\frac{\partial}{\partial R}[(\epsilon_p + P_p)v_R H R] + v_R H\frac{\partial P_p}{\partial R} + Q_p^+ - Q_p^-, \tag{8}$$

$$\frac{\partial}{\partial t}(\epsilon_e H) + P_e\frac{\partial H}{\partial t} = -\frac{1}{R}\frac{\partial}{\partial R}[(\epsilon_e + P_e)v_R H R] + v_R H\frac{\partial P_e}{\partial R} + Q_e^+ - Q_e^-, \tag{9}$$

We note that the term $v_R\partial(PH)/\partial R$ in equation (4.6) of SS76 should read $v_R H\partial P/\partial R$. But the conclusions of SS76 is still valid. The symbols have their usual definitions: Q^+ and Q^- are the heating and cooling functions per unit surface area (per side) and $\Sigma \equiv 2\int_0^H n dz$ the surface (number) density, where H is the half-thickness of the disk and n is the proton (electron) number density.

These equations are to be linearized by the substitution of the following perturbations:

$$\Sigma(R,t) = \Sigma(R)\,[1+\sigma(R,t)]\,; \quad T_p(R,t) = T_p(R)\,[1+\psi(R,t)]\,; \quad T_e(R,t) = T_e(R)\,[1+\phi(R,t)]\,, \tag{10}$$

where $\Sigma(R)$, $H(R)$, $T_p(R)$, and $T_e(R)$ are the unperturbed quantities, which change over the length scale R. To simplify the notations, we use superscript '0' only when there is possibility of confusion.

Heating of protons per unit surface area due to the friction is (SS73),

$$Q_p^+ = -\frac{1}{2} W_{r\phi} R \frac{\partial \omega}{\partial R} = Q_p^+(0)(1+\sigma+\psi), \tag{11}$$

where $Q_p^+(0)$ is the value of Q_p^+ in unperturbed solution. Electrons and protons are coupled by Coulomb interaction alone and

$$\begin{aligned} Q_p^- = Q_e^+ &= \text{constant } n^2(T_p - T_e)\frac{1+(\theta^*)^{1/2}}{(\theta^*)^{3/2}} H \\ &= Q_e^+(0)\Big[1 + 2\sigma - (\frac{1}{2}\frac{T_p}{T^+} - \frac{T_p}{T^-} + \frac{3}{2}\frac{\theta_p^*}{\theta^*})\psi \\ &\quad - (\frac{1}{2}\frac{T_e}{T^+} + \frac{T_e}{T^-} + \frac{3}{2}\frac{\theta_e}{\theta^*})\phi\Big], \end{aligned} \tag{12}$$

where $T^+ \equiv T_p + T_e$ and $T^- \equiv T_p - T_e$. The effect of proton's thermal velocity is incorporated through $\theta^* \equiv \theta_p^* + \theta_e$. However, we only consider the case where $\theta_p^* + \theta_e \ll 1$, otherwise pair processes should be considered (see White & Lightman 1990 for pair-induced instability). Electron cooling is parametrized to deal with various cooling situations,

$$Q_e^- = Q_e^-(0)(1 + k\sigma + l\psi + m\phi), \tag{13}$$

where

$$k \equiv \frac{\partial \ln Q_e^-}{\partial \ln \Sigma}\Big|_{H^0, T_p^0, T_e^0}; \quad l \equiv \frac{\partial \ln Q_e^-}{\partial \ln T_p}\Big|_{\Sigma^0, H^0, T_e^0}; \quad m \equiv \frac{\partial \ln Q_e^-}{\partial \ln T_e}\Big|_{\Sigma^0, H^0, T_p^0}.$$

Unsaturated Comptonization of soft-photons (with fixed soft-photon energy density) corresponds to $k = 1$, $l = 0$, $m = 1$ and optically thin bremsstrahlung to $k = 2$, $l = -1/2$, $m = 1/2$. If Q_e^- has a functional form $Q_e^- = Hn^\delta f(T_e)$, then $k = \delta$, $j = (1-\delta)/2$, and we expect $\delta \geq 0$ and $m \geq 0$ in most optically thin coolings.

(b) With No Perturbation in Σ

When no perturbation in Σ is assumed, i.e., $\sigma = 0$, the anlysis of the linearized equations become quite simple. This turns out to be not a bad assumption when studying only the thermal stability because including the perturbation in Σ will introduce secular, i.e., Lightman-Eardley type, mode which has much longer time scale than thermal time scales (see Pringle 1976 for similar assumption). We also assume hydrostatic equilibrium in the vertical direction.

