■ 연구논문

A Comparison of Technological Growth Models

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Abstract

Various growth models were each fitted onto the data sets in an attempt to determine which growth models achieved the best forecasts for differing types of growth data. Of six such models studied, some models do significantly better than others in predicting future levels of growth. It is recommened that Weibull and the Gompertz growth curve be considered along with Pearl model by those industries presently considering the implementation of substitution analysis in their life analysis. In the early stage of growth, linear estimation should suffice to give reasonable forecasts. In the latter stage, however, as more data become availavle, nonlinear estimation should be used.

1. Introduction

Technological forecasting (TF) methods are tools which are used for planning and decision making in order to obtain insight on the future of a technology, group of technologies or undiscovered technologies, and their direction of change and advance over the longer time. There are various definitions of TF. One of the earliest pioneers, Lenz (1962), has described it as the prediction of the invention characteristics, dimensions, or performance of a machine serving some useful purpose. The difinition given by Bright (1983) is a quantified prediction of timing and of the character or the degree of change of technical parameters and attributes associated with the design, production, and use of devices, materials and processes, according to a specified system of reasoning. Jantsch (1969) Defined the TF as the probabilistic assessment on a relatively high confidence level of future technology transfer. Lanford (1974) defined the TF as the prediction or determination of the feasible to desirable characteristics of performance parameters in future technologies. A widely accepted definition of TF has not been formulated to date. One reason that a single definition has been elusive is that various researchers see different meanings in different disciplines [1, 2]. As a result, some of the terms associated with the TF might not have consistent meaning across various areas. The subject of this study is the application of the TF methods to the process of life analysis. The term of TF within life analysis has come to mean the forecasting of process from birth to death of a product

2. Overview of technological growth models

The TF methods are frequently grouped to explain and predict the future growth in functional capabilty of some specipic technology. Many trend extrapolation methods and techniques are being developed to forecast technological growth. However, only limited attempts have been made to apply quantitative methods to real life data patterns [Oh, 1991]. One of the quantitative forecasting techniques in the realm of TF methods is curve fitting utilizing the growth models. The term growth model refers to a plot over time of some capability or characteristic of a particular technology. It is generally believed that this curve, plotted on arithmetic scales, reflects a slow start followed by exponential growth, and then levels off against some upper limit produced by nature or technical capabilities. This pattern of growth results in, what is termed, an S shaped curve with respect to time. Various growth models have been proposed to represent the time pattern of technological growths. For practical purpose, these curves classified by the degree of skewness: symmetric growth models and nonsymmetric growth models.

2.1 Symmetric growth models

2.1.1 Pearl growth curve

Pearl and Reed (1922) made extensive studies on the growth behavior of organisms and described the mathematical function of their results by what has become known as the Pearl growth curve or the logistic curve. The equation of this curve is,

$$Y = \frac{L}{1 + \alpha e^{-\beta t}} \qquad (1)$$

where Y = the technological variable being achived at time t,

L = the upper limit to that technological capability,
t = the value of time,
$\alpha, \beta(\alpha, \beta > 0)$ = the parameters of the model.

In this equation, α and β are parameters which control the shape of the growth curve. The value of β determines the steepness of the exponential growth portion, while the value of the α determines the position of the curve on the time (t) axis. The curve has an initial values of zero at time $t = -\infty$, and reaches the limiting value L at time $t = +\infty$. By setting the second derivatives of Y with respect to time equal to zero, it can be shown that the inflection point of the curve occurs at $t = (\ln \alpha)/\beta$, where $Y = L/2$. The curve is symmetric with respect to the inflection point.

2.1.2 Fisher-Pry model

Fisher and Pry (1971) explained a technology as a set of substitution processes and showed that substitutions tend to proceed exponentially in the early years, and to follow an S shaped curve. They created a model in which the substitution proceeds at a rate determined by the formula,

$$Y = \frac{1}{2}[1 + \tanh a(t - t_0)] \tag{2}$$

where Y = fraction of growth of the technology,
tanh = hyperbolic tangent function,
a = half of the annual fractional growth in the early years,
t_0 = the time in which the new technology captures 50% of the usage or $Y = 1/2$.

A more convenient form of the above substitution expression is [Oh, 1988],

$$\frac{Y}{1-Y} = e^{2a(t-t_0)} \tag{3}$$

This model can be derived from the Pearl growth curve, but there exist some differences in the fit due to the estimator procedure.

