▩ 연구논문

Test of Fit for the Log-Normalness of the Post Dialling Delay

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Abstract

Delay has influenced customer satisfaction while using a telephone. It has been ranked second after transmission impairments in annoyance to customers. Therefore, it is important to analyze its behavior not only to increase the quality of service for the customers, but to provide its information to network planners and administrators and to designers of equipment and systems.

In this paper, we consider a log-normal model as a candidate for describing the statistical behavior of the post dialling delay, i.e., the time to connect a call to the telephone network after dialling. Based on the real data, the log-normalness of the post dialling delays are convinced for the long distance calls, but not supported for the local calls. Uses of graphical plotting paper and statistical test of fit are discussed.

1. Introduction

Delay has been an important factor for influencing customer satisfaction while using a telephone. Kort (1983) reports that delay-related factors have been ranked second after transmission impairments in annoyance to customers. In order to understand their characteristics several statistical models have been considered and widely used. Among them, a family of log-normal distributions has been popularly adopted as a statistical description for the behavior of delays in connection. Bell Laboratories developed the call-setup rating model based on the log-normal models[Kort, 1983]. Some examples for their use of the log-normal models are the ones for dial-tone delay, time to audible ring(post dialling delay), time to operator intercept, and time-to-no such number announcement. Also, abandonment and retrial before network response — the probability of abandoning within t seconds after end of dialling, given that the customer is still waiting for network response at time t – is reported to be well-modelled by the weighted sum of two log-normals.

In this paper, we search for the brief history of post dialling delay as an annoyance factor to the customer satisfaction, obtained from the surveys conducted at the AT & T Bell Laboratories. Then, our concern will be given to the statistical test for the goodness of fit of the log-normal distribution as a post dialling delay model. The test will be done with the data obtained from the current telephone network in Korea. The modelling issue will be important in a later stage for developing the quality of service model, which can evaluate the customer satisfaction in regards to using the current telephone system.

2. Post Dialling Delay

Bell Laboratories considered the information related to the transmission performance of the switched telecommunication network is essential to the evaluation of toll service quality, necessary for assessing the adequacy of present administrative and maintenance procedures, and important in establishing objectives for new transmission systems and equipment. Bell Telephone Laboratories started to conduct a number of system-wide transmission surveys since 1959. The post dialling delay (PDD) appeared in the 1966 survey as a factor influencing the transmission performance for the first time under the name of 'Time to connect on DDD(Direct Distance Dialling) toll connections'. In a survey of toll connections during 1969 and 1970, 'time to receipt of audible ring' has been shown whose meaning is not much different from PDD. The result of 1966 and 1970 surveys is reproduced in 〈Table 1〉 under the respective titles of 'time to conncet … ' and 'time to audible ringing … '. Currently CCITT (1984) defines the PDD as the elapse of time between the end of sending of the last digit and the reaction of the network, e.g., ringing tone, busy tone, information tone, etc. In both surveys, distributions for time to receipt of audible ringing are considered to be Gaussian in all mileage categories [Duffy and Thatcher, 1971].

From the survey conducted throughout the Bell system in 1974, Duffy and Mercer (1978) report the network performance and customer behavior characteristics for the effect on the call dispositions, setup times, and customer abandonment times associated with direct-distance-dialling(DDD) attempts. They collected the necessary data through the Dial Line Service Observing(DLSO). 〈Table 2〉 shows a summary on the network responses after the end of dialling. Again their purpose is to provide network performance and customer behavior characteristics to network planners and administrators and to designers of equipment and systems which use, and interact with, the telephone network. Owing to DLSO system, they could get the various attempts to route a call to the called station after the end of dialling.