Under this assumption, the linearized form of equations (8) and (9) for $T_p \gg T_e$ cases are

$$\begin{aligned} \frac{\partial \psi}{\partial t} &= \frac{3}{4} t_h^{-1}\Big[(\frac{1}{2} + \frac{3}{2}\frac{\theta_p^*}{\theta^*})\psi + \frac{3}{2}\frac{\theta_e}{\theta^*}\phi\Big] \\ \frac{\partial \phi}{\partial t} &= t_c^{-1}\Big[(\frac{1}{2} - \frac{3}{2}\frac{\theta_p^*}{\theta^*} - l)\psi - (m + \frac{3}{2}\frac{\theta_e}{\theta^*})\phi\Big], \end{aligned} \tag{14}$$

where $t_h = (\alpha\omega)^{-1}$ and $t_c = (\theta_e/\theta_p)t_h \ll t_h$. This is a linear autonomous system whose generic form is

$$\dot{\mathbf{Y}} = \mathbf{MY} \qquad \text{with } \mathbf{Y} = \begin{pmatrix} \psi \\ \phi \end{pmatrix} \tag{15}$$

and has a general solution

$$\mathbf{Y}(t) = c_1 \mathbf{V}_1 \exp(\Omega_1 t) + c_2 \mathbf{V}_2 \exp(\Omega_2 t), \tag{16}$$

where c_1 and c_2 are the arbitrary constants, $\mathbf{V_1}$ and $\mathbf{V_2}$ the eigenvectors of matrix $\mathbf{M}$, and Ω_1 and Ω_2 the corresponding eigenvalues, satisfying

$$\Omega^2 + \Big[t_c^{-1}(m + \frac{3}{2}\frac{\theta_e}{\theta^*}) - \frac{3}{4}t_h^{-1}(\frac{1}{2} + \frac{3}{2}\frac{\theta_p^*}{\theta^*})\Big]\Omega + \frac{3}{8}(t_h t_c)^{-1}\frac{\theta_e}{\theta^*}\Big[-3 + 3l - m - 4m\frac{\theta_p^*}{\theta_e}\Big] = 0. \tag{17}$$

This dispersion equation always has two real roots for Ω as long as $t_c/t_h = T_e/T_p \ll 1$: If $-3+3l-m-4m(\theta_p^*/\theta_e) < 0$ in addition, $\Omega_1 > 0$ and $\Omega_2 < 0$, and the disk has one growing mode and one decaying mode. Otherwise, $\Omega_1 < 0$ and

$\Omega_2 < 0$, and the disk has two decaying modes. Most optically thin cooling mechanisms of the form $Q_e^- = Hn^\delta f(T_e)$ with $\delta > 0$ and $d\ln f/d\ln T_e > 0$, including unsaturated Compton cooling and bremsstrahlung, give one growing and one decaying mode.

Solving equation (17) gives the growth/decay rate for each mode,

$$\begin{aligned}\Omega_1 &= \frac{3}{8}t_h^{-1}\frac{4m+3(1-l-m)(\theta_e/\theta^*)}{m+(3/2)(\theta_e/\theta^*)} + O([\theta_e/\theta_p]^1),\\ \Omega_2 &= -t_c^{-1}(m+\frac{3}{2}\frac{\theta_e}{\theta^*}) + O([\theta_e/\theta_p]^0),\end{aligned} \tag{18}$$

which has the corresponding eigen solution,

$$\begin{pmatrix}\psi\\ \phi\end{pmatrix} \simeq c_1\begin{pmatrix}1+\frac{2}{3}m\theta^*/\theta_e\\ 1-\frac{2}{3}(1+l)\theta^*/\theta_e\end{pmatrix}\exp(\Omega_1 t) + c_2\begin{pmatrix}-1\\ (4/3)(\theta_p/\theta_e)[1+(2/3)m\theta^*/\theta_e]\end{pmatrix}\exp(\Omega_2 t). \tag{19}$$

The first mode grows in heating time scale, $\Omega_1^{-1} \sim t_h$, and the second mode decays in cooling time scale, $\Omega_2^{-1} \sim -t_c$. In hot 2T disk, the latter mode has a much shorter time scale and is superposed onto the heating mode. Depending on the values of (θ_p^*/θ_e), l, and m, sign of ϕ can be equal to or different from that of ψ for the growing mode.

