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$\Omega < 1$ POLAR INFLATION DRIVEN BY NEGATIVE GRAVITY

LA, DAILE
Department of Earth Science
Sejong University, Seoul, 133-747 Korea
AND
LEE, HAE SHIM
Department of Physics, Chungnam University
Daejon, 305-764 Korea
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ABSTRACT

We discuss a model 4-dimensional Friedmann cosmology which may have evolved from a model of 4+D dimensions which admits spontaneous compactification of D dimensions (or N-dimensional variants of the Brans-Dicke (BD) theory). The BD parameter appearing in dimensional reduction is negative $-1 < \omega < 0$ (for the N-dimensional variants of the BD theory, $-1.5 \leq \omega$). We find that if there had been inflationary transtion to the standard big-bang model, the Universe can undergoe a polar-type expansion during when the gravitational coupling becomes negative. The unique feature is that for the negative ω, the density parameter of the post-inflationary Universe falls in a range $0 < \Omega < 1$ even if the Universe is geometrically flat ($k = 0$).

Key Words: Polar Inflation-Higher dimensional Gravity-Negative Gravity-Ω Problem

I. INTRODUCTION

In the standard big-bang model, measuring the the average density ρ_{now} of the Universe is one of the most important task. If we set the critical density ρ_c, which separates those models that recollapse in the future from those that expand forever, the density parameter $\Omega_{now} \equiv (\rho_{now}/\rho_c)$ determines the ultimate fate of the Universe. More precisely, the quantity $\Omega_{now} > 1$ if the Universe will at some future time collapse into a second singularity, whereas if $\Omega_{now} < 1$ the Universe will expand forever. The critical density case corresponds to the ever-expanding Universe: $\Omega_{now} = 1$.

Inflation theory favors the $\Omega_{now} = 1$ Universe. What happens is that due to enormous inflationary growth of the Universe, any spatial curvature in the pre-inflationary Universe is flattened close to the euclidean one. Thus $\Omega = 1$ Universe throughout the radiation- and matter-dominated epoch is one of the most robust prediction of inflation models (Guth 1981). This result however poses a serious problem to standard big-bang model. The dilemma is that most observed data indicates $\Omega_{now} < 1$. In particular, the directly observable luminous (baryonic) matter contributes only a tiny fraction of the critical density: $\Omega_{luminous} \approx 0.005$. Dynamical estimate of the masses of galaxies, for example, rotation curves for spiral galaxies, suggests that galaxies contain more matter (may be large quantities of dark matter) which may increase the cosmological density by a factor ten (Peebles 1986). The problem is clear that even the second estimate cannot yield the average density close to unity. Also standard nucleosynthesis models which depend on many of the parameters such as neutron half-life and the number of light neutrino types, yields the present density (Maddox 1990):

$$0.015 \ < \ \Omega_{Baryonic}h^2 \ < \ 0.01.$$

(Here $0 < h^2 < 1$ is the normalized Hubble parameter.) Thus critical-density baryon-only Universe is not compatible with the observations.

Several prescriptions has been suggested. One can resort to the the famous dark matter. If such 'unseen' matter indeed exists, it does provides a merit that the presence of a 'unseen', non-baryonic dark matter can also enhance the efficiency of mass clumping. Therefore dark matter helps the structure formation of the Universe. However, recent APM survey reports that the presence of the dark matter is not enough. There are cosmic structure greater than $\sim 30Mpc$ where even successful cold dark matter (CDM) model cannot accomodate (Maddox *et al.* 1990). One can conjecture the possible existence of a cosmological constant, known as the Λ-term, in the Universe. But unfortunatley, we hardly understand the nature of the cosmological constant. If a non-zero cosmological constant Λ does exist in the Universe, we should be able to answer why the constant takes a value just right enough to explain the puzzle. Considering these difficulties, in this paper, we investigated whether there exists a different way to resolve the Ω-problem. We report that compactified higher-dimensional gravitation theories can provide a solution to the problem.

