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Determination of the Optimal Target Values for a Canning Process with Linear Shift in the Mean

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Abstract

The problem of selecting the optimal target values in a canning process is considered for situations where there is a linear shift in the mean of the content of a can which is assumed to be normally distributed with known variance. The target values are initial process mean, length of resetting cycle and controllable upper limit. Profit models are constructed which involve give-away, rework, and resetting costs. Methods of finding the optimal target values are presented and a nemerical example is given.

1. Introduction

For an industrial process in which items are produced continuously, suppose there is a lower specification limit L for a quality characteristic X such that items with X 〈 L are rejected(for example, to be reprocessed or sold at a discount). A process parameter T = L + △ for the mean is to be selected so that the expected net profit per item is maximized. The general problem considered is to develop a procedure that takes process variability and production costs into account for determining the optimal value of △ and hence T.

This problem has been studied under various conditions. Bettes [3] treated the problem of simultaneously selecting an optimal process mean and a controllable upper limit U where items with X 〈 L or X 〉 U are reprocessed at a fixed cost. Hunter and Kartha [11]

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considered the problem of selecting optimal process mean with the assumption that items with X $\geq$ L are sold at the regular price and items with X $<$ L are sold at a reduced price. Bisgaard et al. [4] extended the work of Hunter and Kartha [11] to a situation where items with X $<$ L are sold at the price proportional to the amount of ingredient used. Carlsson [6] studied the problem of determining the optimal process mean by maximizing the expected net profit which is a piecewise linear function of X. Glohar [9] considered the problem of selecting optimal process mean in a canning process; cans filled above L are sold at a fixed price and underweight cans are emptied and refilled at the expense of a reprocessing cost. Golhar and Pollock [10] extended the work of Golhar [9] to the case where a controllable upper limit U is also present; underfilled (X $<$ L) and overfilled (X $>$ U) cans are emptied and refilled. Carlsson [7] studied the case of acceptance sampling where the reject criterion was based on the sampling mean. Riew [12] considered the problem of selecting the optimal specification limits by minimizing the total expected cost for a given target value. Boucher and Jafari [5] studied the problem of determining the optimal process mean and rejection criterion which is based on the number of nonconforming items in the sample by maximizing the expected profit when a sampling plan is used. Arcelus and Rahim [2] Considered the problem of determining simultaneously the target values for attribute and variable quality characteristics by maximizing the expected profit per item. In all these studies the quality characteristic X is assumed to be normally distributed with known variance and unknown process mean which is treated as constant over time.

In many cases, however, the quality characteristic is subject to a systematic shift in its mean level. Such a shift may be found in tool wear in machining, drawing, stamping and moulding operations and automatic filling machine, and it makes the process quality level to deterioarate over time. Arcelus and Banerjee [1] extended the work of Bisgaard et al. [4] to the case where there is a linear shift in the mean. Drezner and Wesolowsky [8] considered the problem of finding optimum initial process mean and length of resetting cycle when there is a linear shift in the mean for the case of a quadratic loss function that is symmetrical about the target value.

In this paper we extend the canning problem of Golhar [9] and Golhar and Pollock [10] to the case where there is a linear shift in the mean of the content of a can. The target values to be optimized are : i) initial process mean and length of resetting cycle in Golhar medel, ii) initial process mean and controllable upper limit when length of resetting cycle is fixed, and iii) initial process mean, controllable upper limit and length of resetting cycle in the Golhar - Pollock model. It is assumed that the content of a can is normally distributed with linearly increasing mean over time and known constant variance and has a lower specification limit. Profit models are constructed which involve give-away, rework, and resetting costs and methods of finding optimal target values are presented and a numerical example is given.

The following notation is used.