Take the unsaturated Compton cooling ($l = 0$ and $m = 1$) for example. The coefficient for ψ in growing mode $1+(2/3)m\theta^*/\theta_e > 0$, but that for ϕ, $1-(2/3)\theta^*/\theta_e = 1/3-(2/3)(T_p/T_e)(m_e/m_p)$, can be either positive or negative depending on the value of T_p/T_e: If $T_p/T_e < (1/2)m_p/m_e$, electrons are heated when protons are heated, and cooled when protons cooled. However, if $T_p/T_e > (1/2)m_p/m_e$, electrons get cooled when protons are being heated, and heated when protons cooled. So it is possible that the disk can puff up while electrons are being cooled. This interesting property is due to the effect of the proton temperature in the Coulomb coupling: if protons are too hot, proton velocity become significant, and coupling gets weaker transferring less energy.

Comparison of the components of $\mathbf{V}_1 \equiv (V_{1,\psi}, V_{1,\phi})$ and $\mathbf{V}_2 \equiv (V_{1,\psi}, V_{1,\phi})$ in equation (19) shows how the time-dependent behaviour depends on the initial conditions. Suppose some initial value ψ_0 and ϕ_0 at $t = 0$. Under the condition $|V_{2,\phi}| \gg |V_{1,\psi}|$, $|V_{1,\phi}|$, $|V_{2,\psi}|$, $c_1 \simeq V_{1,\psi}^{-1}\psi_0$ and $c_2 \simeq (V_{1,\psi}V_{2,\phi})^{-1}(-V_{1,\phi}\psi_0 + V_{1,\psi}\phi_0)$. So c_1 is basically determined by ψ_0. But the solution is solely determined by the growing mode after a fraction of t_h, whose amplitude is c_1. Thus, we can see that time-dependent behaviour is determined by the initial value of proton temperature alone: if protons are heated above the equilibrium value, proton temperature keeps increasing (disk puffing up), and if cooled below the equilibrium value, it keeps decreasing (disk collapsing). Electron temperature just follows proton temperature in the same way or in the opposite way according to the relative sign between $V_{1,\psi}$ and $V_{1,\phi}$.

Similarly, we can write down more general criterion for the thermal instability for hot 2T ($\theta_p \gg \theta_e$) disk with Coulomb coupling,

$$3\left(\frac{\partial \ln Q_p^+}{\partial \ln T_p} - \frac{\partial \ln Q_e^-}{\partial \ln T_p}\right) + \left(2\frac{\partial \ln Q_p^+}{\partial \ln T_p} - 1\right)\frac{\partial \ln Q_e^-}{\partial \ln T_e} + 2\frac{\theta_p^*}{\theta_e}\left(\frac{\partial \ln Q_p^+}{\partial \ln T_p} + 1\right) > 0. \tag{20}$$

This is more complex condition than that in 1T case, $\partial\ln Q^+/\partial\ln T > \partial\ln Q^-/\partial\ln T$. Standard α disk corresponds to $\partial\ln Q_p^+/\partial\ln T_p = 1$.

(c) With Perturbation in Σ

Now we allow the surface density Σ to be perturbed but still assuming hydrostatic equilibrium. Linearized form of the mass conservation equation under the assumption $H \ll R$ is

$$\frac{\partial \sigma}{\partial t} = \frac{1}{2}\alpha\omega H^2\frac{\partial^2}{\partial R^2}\left(\sigma + \frac{T_p}{T^+}\psi + \frac{T_e}{T^+}\phi\right), \tag{21}$$

that of the proton energy equation

$$\begin{aligned}&3\frac{\partial \sigma}{\partial t} + (3+\frac{T_p}{T^+})\frac{\partial \psi}{\partial t} + \frac{T_e}{T^+}\frac{\partial \phi}{\partial t}\\ &= \frac{10}{3}\alpha\omega H^2\frac{\partial^2}{\partial R^2}\left(\sigma + \frac{T_p}{T^+}\psi + \frac{T_e}{T^+}\phi\right)\\ &+ 3\alpha\omega\frac{T^+}{T_p}\left[-\sigma + (\frac{3}{2}\frac{T_p}{T^+} - \frac{T_p}{T^-} + \frac{3}{2}\frac{\theta_p^*}{\theta^*})\psi + (\frac{3}{2}\frac{T_e}{T^+} + \frac{T_e}{T^-} + \frac{3}{2}\frac{\theta_e}{\theta^*})\phi\right],\end{aligned} \tag{22}$$

and that of the elctron energy equation

$$
\begin{aligned}
&3\frac{\partial\sigma}{\partial t}+\frac{T_p}{T^+}\frac{\partial\psi}{\partial t}+(3+\frac{T_e}{T^+})\frac{\partial\phi}{\partial t}\\
&=\frac{10}{3}\alpha\omega H^2\frac{\partial^2}{\partial R^2}\left(\sigma+\frac{T_p}{T^+}\psi+\frac{T_e}{T^+}\phi\right)\\
&+3\alpha\omega\frac{T^+}{T_e}\Big[(2-k)\sigma+(-\frac{1}{2}\frac{T_p}{T^+}+\frac{T_p}{T^-}-\frac{3}{2}\frac{\theta_p^*}{\theta^*}-l)\psi\\
&\qquad-(\frac{1}{2}\frac{T_e}{T^+}+\frac{T_e}{T^-}+\frac{3}{2}\frac{\theta_e}{\theta^*}-m)\phi\Big].
\end{aligned}
\tag{23}
$$