〈 Table 1 〉 Post dialling delay times surveyed by Bell Lab, in 1966 and 1970

used titles	time to connect on DDD toll connections in 1966		time to audible ringing on DDD toll connections in 1970	
connection length (airline miles)	mean(sec)	s.d(sec)	mean(sec)	s.d(sec)
all			11.7(1.8)	4.3
0 – 180	11.1(0.9) *	4.6	10.8(1.5)	3.7
180 – 725	15.6(1.0)	5.0	14.7(3.2)	5.0
725 – 2900	17.6(2.1)	6.6	13.9(2.6)	4.5

* Number in ( ) indicates 90% confidence level

〈 Table 2 〉 First DDD system responses after end of dialling in 1974

responses from end of dialling to	delay times(second)	
	mean	s.d
ringing before answer of disconnect	10.9(0.5)	5.0
answering without a ring signal	11.1(0.9)	5.3
busy signal	10.5(0.9)	5.2
no circuit/reorder signal	7.4(2.0)	7.4
no such number tone(NSN)	1.2	1.4
ringing prior to NSN announcement	5.3	4.1
NSN announcement without a ring signal	7.3(1.6)	5.4
ringing prior to intercept for number changing or disconnecting(INT)	10.9(1.1)	4.6
INT without a ring signal	13.9	4.4
customer abandonment without a system response	15.5(2.8)	17.8

3. Influences on PDD

PDD times (and generally call setup times) are influenced by certain attempt characteristics such as class of subscriber service and calling distance. Also, type of switching is known to strongly influence on the performance of the call setup.

In this section we describe influences on PDD by the suvscriber calss, distance and types of switching equipments.

(1) class of subscriber service

The distributions for the time from end of dialling to first system responses are basically the same for business and residential traffic. However the different classes have strong influences on other factors such as dial tone delay, completion rates, dialling times, and holding times. Traffic trends behave differently in weekdays and weekends variations. But class of subscriber service plays a minor role in determining call setup times.

(2) calling distance

Generally, as the distance increases, the call setup process becomes more complex due to additional toll switching, intraoffice processing, and interoffice trunking. This added setup complexity has considerable impact upon call disposition and setup times. From 〈Table 1〉 we see the fact that the interval from the end of dialling to a ring or busy signal increases as calling distances get longer, because additional switching tends to slow down the setup time.

〈 Table 3 〉 DDD call attempt statistics for the time from end of dialling to ring or busy as a function of originating and terminating switching

oringinating switch	Rotary dial customer mean(sec)	s.d(sec)	TOUCH-TONE customer mean(sec)	s.d(sec)
PAN	13.6(3.0)	7.4	–	–
SXS	12.6(1.6)	5.8	16.5(1.7)	4.7
5XB	10.1(0.4)	4.4	10.8(0.6)	4.6
1XB	10.8(0.8)	5.8	12.3(1.9)	4.6
ESS	9.2(0.6)	4.6	9.5(0.9)	3.6

terminating switch	all customers	
SXS	12.7(0.6)	4.6
CDO	12.8(1.0)	4.7
5XB	9.7(0.7)	5.0
1XB	9.5(0.6)	3.8
ESS	7.9(0.5)	3.7

PAN : pannel SXS : step-by-step 5XB : no. 5 cross bar 1XB : no. 1 cross bar
Ess : Electronic Switching System CDO : Community Dial Office

(3) local switching

The average setup time from end of dialling to ring or busy varies substantially for attempts classified by the types of local switching. The average setup times with accompanying 90% confidence intervals and standard deviations are listed in 〈Table 3〉 for originating and terminating local switching classifications [Duffy and Mercer, 1978]. Estimates for attempts which originate from rotary dial and TOUCH-TONE dialling stations are given separately for the originating local switching classifications.

4. Test of fit of the Log-normal model for PDD

The log-normal distribution has been widely used for various fields. Cramer (1946) derives log-normal function from the the biological application such as the growth of organ. Also the same deduction is applicable to the distribution of income in a certain population. In representing life-time distribution, the log-normal family has been found to be serious competitor to the Weibull family [Lawless, 1982].