Notation

X_t weight of the fill of a can at time t

μ_0 initial process mean

σ^2 variance of X_t

θ drift in the process mean per unit time, $\theta \geq 0$

L lower specification limit

U controllable upper limit

a selling price per can in a regular market

c cost of content per unit weight

r refilling cost per can

d resetting cost

τ length of resetting cycle

$\phi(\cdot)$, $\Phi(\cdot)$ density and distribution function of the standard normal distribution

2. The Model

Let X_t be a random variable representing the weight of the fill of a can at time t. It is assumed that X_t is normally distributed with mean $\mu_t = \mu_0 + \theta t$ and variance σ^2. Let P_t be the profit function at time t for a can filled with content X_t. All cans are inspected and $L \leq X_t \leq U$ the can is sold for a and the profit is $a - cX_t$. On the other hand, if $X_t \langle L$ or $X_t \rangle U$ it is emptied. For the model simplification, we assume that an emptied can at time t is refilled at time $t+\tau$ at a cost of r. This refilled can will then realize an expected profit $E(P_t)$. Hence, for the reprocessed can the expected net profit is $E(P_t) - r$. The profit function at time t per can is therefore

$$P_t = \begin{cases} a - cX_t, & L \leq X_t \leq U \\ E(P_t) - r, & \text{otherwise.} \end{cases} \tag{1}$$

Using the relation

$$\int_L^U \frac{x_t}{\sqrt{2\pi}\sigma} e^{-(X_t - \mu_t)^2/2\sigma^2} dx_t$$

$$= \mu_t \left[\Phi\left(\delta_2 - \delta_1 - \frac{\theta t}{\sigma}\right) + \Phi\left(\sigma_1 + \frac{\theta t}{\sigma}\right) - 1 \right]$$

$$- \sigma \left[\phi\left(\delta_2 - \delta_1 - \frac{\theta t}{\sigma}\right) \right.$$

$$\left. - \phi\left(\delta_1 + \frac{\theta t}{\sigma}\right) \right], \tag{2}$$

where $\delta_1 = (\mu_0 - L)/\sigma$ and $\delta_2 = (U - L)/\sigma$, the expected profit per can at time t can be written as

$$E(P_t) = a + r - c\sigma\delta_1 - cL - c\theta t$$

$$+ \frac{c\sigma[\Phi(\delta_2 - \delta_1 - \theta t/\sigma) - \phi(\delta_1 + \theta t/\sigma)] - r}{\Phi(\delta_2 - \delta_1 - \theta t/\sigma) + \Phi(\delta_1 + \theta t/\sigma) - 1} \tag{3}$$

Note that with no drift in the process mean, i.e., $\theta = 0$, formula(3) becomes

$$E(P_t \mid \theta = 0) = a + r - c\sigma\delta_1 - cL$$

$$+ \frac{c\sigma[\Phi(\delta_2 - \delta_1) - \phi(\delta_1 + \theta t/\sigma)] - r}{\Phi(\delta_2 - \delta_1 - \theta t/\sigma) + \Phi(\delta_1 + \theta t/\sigma) - 1},$$

which is the same as the one obtained by Golhar and Pollack[10].

Suppose that process is reset at a cost d for every τ and resetting time is negligible and rate of production is constant. Then the expected profit per unit time is

$$P(\delta_1,\delta_2,\eta)=\frac{1}{\tau}\int_0^{\tau}E(P_t)dt-\frac{d}{\tau}$$

$$=a+r-c\sigma\delta_1-cL-\frac{c\sigma\eta}{2}$$

$$-\frac{c\sigma}{\eta}\log\left\{\frac{\Phi(\delta_2-\delta_1-\eta)+\Phi(\delta+\eta)-1}{\Phi(\delta_2-\delta_1)+\Phi(\delta_1)-1}\right\}$$

$$-\frac{r}{\eta}\int_{\delta_1}^{\delta_1+\eta}\left\{\frac{1}{\Phi(\delta_2-z)+\Phi(z)-1}\right\}dz-\frac{d\theta}{\eta\sigma}$$

where $\eta=\tau\theta/\sigma$.

3. Optimal Solutions

In this section, methods of finding the optimal target values will be given for three cases; i) determination of μ_0 and τ, ii) determination of μ_0 and U with fixed τ, and iii) determination of μ_0, U and τ.