The equations (21)–(23) are to be solved by assuming σ, ψ, and ϕ to have exponential time dependence and sinusoidal radial dependence of wavelength Λ satisfying $H \ll \Lambda \ll R$ (SS76):

$$
\sigma(R,t)\equiv e^{\Omega t}\sin\left(\frac{R}{\Lambda}\right)\overline{\sigma}\quad \psi(R,t)\equiv e^{\Omega t}\sin\left(\frac{R}{\Lambda}\right)\overline{\psi}\quad \phi(R,t)\equiv e^{\Omega t}\sin\left(\frac{R}{\Lambda}\right)\overline{\phi},
\tag{24}
$$

where $\overline{\sigma}$, $\overline{\psi}$, and $\overline{\phi}$ are constants locally because we are looking at the short-wavelength modes. This approach is not any different from the direct method used in section §3.2.

Substituting the functional forms (24) into equations (21)–(23) and demanding the set of equations to have non-trivial solutions gives cubic dispersion equation for Ω:

$$
\begin{aligned}
&12(\frac{\Omega}{\alpha\omega})^3+\left[\frac{23}{2}(\frac{H}{\Lambda})^2-3(3+l)+3(3\frac{T^+}{T_e}+\frac{T_p}{T_e})m+24\frac{T^+}{T^-}-18\frac{T^+}{T_p}\frac{\theta_p^*}{\theta^*}+18\frac{T^+}{T_e}\frac{\theta_e}{\theta^*}\right](\frac{\Omega}{\alpha\omega})^2\\
&+\Bigg[\Big\{-\frac{9}{2}k-7l+(\frac{9}{2}\frac{T^+}{T_e}+7\frac{T_p}{T_e})m+23\frac{T^+}{T^-}-\frac{69}{4}\frac{T^+}{T_p}\frac{\theta_p^*}{\theta^*}+\frac{69}{4}\frac{T^+}{T_e}\frac{\theta_e}{\theta^*}\Big\}(\frac{H}{\Lambda})^2\\
&\quad+\frac{27}{2}(\frac{T^+}{T_p}l-\frac{T^+}{T_e}m)+9(-2+\frac{T^+}{T_p}l+\frac{T^+}{T_e}m)\frac{T^+}{T^-}\\
&\quad+\frac{27}{2}(1-\frac{T^+}{T_e}m)\frac{T^+}{T_p}\frac{\theta_p^*}{\theta^*}-\frac{27}{2}(1-\frac{T^+}{T_p}l)\frac{T^+}{T_e}\frac{\theta_e}{\theta^*}\Bigg](\frac{\Omega}{\alpha\omega})\\
&+\Bigg[\frac{45}{4}(\frac{T^+}{T_p}l-\frac{T^+}{T_e}m)-9\frac{T^+}{T^-}k+\frac{9}{2}(\frac{T^+}{T_p}l+\frac{T^+}{T_e}m)\frac{T^+}{T^-}\\
&\quad+\frac{27}{4}(k-\frac{T^+}{T_e}m)\frac{T^+}{T_p}\frac{\theta_p^*}{\theta^*}-\frac{27}{4}(k-\frac{T^+}{T_p}l)\frac{T^+}{T_e}\frac{\theta_e}{\theta^*}\Bigg](\frac{H}{\Lambda})^2=0.
\end{aligned}
\tag{25}
$$

In the limit $H \ll \Lambda$, the equation reduces to equation (17), with an additional zero-frequency mode. For disks with unsaturated Compton cooling, it can be proved that equation (25) always has 3 real roots for Ω as long as $H < \Lambda$, by transforming the equation into special cubic equation and checking the determinant. A negative root with smaller absolute value correspond to the slowly decaying viscous mode (Lightman & Eardley 1974). Another negative root with larger absolute value is the fast decaying cooling mode and the positive root the growing heating mode. The other cooling mechanisms, like any of optically thin cooling mechanisms, also produce similar stability behaviors. It appears that the advective cooling is not sufficient to suppress the thermal instability.