The log-normal distribution with two parameters has the pobability density function(pdf)

$$f(x) = 1/\{(2\pi)^{1/2}\sigma x\}\exp[(-1/2)\{(\log x-\mu)/\sigma\}^2],\ x > 0, \qquad (1)$$

with the parameters μ and σ. By taking natural logarithm on x, we can transform to the popular normal distribution with the pdf

$$g(y) = 1/\{(2\pi)^{1/2}\sigma\}\exp[(-1/2)\{(y-\mu)/\sigma\}^2],\ -\infty<y<\infty, \qquad (2)$$

where $y=\log x$. Note that σ changes only the scale in normal distribution but it plays as a shape parameter in log-normal distribution. Also it is noteworthy that the shape of log-normal pdf get closer to the shape of normal distribution as σ is smaller and smaller.

Kort(1983) reports the use of the weighted sum of two log-normals for the 'time to audible ring' (RING) distribution in Bell Telephone Laboratories' Call-Setup Model. The model is defined as follows:

$$a\Phi(z_1) + (1-a)\Phi(z_2), \qquad (3)$$

where Φ is the standard normal distribution function and

$$z_1 = (1/\sigma_1)\log_{10}(t/\mu_1), \qquad z_2 = (1/\sigma_2)\log_{10}(t/\mu_2).$$

As we mentioned in section 1, the log-normal model has been favored for describing the statistical nature of delay times. Since the correct model is important not only to summarize the characteristics of the stochastic mechanism, but to analyze and even to forecast its future behavior. Under the rule of parsimony we consider the fitness of the log-normal model defined in Equation (1) for the post dialling delays.

The Korea Telecom has obtained 103 post dialling delays in an effort to find its current level of quality satisfaction by the telephone users. The experiment covered 9 local telephone offices in Seoul and 12 offices in other cities. The five calls have originated from its central office to each of the other offices and hence there were 105 attempts in total. Since 2 calls have lost from the long distance calls because of no responses, 45 local PDDs and 58 long PDD cases have been available for test of fit. The 103 raw data showed that all delay times are greater than 2 seconds and less than 5% has the delay times more than 7 seconds. We summarize the data in 〈Table 4〉, classified into local and long distance groups. The two groups show some different behaviors. The local group is scattered within the spread of 2 to 7 seconds and more than 70% of the calls are concentrated in the interval of 2－3 seconds. Therefore, the shape of the probability density function will have a sharp edge in the left side and a comparatively longer right tail. It is also noteworthy of the possible existence of a hump in the interval of 4－5 seconds. It may be because of random sampling or because of different mechanisms working simultaneously on the delays as shown in the Kort's model in (3). The long-distance group is scattered in the region of 2 to 12 seconds with the range of 10 seconds. Also we can see that the mode of the long-distance group is in the 4 to 5 seconds interval and its density function has a right tail. From these initial summaries, we can say that both groups may have different values in location and shape parameters, but may possess similar right skewed shapes.