II. THE OMEGA PROBLEM IN THE SCALAR-TENSOR GRAVITY

Let us consider the BD-like Universe, which has the negative BD parameter ω. Conventinal BD theory is characterized by the positive ω (Brans and Dicke 1961). The negative ω feature naturally arise in compactified 4-dimensional gravitation theories as well as N-dimensional variants of the BD theory (Freund 1987). Thus in this paper, we will consider specifically a 4-dimensional Friedmann cosmology which may evolve from a model-gravity in 4+D dimensions. The BD fields are known to appear in dimensional reduction. When extra dimensions are compactified into the 4-dimension, the square root of the determinant of the metric in compactified manifold function as a BD field. In this case, the BD parameter:

$$\omega \; = \; -1 + \frac{1}{D}.$$

Since there is no observatinal constraint for the negative ω, it is the weak energy condition which constrains the ω (Hawking and Ellis 1973):

$$-1 < \omega < 0$$

Also for higher dimensional variants of the BD theory, the lower-limit of the omega is constrained as $\omega \geq -1.5$ (Freund 1987). In this work, we will follow both cases, *i.e.*, for the case when

$$-\, 1.5 \; \leq \; \omega \; < \; 0.$$

Now we consider the conventional big-bang models with zero cosmological constant and vanishing dark matter. What will be shown is that in the negative ω BD Universe, the density parameter of the Universe can be fixed uniquely by ω, thereby resolving the Ω-problem. To show this, let us write the 4-dimensional action of the higher-dimensional theories

$$\mathcal{A}(g_{\mu\nu}, \Phi) = \int d^4x\sqrt{-g}\; \left\{ -\Phi\mathcal{R} - \omega\frac{\partial_\mu\Phi\partial^\mu\Phi}{\Phi} - W(\Phi) - \mathcal{L}_{matter}(\sigma) \right\}.$$

Here, $g_{\mu\nu}$ is the metric tensor, and greek indices μ, ν runs from 0 to 3. The quantity g is the determinant of $g_{\mu\nu}$; $M_p = 10^{19}GeV$ is the Planck mass; $\mathcal{R}$ is the Ricci scalar; and Φ is the BD scalar field.

When $W(\Phi) = 0$, the action becomes identical to that of original BD universe, except we are assuming the negative BD parameter: $\omega = -|\omega|$. We are reminded that in the BD Universe, the gravity $G_N = G(t)$ varies monotonically and the BD parameter which is constrained by solar system experiments: $\omega \geq 500$ (Reasenberg 1979). In our case, however, there is no experimental constraint for the negative ω. Thus it is the weak energy condition which constrains ω. In this respect, the inclusion of a non-trivial dilatonic potential $W(\Phi)$ is optional. If an experiment reveals the *negative* ω beyond the range given by the weak energy condition, then one can suppress the variation of the gravitational 'constant' via $W(\Phi)$. As Φ reaches its local minima sometimes during post-inflationary epoch, the theory reduces to conventional FRW Universe in a background of Einstein gravity. In fact, we are reminded that pure BD theory (*i.e.*, $W(\Phi) = 0$) does not seem natural from particle physics point of view. For scale-invariant theories or supersymmetric models, conformal invariance or supersymmetry can be spontaneously broken. In this way, the BD field (gravity) gets its energy scale at low energies (Green Schwarz and Witten 1987). Thus in this work, we will either neglect $W(\Phi)$, or otherwise assume that

$$W(\Phi) \ll -<0|\mathcal{L}_{matter}(\sigma)|0>.$$

In this case the action becomes identical to induced gravity except the negative non-minimal coupling constant $\xi \equiv -(1/|\omega|)$.

Thus far the action takes the same form as extended inflation model (La and Steinhardt 1989). The difference seem quite minor. Only the sign of matter-gravity coupling constant has been changed: $\omega < 0$. But it will be shown that this change yields a dramatic change in inflation scenario. First of all, during inflation, the gravity becomes negative, and there is a polar inflation. Finally, the magnitude of the negative ω as constrained by the weak energy condition fixes the density parameter of 4-dimensinal Friedmann Universe.