(d) With Disk Expansion/Contraction

In all previous linear stability anaysis, hydrostatic equilibrium in vertical direction is always assumed under the justification that the dynamical time scale in the vertical direction, t_{hyd}, is shorter than the heating time scale, t_h, by the factor α. However, for α not much smaller than 1, the two time scales become comparable. Moreover, the cooling time scale, t_c, is shorter than t_h by the factor T_e/T_p, and thus can be shorter than t_{hyd}. Only White and

Lightman (1990) relaxed hydrostatic assumption in studying the time-dependent behavior of the disk with pairs. Here we add the same equation for the time-dependent behavior of disk in vertical direction:

$$\frac{\partial^2 H}{\partial t^2} = -\frac{GM}{R^2}\frac{H}{R} + \frac{P}{\rho H}, \tag{26}$$

which is linearized to

$$\frac{\partial^2 h}{\partial t^2} = \omega^2(-2h + \frac{T_p}{T^+}\psi + \frac{T_e}{T^+}\phi). \tag{27}$$

Now the disk height $H(t)$ is not determined by T_p and T_e, and the electron cooling function should include the dependency on H: $Q_e^- = Q_e^-(0)(1+k\sigma+jh+l\psi+m\phi)$ with $j \equiv \partial \ln Q_e^-/\partial \ln H|_{\Sigma^0,T_p^0,T_e^0}$. Unsaturated Comptonization of soft-photons gives $k = 1$, $j = 0$, $l = 0$, $m = 1$ and optically thin bremsstrahlung $k = 2$, $j = -1$, $l = 0$, $m = 1/2$. For $Q_e^- = Hn^\delta f(T_e)$, $k = \delta$, $j = 1-\delta$, hence $k \geq 0$, $j \leq 1$, and $m \geq 0$ in generic optically thin coolings. Assuming hydrostatic equlibrium is equivalent to maintaining $2h = (T_p/T^+)\psi + (T_e/T^+)\phi$.

With this description, the mass conservation and energy equations are

$$\frac{\partial u}{\partial t} = \frac{\alpha}{2}\omega H^2 \frac{\partial^2}{\partial R^2}(u+2h), \tag{28}$$

$$\begin{aligned} 3\frac{\partial u}{\partial t} + 3\frac{\partial \psi}{\partial t} + 2\frac{\partial h}{\partial t} &= \frac{10}{3}\alpha\omega H^2 \frac{\partial^2}{\partial R^2}(u+2h) \\ &+ 3\alpha\omega\frac{T^+}{T_p}\left[-u + h + (\frac{T_p}{T^+} - \frac{T_p}{T^-} + \frac{3}{2}\frac{\theta_p^*}{\theta})\psi + (\frac{T_e}{T^+} + \frac{T_e}{T^-} + \frac{3}{2}\frac{\theta_e}{\theta})\phi\right], \end{aligned} \tag{29}$$

$$\begin{aligned} 3\frac{\partial u}{\partial t} + 3\frac{\partial \phi}{\partial t} + 2\frac{\partial h}{\partial t} &= \frac{10}{3}\alpha\omega H^2 \frac{\partial^2}{\partial R^2}(u+2h) \\ &+ 3\alpha\omega\frac{T^+}{T_e}\left[(2-k)u - (1+j)h + (-l + \frac{T_p}{T^-} - \frac{3}{2}\frac{\theta_p^*}{\theta})\psi - (m + \frac{T_e}{T^-} + \frac{3}{2}\frac{\theta_e}{\theta})\phi\right] \end{aligned} \tag{30}.$$

Same exponential forms for h, ψ, and ϕ yield quintic dispersion equation, which under the assumptions $T_p \gg T_e$, $\theta_p^* \ll \theta_e$, and $H/\Lambda \ll 1$ can be simplified to:

$$\begin{aligned} &\left(\frac{\Omega}{\alpha\omega}\right)^5 + \frac{T_p}{T_e}(m+\frac{3}{2})\left(\frac{\Omega}{\alpha\omega}\right)^4 + \frac{3}{2}\frac{T_p}{T_e}(l-1)\left(\frac{\Omega}{\alpha\omega}\right)^3 + \alpha^{-2}\frac{T_p}{T_e}(4+\frac{8}{3}m)\left(\frac{\Omega}{\alpha\omega}\right)^2 \\ &+ \alpha^{-2}\frac{T_p}{T_e}(-3+\frac{3}{2}j+3l-m)\left(\frac{\Omega}{\alpha\omega}\right) + \alpha^{-2}\frac{T_p}{T_e}(\frac{3}{4}j - \frac{3}{2}k + \frac{3}{2}l - \frac{3}{2}m)\left(\frac{H}{\Lambda}\right)^2 = 0. \end{aligned} \tag{31}$$

Order of magnitude analyses of the equation show that the first mode is a decaying mode due to the cooling, $\Omega/\alpha\omega \sim -(m+3/2)(T_p/T_e)$, the second one is a growing thermal mode, $\Omega/\alpha\omega \sim [3-(3/2)j-3l+m]/[4+(8/3)m]$. The third one is a decaying viscous mode, $\Omega/\alpha\omega \sim [-(3/4)j+(3/2)k-(3/2)l+(3/2)m]/[-3+(3/2)j+3l-m](H/\Lambda)^2$ with its decay rate depending on Λ. The fourth and last modes are vertically oscillating modes, $\Omega/\alpha\omega \sim \pm[4+(8/3)m]^{1/2}/[(3/2)+m)]^{1/2}\alpha^{-1}i$.