2.1.3 Mansfield-Blackman model

Some of the pioneering work in the modelling of the substitution process was due to Mansfield (1968), who was able to illustrate that the growth in the number of users of an innovation could be approximated by a logistic curve. Blackman (1974) made a revision of Mansfield's model by modifying the definition of substitution to be market share captured by the new technology. The resultant model can be written as,

$$\ln\left[\frac{Y}{L-Y}\right] = \ln\left[\frac{Y_0}{L-Y_0}\right] + \alpha(t-t_1) \qquad (4)$$

where Y = market share captured at time t by new innovations,
L = upper limit of market share which the new innovation can capture in the long run,
Y_0 = market share captured when $t=t_1$,
α = constant which governs substitution rate,
t_1 = the time in which the innovation first captures a portion of the market.

If $\beta_0 = \ln[Y_0/(L-Y_0)] - \alpha t_1$ and $\beta_1 = \alpha$, then equation (4) can be rewritten as,

$$\ln\left(\frac{Y}{L-Y}\right) = \beta_0 + \beta_1 t \qquad (5)$$

Note that the Mansfield-Blackman model is simply the Fisher-Pry model when the upper limit L is set to 1. For this reason, it will not be treated as a separate model.

2.1.4 Bass model

Bass (1969) developed a somewhat different approach to the introduction of new products concerned with the development of the theory explaining the timing of the initial purchase of new products for consumer duables. He proposed the growth model to utilize the derivative dy/dt, rather than deal with the cumulative form of the model:

$$S(t) = pm + (q-p)Y(t) - \frac{q}{m}[Y(t)]^2 \qquad (6)$$

where $S(t)$ = the predicted sales at time t,
$Y(t)$ = the cumulative sales of consumer durables in time interval $(0, t)$,
m = the market potential for first time purchasers,
p = the coefficient of innovation,

q = the coefficient of imitation to reflect the word-of-mouth communication between adopters.

This model is based on the Pearl growth curve and the derivation of the Bass model from the logistic function is given in Appendix A.

2.2 Non-symmetric growth models

2.1.1 Gompertz growth curve

An another growth curve that has been employed extensively in TF is the Gompertz curve (1984), named after Benjamin Gompertz, who originally proposed the Gompertz curve as a law governing mortality rates. The mathematical form of this curve is,

$$Y = Le^{-Me^{-kt}} \tag{7}$$

where Y = the technological variable being achived at time t,
L = the upper limit to that technological capability,
t = the value of time,
$M, k(M, k > 0)$ = the parameters of the model.

Similar to the Pearl growth curve, the Gompertz growth curve ranges from zero at $t=-\infty$ to L at $t=+\infty$. By setting the second derivatives of Y with respect to time equal to zero, the point of inflection occurs at $t=(\ln M)/k$, where $Y=L/e$. The curve is not symmetric with respect to the inflection point.

2.2.2 Floyd model

Floyd (1986) wished to develop a means of analyzing historical technological growth and to forecast growth trends in improved functional capability. This capability is translated in "figures of merit" which serve as the dependent variables. He assumed that there is a fixed number of techniques that can be tried to accomplish a specific goal and only a fraction of these will be successful. His model can be summarized as,

$$P(Y, t) = 1 - \exp[-0.6941 \frac{(C_1 t + C_2)}{F + \ln(F-1) + C_2}] \tag{8}$$

where $P(Y, t)$ = probability of achieving figure of merit level Y by time t,
L = limiting value of figure of merit,
Y = level of figure of merit for new technology,

Y_c = level of figure of merit for competitive technology,

C_1, C_2 = constants,

$F = (1-\frac{Y_c}{L}) / (1-\frac{Y}{L})$.

Sharif and Uddin (1975) developed a procedure for adapting available mathematical models for forecasting technological substitution and revised the Floyd model as,

$$\ln[\frac{Y}{L-Y}] + \frac{L}{L-Y} = C_1 + C_2 t \qquad (9)$$

where L = upper limit of the market share,

Y = market share of a substitute product at time t,

C_1, C_2 = constants.

In this form, it can be seen that the Floyd model is a modification of the Mansfield-Blackman model. The term $L/(L-Y)$ has been inserted to allow for a time-decreasing coefficient of delay, where as the Mansfield-Blackman assumes that this coefficient of imitation is time-invariant.