Now we proceed to the main issue. In the Friedmann-Robertson-Walker (FRW) universe, where line element $ds^2 = -(dt)^2 - R(t)^2(\frac{(dr)^2}{1-kr^2} + r^2 d\Sigma^2)$ and the matter energy-momentum tensor represented in a perfect fluid form $T_{\mu\mu} = p_\sigma g_{\mu\nu} + (\rho_\sigma + p_\sigma)U_\mu U_\nu$, the equations of motion for the cosmic scale factor R and the BD field, Φ-field is

$$H^2 = \frac{8\phi\rho_\sigma}{3\Phi} - \frac{k}{R^2} + \frac{\omega}{6}\left(\frac{\dot{\Phi}}{\Phi}\right)^2 - H\left(\frac{\dot{\Phi}}{\Phi}\right),$$

$$\dot{\rho}_\sigma = -3H\,(\rho_\sigma + p_\sigma),$$

$$\ddot{\Phi} + 3H\dot{\Phi} = \frac{8\pi[\rho_\sigma(t) - 3p_\sigma(t)]}{3+2\omega}.$$

Here, dot represents the derivative with respect to cosmic time; $H = \dot{R}/R$ is the Hubble parameter, and $k \pm 1, 0$ represents the 3-space curvature $r^2 d\Sigma$. The quantities $\rho_\sigma(t)$, $p_\sigma(t)$ are energy density and pressure of cosmic matter, respectively.

For $\omega > 0$, as the universe supercools in the false phase, the energy density approaches a constant value so that $\rho_\sigma \to \rho_F$. This acts as an effective cosmological constant. Then $-p_\sigma = +\rho_F$, and in this case (for $k = 0$),

$$\Phi = m_p^2\left(1 + \frac{H_B t}{\alpha}\right)^2,$$

$$R(t) = \left(1 + \frac{H_B t}{\alpha}\right)^{\omega+\frac{1}{2}}.$$

We are familiar with this solution (La and Steinhardt 1989). Here, H_B is the Hubble parameter at the beginning of inflation, and $\alpha \equiv (3+2\omega)(5+6\omega)/12$. The quantity m_p is an arbitrary constant corresponding to the effective Planck mass at the beginning of inflation, $t = 0$. The (Planck mass)^2 today is the inverse of the Newtonian gravitational constant $(1/G_N)$.

This solution, which is a basis of extended inflation models, describes the inflationary expansion ($\ddot{R} > 0$) of the Universe. The nature of the power-law expansion, in particular, enables the Universe exit 'gracefully' from false phase via bubble nucleation process. The density parameter of the post-inflationary Universe is

$$\Omega_{total} \equiv \frac{8\pi\rho_\sigma}{3\Phi H^2} = 1 + \frac{6\omega + 4}{3(2\omega+1)^2}.$$

Thus for $\omega > 500$ as constrained by solar-system experiments, (or $1.5 < \omega \leq 20$ as allowed in the EI model), Ω_{total} deviate from unity insignificantly. Also Ω_{total} is greater than one. This is a disappointing result. Of course, we well-aware why this happens. The main culplit is a rather weak coupling of the BD field with matter fields.

III. POLAR INFLATION

For $\omega < 0$, we must seek a new type of inflation solution. Above pair of solution is useless in that there is no inflation for the negative ω: $\ddot{R} < 1$. Thus it is necessary to seek a new pair of inflation solution. Now then, could a new type of inflationary solution exist? There is a clue, fortunately. We are reminded that it was the *time-increasing* nature of the BD field (or time-decreasing gravitational coupling) which yields the inflationary power-law solution. Then, what happens if we consider a time-decreasing gravitational coupling plus when its sign is negative. In this case, gravity is still time-decreasing. Thus one finds that there indeed exist an inflation solution for $-p_\sigma = +\rho_F = constant$:

$$\Phi(t) = -\mathcal{A}(1-\chi t)^2$$
$$R(t) = (1-\chi t)^{-1}.$$

Here

$$\mathcal{A} \equiv \frac{8\pi\rho_F}{\chi^2\,(3+2\omega)},$$

and

$$\chi = \sqrt{\frac{8\pi\rho_F}{\beta_p^2(3+2\omega)}}.$$

Here we set $\Phi(0) = -\beta_p^2$, where β_p^2 is the Planck mass at the onset of inflation. During inflation, β_p^2 decreases until $\beta_p^2 \to M_p^2$. Thus inflation terminates when $\Phi \to -M_p^2$. In the process, in order that χ remain a real quantity, we need a non-trivial condition

$$\omega > -1.5,$$

which changes the lower-limit of ω as $-1.5 < \omega < 0$.

Even with the negative gravity, we find that there exists a polar-type inflationary expansion for any $0 < \chi t < 1$. (Here, $\chi t = 0, \chi \to 1$ refers to the moment when the Universe enters to, and exit from the inflationary epoch, respectively.) We are reminded that the negative gravity occurs strictly during inflation.

The polar inflation is much faster than conventional exponential or power-law expansion. Thus the polar inflation will not likely to be terminated via bubble nucleation and percolation processes. This feature comes from the observation that the Hubble parameter during the polar inflation increase as $\propto (1-\chi t)^{-1}$. Thus bubble nucleation parameter $\epsilon \propto H^{-4}$ which determines whether the Universe can percolate or not, will be decreasing, makes it harder for the Universe to percolate (Guth and Weinberg 1982; La , Steinhardt and Bertschinger 1989). Therefore, in this scenario, the Universe will exit from the inflationary epoch via a second-order phase-transition of the inflaton field.

IV. THE OMEGA PROBLEM

The most interesting feature emerging is that the density parameter of the post-inflationary Universe is always less than unity. This is the unique featue of the model. As the polar inflation ends, the Universe becomes radiation-dominated. In epochs following, the Φ-field is: (1) fixed to a constant (*i.e.* to Newtonian value) when the Φ-field reaches local minima of $W(\Phi)$, or (2) varies monotonically when $W(\Phi) \neq 0$. For the former case, the Φ-field can further oscillate with respect to its minima which may produces several astrophysically instresting effects (Steinhardt and Will 1994). For the latter case, the Φ-field will grow like conventional BD gravity, but with different speed. (The range of negative ω experimentally acceptable will remain as an open question at the moment.)

For the density of matter in the Universe, for $\Phi(t) = -\mathcal{A}(1-\chi t)^2$ and $R(t) = (1-\chi t)^{-1}$, the density parameter $\Omega_{total} \equiv \frac{8\pi\rho}{3M_p^2H^2}$ becomes

$$\Omega_{total} = 1 - \frac{2|\omega|}{3}.$$

Therefore, for $-1 < \omega < 0$,

$$\frac{1}{3} < \Omega_{total} < 1,$$

and for $-1.5 < \omega < 0$,

$$0 < \Omega_{total} < 1.$$

Interestingly, Ω_{total} of 4-dimensional Universe is fixed by the number of compactified higher-dimensions. Depending on $|\omega|$, *i.e.,* the number of compactified higher-dimensions, the density parmeter Ω can be made quite small. For example, if the 4-dimensional Friedmann Universe is evolved from 11-dimension, then $D = 7$ so that the density parameter $\Omega = (3/7) \approx 0.43$. In this way, we find the average density of the baryonic matter can be arbitrary small even if the Universe is geometrically flat ($k = 0$).

V. CONCLUSION

We considered 4-dimensional BD cosmology characterized by negative BD parameter. We find that as the Universe enters the inflationary epoch, gravity becomes negative, and there is a polar-type inflation. The polar inflation will be terminated via continuous second-order phase transion. It is shown that the 4-dimensional density parameter is fixed by the negative ω (or number of compactified dimension). For $-1.5 < \omega < 0$, $0 < \Omega_{total} < 1$.

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