Case i

There is only a lower specification limit, and and initial process mean and length of resetting cycle are to be selected. If $X_t \geq L$, the can is sold for a, and if $X_t < L$, the can is emptied and refilled at a cost of r. Hence, the expected profit per unit time $P_1(\delta_1,\eta)$ is, by letting $\delta_2=\infty$ in(4),

$$P_1(\delta_1,\eta)=a+r-c\sigma\delta_1-cL-\frac{c\sigma\eta}{2}-\frac{d\theta}{\eta\sigma}-\frac{c\sigma}{\eta}$$

$$[\log\Phi(\delta_1+\eta)-\log\Phi(\delta_1)]$$

$$-\frac{r}{\eta}\int_{\delta_1}^{\delta_1+\eta}\frac{1}{\Phi(z)}dz. \qquad (5)$$

Equating the first derivatives of $P_1(\delta_1, \eta)$ with respect to δ_1 and η to zero yields

$$M[\Phi(\delta_1+\eta)-\Phi(\delta_1)]$$
$$+[\Phi(\delta_1+\eta)\phi(\delta_1)-\Phi(\delta_1)\phi(\delta_1+\eta)]$$
$$=\eta\Phi(\delta_1+\eta)\Phi(\delta_1), \qquad (6)$$

and

$$\frac{1}{\Phi(\delta_1+\eta)}\left\{\frac{\eta}{2}\Phi(\delta_1+\eta)+\phi(\delta_1+\eta)+M\right\}$$
$$=\frac{1}{\eta}[\log\Phi(\delta_1+\eta)-\log\Phi(\delta_1)+K+M$$
$$\int_{\delta_1}^{\delta_1+\eta}\frac{1}{\Phi(z)}dz],$$

where $M=\frac{r}{c\sigma}$ and $K=\frac{d\theta}{c\sigma^2}$

It is difficult to show analytically that equations (6) and (7) have a unique solution, or $P_1(\delta_1, \eta)$ is a unimodal function of δ_1 and η. Numerical Procedure study over wide ranges of M($0\leq M\leq 6.0$) and K ($0.005\leq K\leq 5.0$), however, indicates that it is indeed unimodal. Hence, the optimal values δ_1^* and η^* can be obtained by solving equations (6) and (7) simultaneously. No closed form solutions for equations (6) and (7) can be obtained and a numerical such as Gauss-Seidel's iterative method can be used to obtain δ_1^* and η^*. Values of (δ_1^*, η^*) for selected combinations of M and K are shown in Table 1. The optimal process mean μ_0^* and length of resetting cycle τ^* are then obtained by

$$\mu_0^* = L + \delta_1^* \sigma, \quad (8)$$

$$\tau^* = \eta^* \sigma / \theta \cdot \quad (9)$$

Case ii

In the above case, an overfilled can is sold at a fixed price. In situations where the content is expensive or the excess is too much, however, it may be more profitable to reprocess the overfilled cans. Therefore, a controllable upper limit U is considered to reprocess the overfilled cans weighing above this limit. The process is reset for every τ which is fixed. The expected profit per unit time $P_2(\delta_1, \delta_2)$ is the same as formula (4) with η fixed, and δ_1 and δ_2 to be optimally determined. Equating the first derivatives of $P_2(\delta_1, \delta_2)$ with respect to δ_1 and δ_2 to zero yields

Table 1. Values of δ^*_1 and η^* for Selected Combinations of M and K.