2.2.3 Sharif and Kabir model

Sharif and Kabir (1976) developed a generalized mathematical model for forecasting technological substitution under a wide variety of circumstances. The Floyd model gives an underestimation of the forecast, while the Mansfield-Blackman and Fisher-Pry models gives an overestimation of the forecast (1975). Since it is likely that the correct estimation lies between these two extremes, Sharif and Kabir suggested that the models of Floyd and of Mansfield-Blackman be linearly combined, as follow :

$$\ln[\frac{Y}{L-Y}] + \sigma[\frac{L}{L-Y}] = C_1 + C_2 t \qquad (10)$$

where σ = 1, the Floyd model results,

σ = 0, the Mansfield-Blackman model results,

σ = 0 and L=1, the Fisher-Pry model is evoked.

The term $L/(L-Y)$ is labelled a delay factor and σ is termed the delay coefficient. Since σ can be take a value between zero and one, a set of smoothed S shaped curves can be obtained, ranging from the most optimistic to the most pessimistic forecast.

2.2.4 Weibull growth curve

The Weibull distribution (1951), named after its conceiver, Waloddi Weibull, has been found experimentally to describe industrial property mortality characteristics. Sharif and Islam (1980) proposed the empirical Weibull growth curve as a general model for technological forecasting as follow:

$$Y = L - Le^{-\left(\frac{t-u}{\alpha}\right)^{\beta}} \qquad (11)$$

where u = a threshold of shift parameter,

$\alpha(\alpha > 0)$ = a scale parameter,

$\beta(\beta > 0)$ = a shape parameter.

For technological growth cases, the upper limit L is taken to be any desired value less than or equal to unity. Depending on the value of β, the Weibull curve becomes left skewed, symmetrical, or right skewed. In a way, β is similar to the delay coefficient in the Sharif and Kabir model, thus, the Weibull curve effectively models TF for a wide variety situations.

3. The models studied and the statistical analysis

Six mathematical forms of an S shaped growth curves are considered as models for yearly percentages of technological attainment. The upper limit is set at 100 percent attainment. Given that Y is the cumulative percentage of technological attainment at time t (i.e., penetration level achived at time t), the three nonlinearized growth models employed are,

1) Pearl growth curve

$$Y = \frac{1.0}{1 + \alpha e^{-\beta t}} + \varepsilon(t) \qquad (12)$$

2) Gompertz growth curve

$$Y = e^{Me^{-\beta t}} + \varepsilon(t) \qquad (13)$$

3) Weibull growth curve

$$Y = 1.0 - e^{-(\frac{t}{\alpha})^{\beta}} + \varepsilon(t) \quad (14)$$

The three linear version of growth models are,

1) linearized Fisher-Pry model

$$\ln\left[\frac{Y}{1-Y}\right] = \beta_0 + \beta_1(t) + \varepsilon(t) \quad (15)$$

2) linearized Gompertz growth curve

$$-\ln[-\ln(Y)] = \beta_0 + \beta_1(t) + \varepsilon(t) \quad (16)$$

3) linearized Weibull growth curve

$$\ln[-\ln(1.0 - Y)] = \beta_0 + \beta_1(t) + \varepsilon(t) \quad (17)$$

where $\varepsilon(t) \sim i.i.d.\ N(0, \sigma^2)$.

3.1 Data

The data used in this study are twenty-two historical growth cases from various industries. The main requisite for inclusion in the data set was that any case has at least two points before the 5% penetration level, an increasing number of points through 10%, 25%, 50%, and several points beyond the 75% penetration level. This was because the analysis was designed to check for fitting ability at each and every one of those levels. A list of the cases is given in Appendix B.

3.2 Comparison of the models

Prediction achivement is the criteria which will be used to determine if any particular model was or a group of models were dominantly superior to other models as forecasters of technological growth at different penetration levels. A measure of mean square error(m.s.e.) is used to determine the adequacy of the fit. The smallest m.s.e. may be used as an indication of which model to use. Since the fitted data of linearized models must be transformed back into y_t(predicted penetration at time t) values, the R^2 (coefficient of multiple determination) provided in the computer fitting process cannot be utilized; for this reason a modified m.s.e. (i.e., mean estimate error) is used to align for model comparisons,

$$\text{m.e.e.} = \sum_{t=1}^{N} \frac{(Y_t - y_t)^2}{N} \times 1000 \quad (18)$$

where Y_t is the actual penetration achived at time t, y_t is the predicted penetration at time t, N is the number of terms in the series, and multiplying by 1000 is only to avoid working with very small number. This procedure is not unique and it has been used by other investigators such as Nagar (1962) and Eilon (1973). Comparison of the models is accomplished by testing the following hypothesis:

H_0 : No significant difference exists among the fitting errors of technological growth models at each penetration level.