For general cases, the quintic dispersion equation has to be solved numerically and the roots depend on the value of H/Λ. The $\Omega/\alpha\omega$ of each mode is shown in Figure 2 for the steady-state solution shown in Figure 1 ($L/L_E = 0.1$, $\alpha = 0.1$; unsaturated Comptonization). The result confirms the dimensional analyses. The decay rate of the cooling mode is independent of Λ, yet shorter wavelength viscous mode decays faster. And longer wavelengh heating mode grows slightly faster that shorter wavelength one. The oscillating mode's growth rate increases as Λ decreases. For perturbation of $\Lambda > H$, there are three growing modes: heating mode grows in thermal time scale of $\sim (5/3)(\alpha\omega)^{-1}$ and two oscillating modes in time scale of $\gg (\alpha\omega)^{-1}$. So we can see that the expansion/contraction of the disk in the vertical direction does not affect the basic stability property of the disk, only adding the growing oscillatory modes.

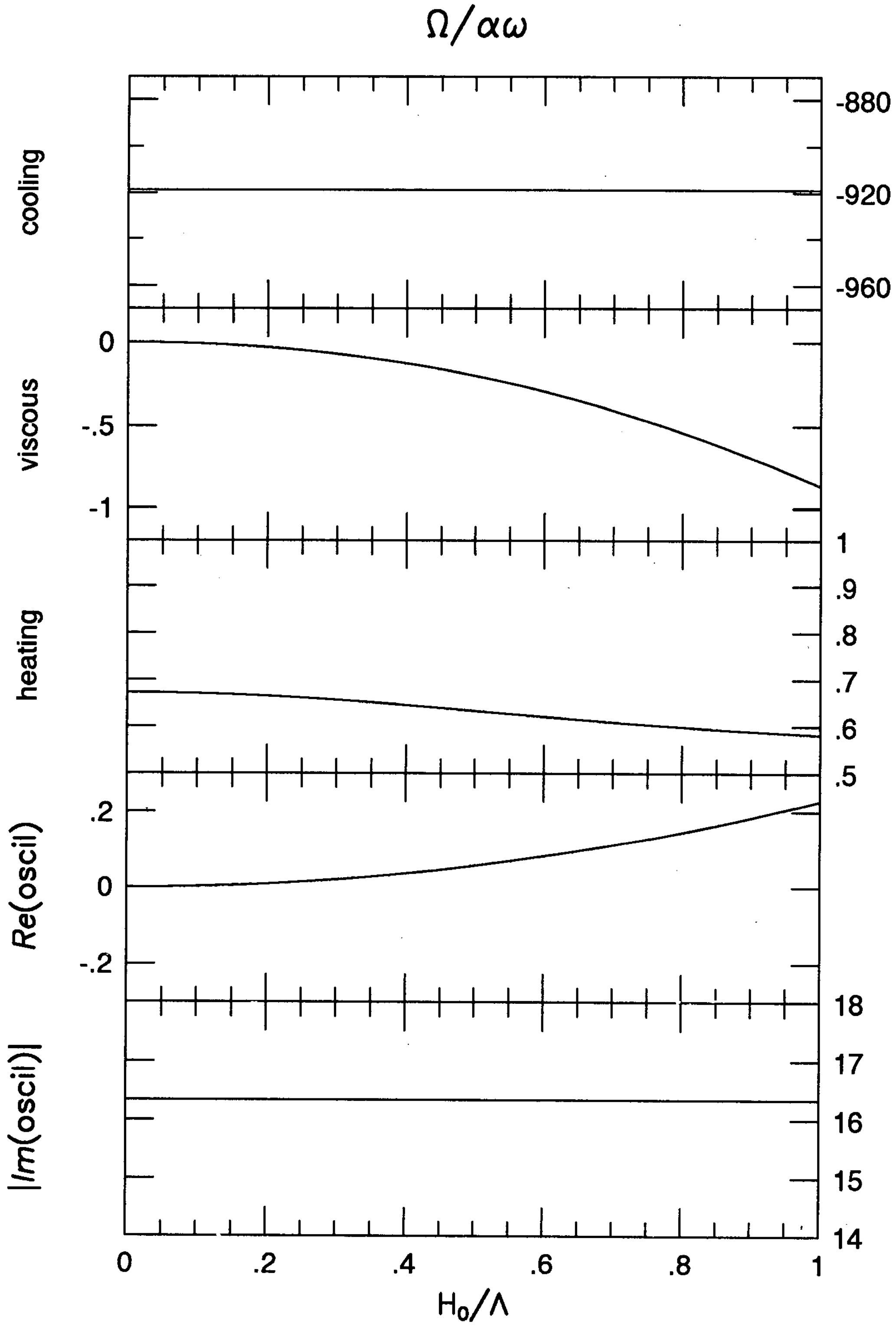

Fig. 2. Five modes of perturbation for two-temperature hot accretion disk cooled by unsaturated Comptonization. Mode frequency Ω in units of $\alpha\omega$ is expressed as a function of disk height-to-wavelength ratio. The last two frames show the real and imaginary parts of the oscillating mode frequencies.

IV. NON-LINEAR BEHAVIOUR