M	K									
	0.05		0.1		0.5		1.0		2.0	
	δ_1^*	η^*	δ_1^*	η^*	δ_1^*	η^*	δ_1^*	η^*	δ_1^*	η^*
					-1.174					
0.1	-.794	.956	-.889	1.201	-.859	2.031	-1.323	2.538	-1.484	3.169
0.2	-.476	.914	-.571	1.150	-.664	1.951	-1.011	2.446	-1.178	3.069
0.3	-.282	.890	-.376	1.119	-.521	1.903	-.818	2.391	-.988	3.012
0.4	-.141	.872	-.234	1.097	-.408	1.870	-.676	2.354	-.848	2.972
0.5	-.030	.858	-.123	1.080	-.315	1.844	-.563	2.324	-.736	2.941
0.6	.062	.847	-.030	1.066	-.234	1.823	-.470	2.301	-.643	2.916
0.7	.140	.837	.049	1.054	-.164	1.805	-.389	2.280	-.563	2.895
0.8	.208	.829	.118	1.044	-.102	1.789	-.319	2.263	-.492	2.877
0.9	.268	.821	.179	1.035	-.046	1.776	-.257	2.248	-.430	2.861
1.0	.323	.815	.233	1.027	-.102	1.764	-.200	2.234	-.374	2.847
1.2	.417	.803	.328	1.013	-.046	1.743	-.102	2.211	-.275	2.823
1.4	.496	.794	.409	1.002	.051	1.725	-.108	2.192	-.191	2.803
1.6	.566	.786	.479	.992	.134	1.711	.055	2.175	-.117	2.786
1.8	.627	.779	.541	.983	.207	1.697	.119	2.160	-.052	2.771
2.0	.681	.773	.569	.975	.270	1.686	.177	2.147	.007	2.757
2.5	.796	.760	.713	.959	.328	1.662	.301	2.121	.132	2.729
3.0	.791	.749	.808	.947	.449	1.643	.401	2.099	.234	2.707
3.5	.890	.741	.888	.936	.547	1.627	.486	2.081	.321	2.688
4.0	1.037	.734	.956	.927	.630	1.613	.560	2.066	.396	2.673
5.0	1.149	.722	1.070	.913	.702	1.591	.624	2.053	.521	2.647
					.821					

$$\frac{\phi(\delta_2-\delta_1-\eta)-\phi(\delta_1+\eta)-\mathrm{M}}{\Phi(\delta_2-\delta_1-\eta)+\Phi(\delta_1+\eta)-1} - \frac{\phi(\delta_2-\delta_1)-\phi(\delta_1)-\mathrm{M}}{\Phi(\delta_2-\delta_1)+\Phi(\delta_1)-1}=\eta, \tag{10}$$

and

$$\left(\frac{\phi(\delta_2-\delta_1-\eta)}{\Phi(\delta_2-\delta_1-\eta)+\Phi(\delta_1+\eta)-1} - \frac{\phi(\delta_2-\delta_1)}{\Phi(\delta_2-\delta_1)+\Phi(\delta_1)-1}\right) = \mathrm{M}\int_{\delta_1}^{\delta_1+\eta}\frac{\phi(\delta_2-z)}{[\Phi(\delta_2-z)+\Phi(z)-1]^2}dz. \tag{11}$$

The optimal values δ_1^* and δ_2^* can be obtained by solving equations (10) and (11) simultaneously. As in the previous case, no closed form solutions for equations (10) and (11) can be obtained and δ_1^* and δ_2^* can be found by a numerical search method. Values of (δ_1^*, δ_2^*) for selected conbinations of M and η are shown in Table 2. The optimal process mean μ_0^* is obtained by formula (8) and the optimal controllable upper limit U* by

$$U^*=L+\delta_2^*\sigma. \tag{12}$$

Table 2. Values of δ^*_1 and δ^*_2 for Selected Combinations of M and η.

M	η									
	1.0		1.5		2.0		1.0		3.0	
	δ_1^*	δ^*_2	δ_1^*	δ^*_2	δ_1^*	δ^*_2	δ_1^*	δ^*_2	δ_1^*	δ^*_2
0.1	-.249	.728	-.482	.748	-.707	.780	-.925	.825	-1.134	.891
0.2	-.145	1.037	-.371	1.066	-.587	1.111	-.792	1.178	-.984	1.275
0.3	-.066	1.279	-.286	1.314	-.495	1.371	-.691	1.455	-.871	1.578
0.4	.001	1.487	-.215	1.528	-.418	1.594	-.607	1.694	-.778	1.839
0.5	.060	1.674	-.153	1.721	-.351	1.796	-.534	1.909	-.697	2.074
0.6	.113	1.846	-.097	1.898	-.291	1.982	-.469	2.108	-626	2.291
0.7	.162	2.009	-.046	2.065	-.237	2.156	-.410	2.295	-.562	2.494
0.8	.207	2.163	.001	2.223	-.187	2.323	-.356	2.473	-.504	2.686
0.9	.249	2.310	.045	2.375	-.140	2.482	-.306	2.643	-.451	2.869
1.0	.289	2.453	.086	2.522	-.097	2.636	-.260	2.807	-.402	3.045
1.2	.361	2.726	.161	2.803	-.018	2.930	-.177	3.120	-.314	3.379
1.4	.426	2.987	.229	3.072	.052	3.212	-.103	3.420	-.237	3.695
1.6	.486	3.240	.290	3.332	.116	3.484	-.037	3.706	-.169	3.996
1.8	.540	3.486	.345	3.585	.173	3.749	.022	3.985	-.108	4.286
2.0	.589	3.728	.396	3.833	.225	4.008	.076	4.255	-.052	4.567
2.5	.697	4.316	.506	4.483	.338	4.637	.192	4.908	.066	5.239
3.0	.786	4.890	.597	5.027	.431	5.245	.288	5.535	.164	5.878
3.5	.862	5.454	.674	5.605	.511	5.839	.369	6.142	.247	6.494
4.0	.927	6.011	.741	6.174	.579	6.421	.440	6.734	.319	7.093
5.0	1.036	7.108	.852	7.291	.694	7.557	.557	7.855	.440	8.253