H_A : At least one significant difference exists between the fitting errors of technological growth models at each penetration level.

4. Results for comparison of the fitting ability

4.1 Results of Kruskal-Wallis test

To test this hypothesis and determine if any one growth model outperforms any other of the growth models at each penetration level, a nonparametric analysis is performed. The Kruskal-Wallis test [Concover, 1980] is chosen rather than the median test since the former uses more information contained in the observations than does the latter. The Kruskal-Wallis test statistic is a function of the ranks of the observations in the combined sample while the median test statistic is dependent only on the knowledge of whether the observations were below or above the grand median. Therefore, the data are tested using the nonparametric Kruskal-Wallis test available in the SAS package of computer programs by NPARIWAY procedure. All statistical siginificance tests were performed at 95% condidence level.

Table 1. Results of Kruskal-Wallis

penetration level	test value	probability
5 %	8.55	0.1282
10 %	11.83	0.0373 *
25 %	15.84	0.0073 *
50 %	22.57	0.0004 *
75 %	22.13	0.0005 *
100 %	44.87	0.0001 *

key : * = indicates significantly different with 5% confidence

The results of Table 1 suggest that no siginificant difference is detected among the fitting errors of the six growth models at 5% penetration level; however, a siginificant difference seems to exist among the fitting errors of six growth models at 10%, 25%, 50%, 75%, and 100% penetration level.

4.2 Results of Tukey test

An F-test provides evidence that a difference may exist among means (m.e.e. or m.s.e.) of several groups. However, if there are k different means being compared, there are $k(k-1)/2$ potential differences, and the F-test does not indicate between which means these differences may be. Therefore, a modified Tukey's test [Snedecor and Cochran, 1980] is performed to test all comparisons among means. The test is made by computing a difference, D, which is significant at the 5% level, when comparing it with the $k(k-1)/2$ sample differences in the experiment. D is the product of a square root of s^2/n and a factor, Q, where s^2 is the estimate of the variance within the groups, and Q is given in the table of "studentized range" [May, 1952]. If any sample differences exceed the calculated D-value, then a significant difference may exist between the means of those two groups and may indicates that one growth model may perform significantly different from another growth model.

4.2.1 Results of fitting ability at 10% level

Table 2 contains the analysis of variance (ANOVA) results when Tukey's test was applied to the relative fitting errors of growth models for twenty-two cases at 10% penetration level. The factor $Q(=4.093)$ is obtained from Biometrika [May, 1952] for 6 treatments and 126 degree of freedom. $D(=Q\times s)$ represents the largest difference between the means of any two treatments that may exist and still be considered sampling error rather than a difference between treatment means.

Table 2. ANOVA of fitting error at 10% level

source of variation	d. f.	sum of squres	mean square
model	5	0.03888254	
error	126	0.39649599	0.00314679
total	131	0.43537853	

Table 3 contains the difference between sample means of six growth models which are compared to Tukey's D of 0.04895. The brackted number after the name of model is the mean for the twenty-two cases of the mean estimate error using each growth model as described in equation (18). As can be seen from Table 3, the H_0 hypothesis is rejected at 5% of significance, indicating that a statistical difference exists between the linearized Fisher-Pry model and the Weibull growth curve. Since this difference is too large to be due to sampling error alone, the linearized Fisher-Pry model appears to have a larger fitting error than Weibull growth curve at 10% penetration level.

Table 3. Fitting differences between models at 19% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP (0.076)	–	0.03209	0.03359	0.04537	0.04764	0.05127 *
LGZ (0.044)		–	0.00150	0.01328	0.01555	0.01918
LWB (0.042)			–	0.01178	0.01405	0.01768
PL (0.030)				–	0.00227	0.00590
GZ (0.028)					–	0.00362
WB (0.025)						–

key : LFP = linearized Fisher-pry model
LGZ = linearized Gompertz growth curve
LWB = linearized Weibull growth curve
PL = Pearl growth curve
GZ = Gompertz growth curve
WB = Weibull growth curve
* = indicates significant difference exists between models

4.2.2 Results of fitting ability at 25% level

Table 4 contains the ANOVA results when Tukey's test was applied to relative fitting errors of growth models at 25% penetration level. Six treatments and 126 degrees of freedom are used to determine Q.