Very important question unanswered so far is the final state of the disk when it gets thermally unstable. To get some insights, we explicitly integrated the energy equations (8) and (9) without v_R term for unsaturated Compton disk. To deal with low temperature state we used hybrid cooling law of Wandel and Liang (1991) modified for unsaturated Comptonization. The result is shown in Figure 3: the disk either expands or collpases depending on the sign of the initial proton temperature perturbation. Upper curves show the proton temperature evolution and lower ones the electron. Solid curves are for positive initial proton temperature perturbation and dotted for negative one.

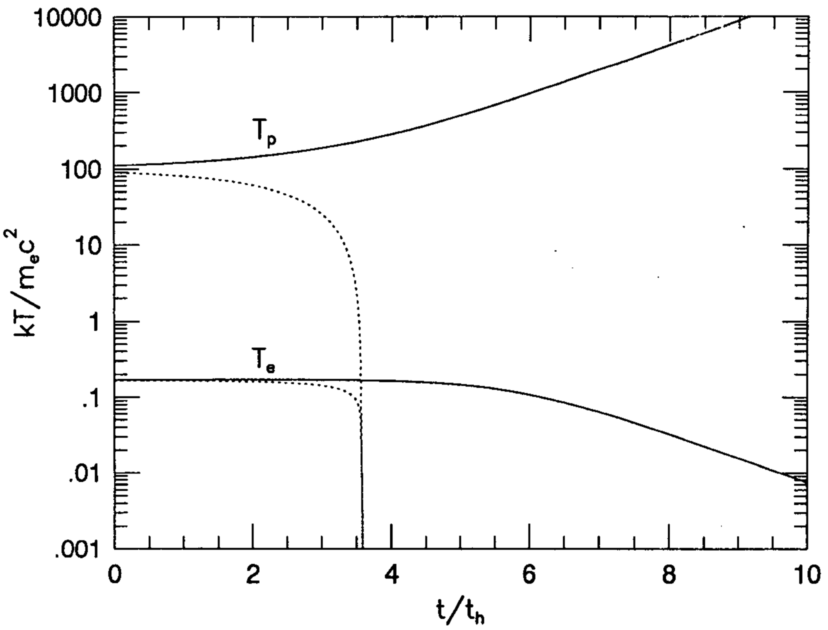

Fig. 3. Evolution of proton and electron temperature. *Solid* curves represent proton (upper) and electron (lower) temerature when initial proton temperature perturbation is positive. *Dotted* curves represent the same when initial proton temperature perturbation is negative. Time is in unit of $t_h = (\alpha\omega)^{-1}$.

Integrations for various initial conditions confirm the linear analysis result (§2b): electron temperature perturbation is unimportant and can have the same or opposite sign as proton temperature perturbation, depending on the value of T_p/T_e. However, we also find that electron temperature always decreases after a few t_h regardless of initial increse or decrease: If protons get heated, the Coulomb coupling gets weaker and electrons get cooled. If protons get cooled, sometimes electron temperature initially increases slightly due to the stronger coupling as discussed in §2b, but afterwards protons get cooled too much and transfer of energy decreases, thereby bringing down the electron temperature.