Case iii

In this case, and initial process mean, controllable upper limit and length of resetting cycle are simultaneously to be selected. The expected profit per unit time $P_3(\delta_1, \delta_2, \eta)$ is the same as formula (4) with all of δ_1, δ_2 and η to be optimally determined. Equating the first derivatives of $P_3(\delta_1, \delta_2, \eta)$ with respect to δ_1, δ_2 and η to zero yields formulas (10) and (11) and

$$\frac{\phi(\delta_1+\eta)-\phi(\delta_2-\delta_1-\eta)+M}{\Phi(\delta_2-\delta_1-\eta)+\Phi(\delta_1+\eta)-1}$$
$$-\frac{1}{\eta}[K+\log\Phi(\delta_2-\delta_1-\eta)+\Phi(\delta_1+\eta)-1$$
$$-\log[\Phi(\delta_2-\delta_1)+\Phi(\delta_1)-1]+\frac{\eta}{2}$$
$$=\frac{M}{\eta}\int_{\delta_1}^{\delta_1+\eta}\frac{1}{\Phi(\delta_2-z)+\Phi(z)-1}dz. \qquad (13)$$

As in the previous cases, no closed form solutions for equations (10)−(11) and (13) can be obtained and a numerical search method

Table 3. Values of δ^*_1, δ^*_2 and η^* for Selected Combinations of M and K.

M	K														
	0.05			0.1			0.5			1.0			2.0		
	δ^*_1	δ^*_2	η^*	δ^*_1	δ^*_2	η^*	δ^*_1	δ^*_2	η^*	δ^*_1	δ^*_2	η^*	δ^*_1	δ^*_2	η^*
0.1	-.328	.734	1.168	-.461	.746	1.454	-.862	.810	2.353	-1.072	.869	2.849	-1.298	.967	3.414
0.2	-.168	1.039	1.050	-.287	1.053	1.312	-.650	1.128	2.152	-.844	1.200	2.632	-1.055	1.323	3.195
0.3	-.061	1.278	.989	-.170	1.293	1.235	-.513	1.377	2.046	-.697	1.458	2.518	-.899	1.602	3.083
0.4	.025	1.484	.947	-.080	1.500	1.185	-.408	1.590	1.976	-.586	1.680	2.443	-.782	1.843	3.012
0.5	.097	1.668	.917	-.005	1.685	1.150	-.322	1.782	1.924	-.494	1.880	2.389	-.685	2.060	2.964
0.6	.160	1.839	.894	.061	1.857	1.122	-.248	1.959	1.884	-.416	2.064	2.348	-.604	2.260	2.927
0.7	.216	1.999	.874	.119	2.018	1.099	-.182	2.125	1.853	-.348	2.238	2.316	-.533	2.448	2.899
0.8	.268	2.151	.859	.173	2.170	1.080	-.124	2.283	1.827	-.287	2.403	2.290	-.469	2.627	2.877
0.9	.315	2.297	.846	.221	2.317	1.064	-.070	2.435	1.805	-.231	2.560	2.268	-.412	2.798	2.859
1.0	.359	2.438	.835	.267	2.458	1.051	-.022	2.581	1.788	-.181	2.713	2.249	-.360	2.963	2.844
1.2	.439	2.708	.817	.349	2.729	1.029	.066	2.861	1.758	-.090	3.005	2.219	-.266	3.277	2.819
1.4	.510	2.966	.802	.421	2.989	1.012	.143	3.130	1.734	-.012	3.285	2.196	-.186	3.576	2.799
1.6	.573	3.216	.792	.486	3.240	.999	.211	3.390	1.716	.059	3.555	2.178	-.114	3.862	2.783
1.8	.631	3.460	.783	.545	3.485	.988	.273	3.643	1.702	.122	3.817	2.162	-.050	4.139	2.769
2.0	.684	3.699	.774	.598	3.725	.978	.329	3.891	1.688	.179	4.074	2.149	.008	4.408	2.757
2.5	.797	4.282	.761	.713	4.310	.961	.448	4.495	1.663	.301	4.696	2.121	.132	5.054	2.730
3.0	.890	4.850	.750	.808	4.880	.947	.547	5.082	1.643	.401	5.297	2.099	.234	5.671	2.707
3.5	.969	5.408	.742	.887	5.441	.936	.630	5.657	1.627	.486	5.884	2.082	.321	6.269	2.688
4.0	1.036	5.960	.734	.956	5.995	.928	.702	6.223	1.614	.559	6.459	2.067	.396	6.854	2.673
4.5	1.096	6.506	.728	1.016	6.543	.919	.765	6.782	1.602	.624	7.025	2.054	.462	7.426	2.659
5.0	1.148	7.047	.722	1.070	7.086	.913	.821	7.335	1.592	.681	7.583	2.041	.521	7.990	2.647