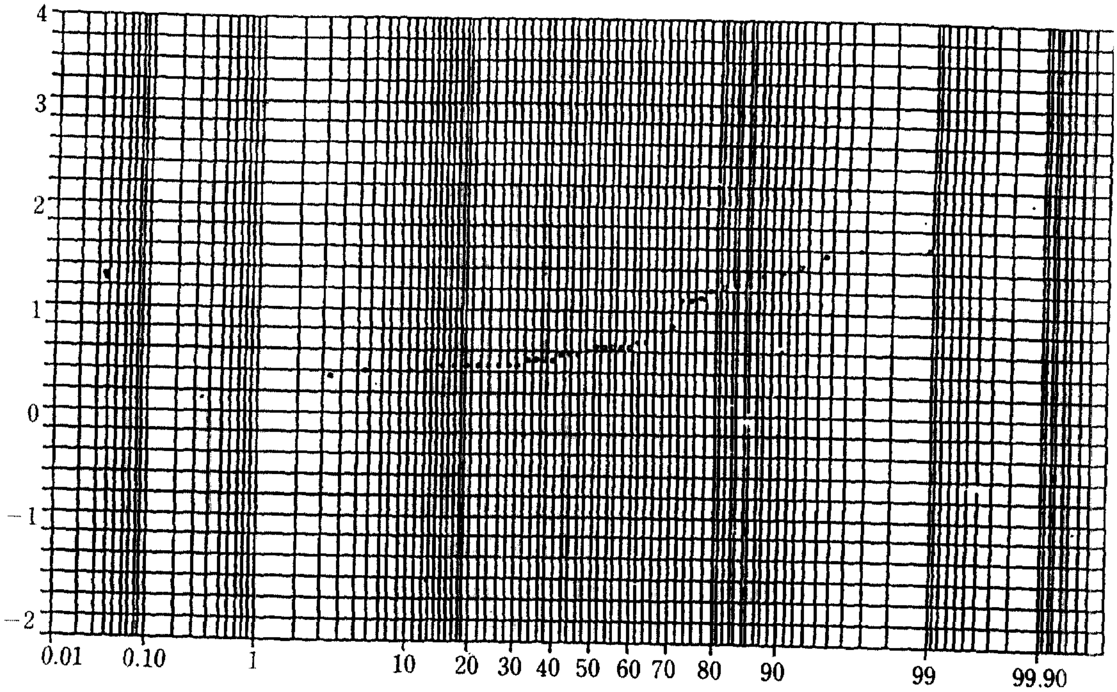
The Gaussian model has been popular owing to its frequent happenings in the nature and also well-developed theory. Therefore, if any data, not fitted at the first hand, could be transformed to justify the Gaussian assumption, it makes easy to draw information in the data, summarized by two parameters, its mean and variance. The Box-Cox power tranformation has been used popularly for this purpose [Draper and Smith, 1981]. When we applied the power transformation on the data, we obtain minimum values both at lambda equal to 0. The value of lambda equal to 0 means that if we take the logarithms on the raw data, the transformed data will be well modelled by the Gaussian distribution. In other words, it is possible that the raw data could be successively described by the log-normal model. The Box-Cox transformation supports the use of log-normal model for each of the PDD groups.

Plotting on a probability paper is the simplest way to make decision on our supposed assumption, i.e., the null hypothesis. By looking at straightness of plotted points, we can judge whether the data fits well our tentative model. Also deviations from the straightness can lead us to other tentative models. PC-Grapher developed by Won, et.al. (1991) has the facility to generate the normal probability paper. When we feed the PDD-local and the PDD-long data to PC-Grapher, after taking logarithmic transformation on the data, 〈Figure 1〉 and 〈Figure 2〉 are obtained from the plotter. It is easy to draw a linear line through the points in 〈Figure 2〉 of the PDD-long data, but is difficult to forge artificial linear line in 〈Figure 1〉 of the PDD-local data. Based only on the currently available data, the mechanism behind the local thelephone calls behave very differently from the one behind the long distance calls and hence we can not treat them under the same distributional assumption. This is what we failed to find when we applied the power transformation.

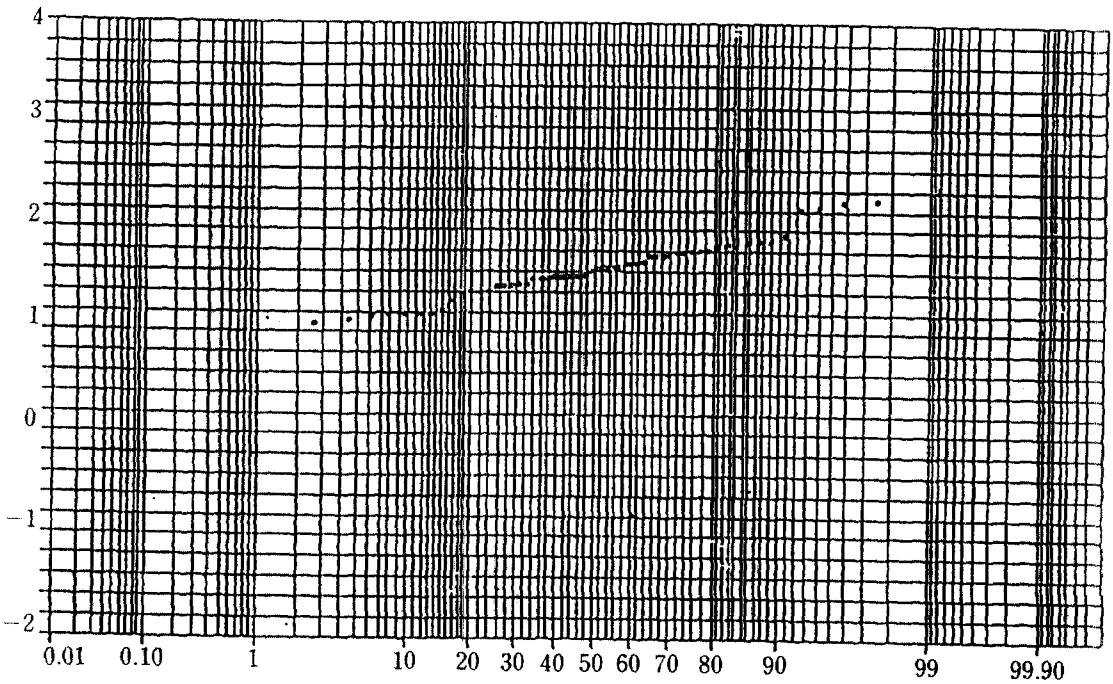
〈 Table 4 〉 Frequency table for the PDD data