Table 4. ANOVA of fitting error at 25% level

source of variation	d. f.	sum of squres	mean square
model	5	3.43363439	
error	126	19.56926028	0.15531159
total	131	23.00289467	

Table 5 contains the difference between sample means of the six growth models which are compared to Tukey's D of 0.34389. The H_0 hypothesis is rejected at the 5% of significance, indicating that a statistical difference exists between the linearized Fisher-Pry model and the other growth models. This result indicates that the linearized Fisher-Pry model appears to have a larger fitting error than the other growth models at 25% penetration level. Therefore, considering only the factor of fitting ability, the linearized Fisher-Pry model would probably provide the analyst with the worst results at 25% penetration level.

Table 5. Fitting differences between models at 25% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP (0.62)	–	0.3640 *	0.3746 *	0.4317 *	0.4612 *	0.4672 *
LGZ (0.26)		–	0.0106	0.0677	0.0972	0.1032
LWB(0.24)			–	0.0571	0.0866	0.0926
PL (0.19)				–	0.0295	0.0355
GZ (0.16)					–	0.0060
WB (0.15)						–

key : * = indicates significant difference exists between models

4.2.3 Results of fitting ability at 50% level

Table 6 contains the ANOVA results when Tukey's test was applied to relative fitting errors of growth models at 50% penetration level. Six treatments and 126 degrees of freedom are used to determine Q.

Table 6. ANOVA of fitting error at 50% level

source of variation	d. f.	sum of squres	mean square
model	5	45.6547054	
error	126	122.6851395	0.97369158
total	131	168.3398453	

Table 7 contains the difference between sample means of the six growth models which are compared to Tukey's D of 0.86104. The H_0 hypothesis is rejected at the 5% of significance, indicating that a statistical difference exists between the linearized Fisher-Pry model and the other growth models. This result indicates that the linearized Fisher-Pry model appears to have a larger fitting error than the other growth models at 50% penetration level. Therefore, considering only the factor of fitting ability, the linearized Fisher-Pry model would probably provide the analyst with the worst results at 50% penetration level.

Table 7. Fitting differences between models at 50% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP (2.17)	-	1.3461 *	1.1311 *	1.5496 *	1.6743 *	1.7196 *
LGZ (0.82)		–	0.2150	0.2035	0.3282	0.3736
LWB (1.04)			–	0.4186	0.5432	0.5886
PL (0.62)				–	0.1247	0.1700
GZ (0.49)					–	0.0453
WB (0.45)						–

key : * = indicates significant difference exists between models

4.2.4 Results of fitting ability at 75% level

Table 8 contains the ANOVA results when Tukey's test was applied to relative fitting errors of growth models at 75% penetration level. Six treatments and 126 degrees of freedom are used to determine Q.

Table 8. ANOVA of fitting error at 75% level

source of variation	d. f.	sum of squres	mean square
model	5	117.128706	
error	126	296.941896	2.35668171
total	131	414.070601	

Table 9 contains the difference between sample means of six growth models which are compared to Tukey's D of 1.3396. The H_0 hypothesis is rejected at the 5% of significance, indicating that a statistical difference exists between the linearized Fisher-Pry model and the other growth models. This result indicates that the linearized Fisher-Pry model appears to have a larger fitting error than the other growth models at 75% penetration level. Therefore, considering only the factor of fitting ability, the linearized Fisher-Pry model would probably provide the analyst with the worst results at 75% penetration level.

Table 9. Fitting differences between models at 75% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP (3.75)	-	2.2487 *	1.7194 *	2.2822 *	2.7970 *	2.6927 *
LGZ (1.50)		–	0.5293	0.0335	0.5483	0.4440
LWB (2.03)			–	0.5629	1.0776	0.9733
PL (1.47)				–	0.5148	0.4104
GZ (0.95)					–	0.1043
WB (1.05)						–

key : * = indicates significant difference exists between models

4.2.5 Results of fitting ability at 100% level

Table 10 contains the ANOVA results when Tukey's test was applied to relative fitting errors of growth models at 100% penetration level. Six treatments and 126 degrees of freedom are used to determine Q.

Table 10. ANOVA of fitting error at 100% level

source of variation	d. f.	sum of squres	mean square
model	5	516.623533	
error	126	1219.772396	9.68073285
total	131	1736.395873	

Table 11 contains the difference between sample means which are compared to Tukey's D of 2.715. The H_0 hypothesis is rejected at the 5% of significance, indicating that a statistical difference exists between the linearized Fisher-Pry model and the other nonlinearized growth models and between the linearized Weibull growth curve and the other nonlinearized growth models. This result indicates that the linearized Fisher-Pry model and the linearized Weibull curve appears to have a larger fitting error than the other nonlinearized growth models at 100% penetration level. Therefore, considering only the factor of fitting ability, the linearized Fisher-Pry and the linearized Weibull curvel would probably provide the analyst with the worst results at 100% penetration level.