can be used to obtain δ_1^*, δ_2^* and η^*. Values of (δ_1^*, δ_2^*, η^*) for selected combinations of M and K are shown in Table 3. The optimal process mean μ_0^*, length of resetting cycle τ^* and controllable upper limit U^* are then obtained by formulas (8)-(9) and (12), respectively.

4. Numerical Example

Let L=10 Kg and a= ₩17,000, r=₩700, c=₩1,400 per kg, and σ=0.35 kg. Suppose that θ is 0.001σ per unit time and d= ₩50,000. Then the corresponding constants are M≃1,4 and K≃0,1.

For case i), we obtain δ_1^*=0.409 and η^*=1.002 from Table 1. Hence,

$$\mu_0^* = L + \delta_1^*\sigma = 10 + 0.409 \times 0.35 = 10.143\text{kg},$$
$$\tau^* = \eta^*\sigma/\theta = 1.002/0.001 = 1{,}002 \text{ unit times.}$$

For case ii), suppose that τ is 2,000 unit times. Then δ_1^*=0.052 and δ_2^*=3.212 from Table 2. Hence,

$$\mu_0^* = L + \delta_1^*\sigma = 10 + 0.052 \times 0.35 = 10.018\text{kg},$$
$$U^* = L + \delta_2^*\sigma = 10 + 3.212 \times 0.35 = 11.124\text{kg}.$$

