Optimum Screening Procedures Using Prior Information

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Abstract

Optimum screening procedures using prior information are presented. An optimal cutoff value on the screening variable X minimizing the expected total cost is obtained for the normal model; it is assumed that a continuous screening variable X given a dichotomous performance variable T is normally distributed and that costs are incurred by screening inspection and misclassification errors. Methods for finding optimal cutoff values based on the prior distributions for unknown parameters are presented.

1. Introduction

Screening procedures are widely used in industries to improve outgoing quality of products. In some situations where the inspection of the major quality characteristic (it is referred to as performance variable) involves destructive or expensive testing, however, it is not feasible to screen items on the performance variable itself. Advances in testing equipment using laser, X-ray, etc., however, enable us to inspect items without destroying them. For example, the strength of welding by which an automobile seat is attached to the frame may be the major qaulity characteristic to be controlled. The measurement of the strength of welding requires destructive testing. It is, however, possible to screen items by measuring X-ray penetration of the weld which is negatively correlated with the strength of the weld; see Owen, Li and Chou (1981). Therefore, we can inspect with a quality characteristic (it is referred to as screening variable) correlated with major quality characteristic of interest rather than the performance variable directly. Inspection with the screening variable may be performed with less cost

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but less accuracy compared with screening with the performance variable.

There has been much attention on screening procedures with screening variables; see for example, Owen, McIntire, and Seymour (1975), Owen and Boddie (1976), Owen and Su (1977), Li and Owen (1979), Owen, Li and Chou (1981), Madsen (1982), Menzefricke (1984), Boys and Dunsmore (1986), Tang (1987, 1988), Bai, Kim and Riew (1990), Moskowitz, Plate and Tsai (1991), Liu (1992) and Kim and Bai (1992). Most of these works deal with the screening methods of increasing the proportion of items within specification from the current value of γ to a specified higher proportion δ after screening, i.e., improving the outgoing quality or the economic design of screening procedures in various situations. They also assume a bivariate or multivariate normal structure between the performance and screening variables. However, when the major quality characteristic is the existence or nonexistence of nonconformity such as flaws in the sheet of steel, cracks in the steel bars, and incidences of certain disease, etc., those normal structures are no longer valid. Boys and Dunsmore (1987) considered screening methods with dichotomous performance and continuous screening variables to raise the predictive success probability to a given level for both diagnostic and sampling paradigms. Bai, Kim and Ahn (1988) considered optimal screening procedures with dichotomous performance and continous screening variable for assuring with a specified degree of confidence that at least l out of m items found acceptable in screening inspection are conforming. Kim and Bai (1990) presented economic screening procedures based on a continuous screening variable in place of a dichotomous performance variable for normal and logistic models.

In this article, optimum screening procedures for the normal model using prior information are presented. Optimal cutoff values on the screening variable are obtained by minimizing the expected cost which includes three cost components, screening inspection cost and the costs due to misclassification errors. Two types of misclassification errors are considered; a conforming item may be classified as of nonconforming (type I error), or a nonconforming one as conforming (type II error).

2. The Model

Suppose that T is the dichotomous performance variable of interest and X is a continuous screening variable. Let the proportion of conforming items in the population before screening be $\gamma = P[T=1]$. If a larger value of X produces higher probability of being a conforming item, a logical screening procedure would be to accept any item satisfying $X \geq \omega$, where ω is the cutoff value to be determined optimally. It is assumed that the conditional distributions of X given $T=i$, $i=0, 1$, are normal with means μ_i and variances σ_i^2, respectively, and without loss of generality $\mu_1 > \mu_0$.