Table 11. Fitting differences between models at 100% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP (6.23)		2.0901	0.3769	4.1826 *	4.7027 *	4.7535 *
LGZ (4.14)		–	1.7132	2.0925	2.6125	2.6634
LWB (5.86)			–	3.8057 *	4.3258 *	4.3766 *
PL (2.05)				–	0.5200	0.5709
GZ (1.53)					–	0.0508
WB (1.48)						–

key : * = indicates significant difference exists between models

5. Conclusion

There is a justifiable need to incorporate technological forecasting in the overall life analysis framework especially in those industries experiencing fast technological changes. Technological growth models provide a valuable way of predicting future obsolescence due to technological improvements.

Of six such models studied, some models do significantly better than others, especially at high penetration levels in predicting future levels of growth, although that performance cannot easily be linked to fitting ability. The lack of a direct relationship between fitting and forecasting ability implies that fitting alone should not be used a *priori* to select among different models for the purposes of predicting.

The models are hardly distinguishable at lower penetration levels. If statistical fitting is the criterion used to select a forecasting model, one would expect the linearized Fisher-Pry model to forecast poorly compared to other linear models, at all levels. It is recommended that the Weibull and the Gompertz growth curve be considered along with the Pearl model by those industries presently considering the implementation of substitution analysis in their life analysis. Nonliner estimation improves the forecasting ability of most of the models especialliy at higher levels.

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APPENDIX A. Derivation of the Bass model

Given the form of the Pearl growth curve,

$$Y = \frac{L}{1+\alpha e^{-\beta t}}$$

the Bass model may be obtained by considering the derivative of Y with respect to time. It follows that,

$$\begin{aligned} S(t) &= \frac{dy}{dt} \\ &= \frac{L\alpha\beta e^{-\beta t}}{[1+\alpha e^{-\beta t}]^2} \\ &= \frac{\beta}{L}\left[\frac{L}{1+\alpha e^{-\beta t}}\right] \times \left[L - \frac{L}{1+\alpha e^{-\beta t}}\right] \\ &= \frac{\beta}{L} Y[L-Y] \\ &= \beta Y - \frac{\beta}{L} Y^2 \\ &= C_0 + C_1 Y + C_2 Y^2 \end{aligned}$$

where $C_0 = 0$, $C_1 = \beta$ of the Pearl growth curve, and $C_2 = (-1/L)\beta$ of the Pearl growth curve for the error free Bass model.

APPENDIX B. List of data

1. Rayon and nylon for cotton as tire cord in tire manufacture(1938-1962)
2. Nylon, polyester and fiberglass for rayon and cotton as tire cord in tire manufacture(1962-1972)
3. Catalytic and hydro-cracking for thermal cracking in crude oil processing (1938-1966)
4. Steam and motor for sail in the United Kindom registered shipping(1918-1938)

4. Steam and motor for sail in the United Kindom registered shipping(1918 – 1938)
5. Percent of underground bituminuous coal automatically loaded(1923 – 1970)
6. Diesel for coal and fuel oil consumption on American railroads(1938 – 1970)
7. Percent of independent telephone companies connecting with the Bell system (1899 – 1957)
8. Open hearth for bessmer in raw steel production in the United States (1876 – 1960)
9. Percentage of U.S. corn acreage planted with corn hybrids(1933 – 1960)
10. Diesel for steam locomotives(1939 – 1962)
11. Percentage of Pennsylvania anthracite mined by stripping(1927 – 1976)
12. Steam and motor for sail in the U.S. Merchant Marine(1820 – 1960)
13. Basic oxigen process for bessemer and open hearth in raw steel production in the U.S.(1955 – 1981)
14. Color for B & W television in the United Kindom(1968 – 1984)
15. Percentage of iron ore pelletized in the U.S.(1953 – 1973)
16. Percentage of farm dwelling units with electric service(1920 – 1956)
17. By-product coke for dven coke in the U.S.(1900 – 1962)
18. Percentage of households in the U.S. with a television set(1946 – 1980)
19. Percentage of households in the U.S. with a radio receiver(1927 – 1970)
20. Percentage of households in the U.S. with a color television set(1955 – 1984)
21. Percentage of homes in the U.S. with at least a mechnical refrigerator (1925 – 1952)
22. Basic oxygen for bessmer and open hearth pig iron total consumption in the U.S.(1957 – 1984)