For case iii), δ_1^*=0.421, δ_2^*=2.989, η^*=1.012 from Table 3. Hence,

$$\mu_0^* = L + \delta_1^*\sigma = 10 + 0.421 \times 0.35 = 10.147\text{kg},$$

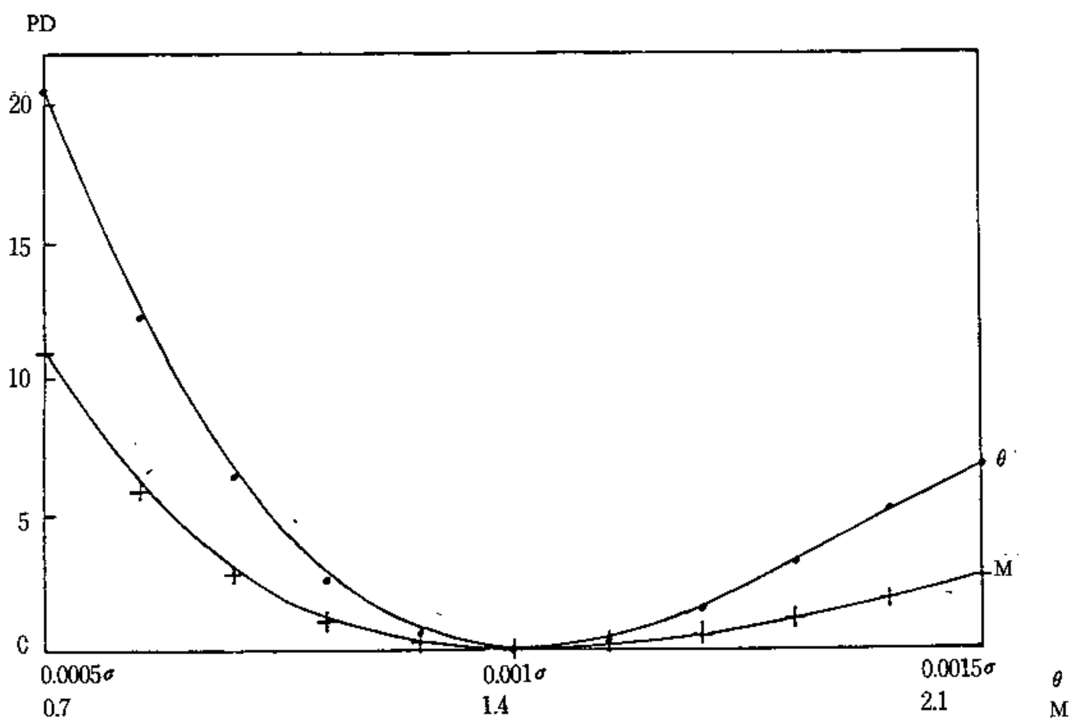

Figure 1. Percentage Decrease of Expected Profit as a Function of Mand θ in Case iii.

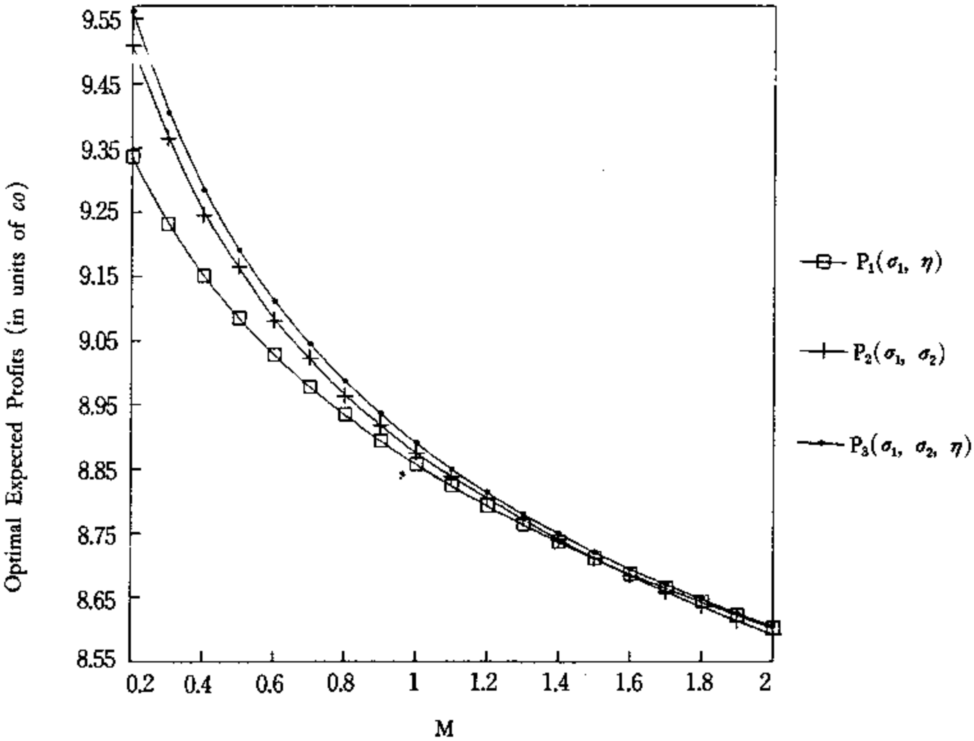

Figure 2. Optimal Expected Profits as a Function of M (K=0.5)

$U^* = L + \delta_2^* \sigma = 10 + 2.989 \times 0.35 = 11.046$kg,
$\gamma^* = \eta^* \sigma / \theta = 1.012/0.001 = 1{,}012$ unit times.