	PDD-local		PDD-long	
interval	frequency	cum.freq.	frequency	cum.freq.
2 - 3 (sec)	71.1 (%)	71.1 (%)	3.5 (%)	3.5 (%)
3 - 4	6.6	77.7	15.5	19.0
4 - 5	13.3	91.0	31.0	50.0
5 - 6	4.5	95.5	24.1	74.1
6 - 7	4.5	100.0	17.3	91.4
> 7	0.0	100.0	8.6	100.0

To avoid any possible subjective errors on our decision based on the straight line in the plotting paper, a formal statistical test based on the empirical distribution function has been computed and compaired with the published table values in Lawless (1982). The Anderson-Darling test statistic has been taken for our purpose, because it is generally known to be the most powerful of the modified test statistics based on the empirical distribution function. We reproduce the table in 〈Table 5〉 with the values of the statistic. The test shows that the PDD-local is not accetable with the significance level of 1% and PDD-long is accetable



〈 Figure 1 〉 Probability plot for PDD local data



〈 Figure 2 〉 Probability plot for PDD long-distance data at the significance level of 10%. Therefore, the result is not much different from what we made with the probability plotting paper.

Other popular parametric models such as log-gamma, Webull, exponential distributions have been further experimented. But our final conclusion is that the PDD-long data is fitted well with the log-normal model, but PDD-local needs more elaborate search for its model.

〈 Table 5 〉 Anderson-Darling test for PDD data

	Quantile				
Function	.75	.90	.95	.975	.99
$(1+4/n-25/n**2)A_n^2$	.576	.656	.787	.918	1.092

(A) For the local : $A_n^2 = 2.656$

$(1+4/45-25/45**2)A_n^2 = 2.859$

(B) For the long : $A_n^2 = 0.506$

$(1+4/58-25/58**2)A_n^2 = 0.537$

5. Conclusion

In this paper, we mentioned the historical trait of the post dialling delay. It has been an important factor influencing the customer service while setting up a phone call. A number of variables have been mentioned as causes of changes in the characteristics of the post dialling delay. As a model for describing the behavior of the delay, Bell Laboratories applied the normal distributions in the early stage of the survey, but they favored a mixture of two log-normal distributions in the later stage.

Accordng to our recent test of fit for log-normalness of the post dialling delay times in the Korea Telecom network, the local delays behave differently from the long delays. The two calls can not be explained by the same statistical model For the long distance calls the log-normal model is statistically acceptable, but for the local calls any single model is not readily acceptable. We may fit a mixture such as the Kort's model. But because of the increase in complexity in a mixture model, increase of the sample size should be made. Also for the sensitivity of the test, sampling methods should be schematically devised for randomness of the sample in the further experimentations. Because finding of the correct model is useful in various ways, it will be helpful to collect more data using experimental design techniques. We believe that the design techniques will clarify the influecnces on the delays by the factors in a systematic and efficient way and therefore will lead to better understanding of the real nature of the delay behaviors.

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