When protons get heated above the equilibrium value, the proton temperature runs away while the electron temperature drops because Coulomb coupling gets weaker for higher proton temperature yet viscous heating increasing (solid line in Fig. 3). When the proton temperature reaches the virial temperature, H becomes comparable to R and the thin disk assumption is violated and our analyses do not apply. But we expect if the expanding disk becomes geometrically thick the advective cooling will be highly efficient and stabilize the disk as in the spherical accretion. Recently Chen and Taam (1993), Narayan and Yi (1994), Abramowicz et al. (1994), and Chen et al. (1995) find the new branch of one-temperature disks stabilized by the advective cooling. The same will be true with 2T disk, and the final configuration of the perturbed disk will be advection dominated slim/thick 2T disk with near virial proton temperature while electron temperature being slightly lower than steady-state value.

In the opposite case when protons get cooled, the disk will collpase and become one-temperature (dotted line in Fig. 3). But it is not the same as ususal SS73 solution because the 2T disk started with different Σ for given $\dot{M}$ and, therefore, generally no steady-state SS73 solution exists with the same Σ. So the disk does not evolve to stedy-state solution of SS73. To find out the final configuration of the cooled disk, we need to follow the change of Σ which requires to integrate all three equations (7)–(9). That is not attempted in this work.

V. DISCUSSIONS

We confirm the basic thermal instability of the 2T hot accretion disk of SLE (Pringle 1976, Piran 1978) even after various improvements in microphysics and dynamics of disk: the disk is unstable mainly by the heating mode. However, we note that the condition for the thermal instability is not so simple as in one-temperature case. Also, by considering the dynamics in the vertical directions, we find two oscillating modes which grow in thermal time scale. The frequency of this mode in the inner region of the disk is probably too high for QPO's we see in galactic X-ray sources. But if somehow this mode is being damped in the inner region by some causes, e.g., by advection, the mode might have the right frequencies at the thermally unstable region of the disk.

Although the result of §3 is valid for any coolings as long as we correctly specify the cooling parameters, the analysis regarding the unsaturated Comptonization cooling is less than satisfactory, the main reason being our ignorance of the Comptonizing soft-photons. For the steday-state solutions, we assumed $Y = 1$, and in time-dependent studies the soft photon field is assumed to be not affected by other disk variables, e.g., temperature, disk height and so on. For steady-state solutions, $Y = 1$ may not be a bad assumption even when we do not know the origin and spectrum of the soft photons, mainly because we see the spectra of high-energy sources are well approximated by the unsaturated Comptonization of soft photons. However, to study the stability of the disk in rigour, we have to specify how the local soft photon radiation field adjusts to the variations of the disk variables. This is unknown unless we know where the soft photons are produced by what mechanism and how they are transferred to the other parts of the disk. This basic difficulties exist in any previous works on Comptonized disk as well as this one. We hope to cope with this problem in future work.

Another uncertainty is the coupling between protons and electrons. The Coulomb coupling as we describe always exists. However, if there can be some plasma instablities or other coupling mediated by photons, the proton-electron coupling will be enhanced. So we artificially strengthened the Coulomb coupling by a factor of 10 or 100 with the same functional dependence on disk variables. Protons and electrons have factor of few higher temperatures compared to original Coulomb coupled disk. Yet, the linear analysis produce essentially the same modes. On the other hand, if $T_p = T_e$ exactly at any time, i.e., 1T disk, by some unknown process, similar linear anaysis shows two modes exist for unsaturated Compton disk: one is decaying viscous mode and the other neutral thermal mode, $\Omega = 0$. Hence, the thermal instability in 2T disk can disappear if electrons and protons are coupled by some process whose functional form is different from Coulomb one.

REFERENCES

Abramowicz, M. A., Chen, X., Kato, S., Lasota, J.-P., & Regev, O. 1994, preprint
Chen, X., Abramowicz, M. A., Lasota, J.-P., Narayan, R., & Yi, I. 1995, preprint
Chen, X., & Taam, R. E. 1993, ApJ, 412, 254
Kazanas, D., & Ellison, D. C. 1986, ApJ, 304, 178
Lightman, A. P., & Eardley, D. M. 1974, ApJ, 187, L1
Mészáros, P., & Ostriker, J. P. 1983, ApJL, 273, L59
Piran, T. 1978, ApJ, 221, 652
Pringle, J. E. 1976, MNRAS, 177, 65
Pringle, J. E., Rees, M. J., & Pacholczyk, A. G. 1973, A&A, 29, 179
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337 (SS73)
Shakura, N. I., & Sunyaev, R. A. 1976, MNRAS, 175, 613 (SS76)
Shapiro, S. L., Lightman, A. P., & Eardley, D. M. 1976, ApJ, 204, 187 (SLE)

Stepney, S. 1983, MNRAS, 202, 467
Wandel, A., & Liang, E. P. 1991, ApJ, 380, 84
White, T. R., & Lightman, A. P. 1990, ApJ, 352, 495