For case iii), Figure 1 shows how sensitive the expected profit is to the use of incorrect values of M and $\theta = \eta\sigma/\tau$ in the above example in terms of the percentage decrease (PD) which is defined as

$$PD = \frac{P_3(\delta_1^*, \delta_2^*, \eta^*) - P_3(\delta'_1, \delta'_2, \eta')}{P_3(\delta_1^*, \delta_2^*, \eta^*)} \times 100(\%),$$

where

$P_3(\delta_1^*, \delta_2^*, \eta^*)$ = the optimal expected profit using correct values of M and θ; in this example M = 1.4 and $\theta = 0.001\sigma$.

$P_3(\delta'_1, \delta'_2, \eta')$ = the optimal expected profit using incorrect values of M and θ.

Figure 1 indicates that the expected profit is more sensitive to θ than M. Similar results can also be obtained for cases i) and ii).

For the three cases, Figures 2 and 3 give the expected profits as a function of M and K. They show that the optimal expected profit of case iii) is somewhat larger than those of cases i) and ii), and the optimal expected profits tend to decrease as M and K increase.

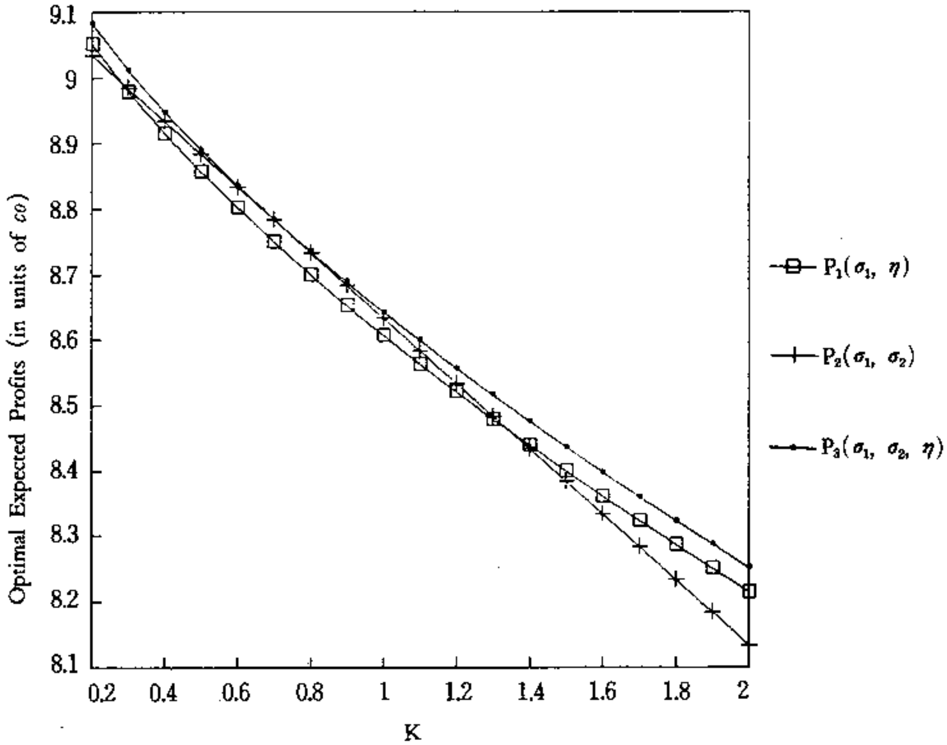

Figure 3. Optimal Expected Profits as a Function of K (M=1.0)

5. Concluding Remarks

The paper considers the problem of selecting the optimal target values for a canning process in which there is a linear shift in the mean value of content of a can. Profit models are constructed and methods of finding the optimal target values are presented. Empirical results indicate that the case with upper limit is somewhat more profitable than the case with no upper limit and that the expected profit tends to decrease as M increases and that the expected profit is more sensitive to θ than M. In this study, the variance of the quality characteristic is assumed to be known and fixed. It will be of interest to consider the case in which there are shifts in both mean and variance of the quality characteristic.

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