The unit costs incurred by type I and type II misclassification errors are assumed to be known constants C_r and C_a ($C_r < C_a$), respectively, and the screening inspection cost per item is C_s. Then the expected cost per item due to type I error is

$$\begin{aligned} EC_1 &= C_r P[X < \omega \text{ and } T = 1] \\ &= C_r \gamma P[X < \omega \mid T = 1], \end{aligned} \tag{1}$$

and that due to type II error is

$$\begin{aligned} EC_2 &= C_a P[X \geq \omega \text{ and } T = 0] \\ &= C_a (1-\gamma) \{1 - P[X < \omega \mid T = 0]\}. \end{aligned} \tag{2}$$

Therefore, the total expected cost per item is

$$\begin{aligned} ETC &= EC_1 + EC_2 + C_s \\ &= C_r \gamma P[X < \omega \mid T=1] + C_a(1-\gamma)\{1 - P[X < \omega \mid T=0]\} + C_s. \end{aligned} \tag{3}$$

The optimal cutoff value of X can be obtained by minimizing (3). The resulting screening procedure can be used only when ETC is less than the cost of acceptance without screening inspection, which is $C_a(1-\gamma)$.

3. Optimal Solution Procedures

In the cases where some parameters of the conditional distributions of X given $T=i$, $i=0, 1$, are unknown, if the prior information on the unknown parameters and a sample observation are available, Bayesian method can be used to obtain the optimal screening procedure. Let the observed values of size $n+m$ which consists of n conforming items and m nonconforming items be $D_0 = \{X_{01}, X_{02}, \cdots, X_{0n}\}$ and $D_1 = \{X_{11}, X_{12}, \cdots, X_{1m}\}$, respectively.

Case where (μ_0, μ_1) are wnknown

If the unknown parameter μ_i has normal prior distribution with mean ξ_i and variance τ_i^2, $i=0, 1$, which is

$$h_i(\mu) = (2\pi\xi_i^2)^{1/2} \exp\left[-\frac{(\mu-\xi_i)^2}{2\tau_i^2}\right], \qquad -\infty < \mu < +\infty,$$

then the posterior distribution of μ_i can be obtained, from the prior distribution and sample result, as follows. Since the posterior distribution, $p_i(\mu_i \mid D_i)$, $i=0, 1$ is proportional to the product of the likelihood function and prior dstribution,

$$p_0(\mu_0 \mid D_0) \propto \frac{1}{[\sigma_0^2(2\pi)]^{n/2}} \exp\left[-\frac{\sum_{i=1}^{n}(X_{0i}-\mu_0)^2}{2\sigma_0^2}\right] \frac{1}{[\tau_0^2(2\pi)]^{1/2}} \exp\left[-\frac{(\mu_0-\xi_0)^2}{2\tau_0^2}\right]$$

$$\propto \exp\left[-\frac{\left(\mu_0 - \frac{\tau_0^2 \sum_{i=1}^{n} X_{0i} + \xi_0\sigma_0}{n\tau_0^2+\sigma_0^2}\right)^2}{\frac{2\sigma_0^2\tau_0^2}{n\tau_0^2+\sigma_0^2}}\right],$$

the distribution of $\mu_0 \mid D_0$ is normal with mean μ_0' and variance $\sigma_0'^2$, where $\mu_0' = (\tau_0^2 \sum_{i=1}^{n} X_{0i} + \xi_0\sigma_0^2)/(n\tau_0^2+\sigma_0^2)$ and $\sigma_0'^2 = \sigma_0^2\tau_0^2/(n\tau_0^2+\sigma_0^2)$. Similarly, it can be easily obtained that $\mu_1 \mid D_1$ is normal with mean μ_1' and variance $\sigma_1'^2$, where $\mu_1' = (\tau_1^2 \sum_{j=1}^{m} X_{1j} + \xi_1\sigma_1^2)/(m\tau_1^2+\sigma_1^2)$ and $\sigma_1'^2 = \sigma_1^2\tau_1^2/(m\tau_1^2+\sigma_1^2)$.

Therefore, ETC becomes

$$ETC = C_r\gamma \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_1(z_1 \mid \mu_1, D_1)p_1(\mu_1 \mid D_1)d\mu_1\, dz_1$$
$$+(1-\gamma)C_a\left\{1-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_0(z_0 \mid \mu_0, D_0)p_0(\mu_0 \mid D_0)d\mu_0\, dz_0\right\}+C_s, \quad (4)$$

where Z_i is $X \mid T=i$ and $f_i(z_i \mid \mu_i, D_i)$ is the probability density function of Z_i. Using the following relations,

$$\int_{-\infty}^{\infty} f_0(z_0 \mid \mu_0, D_0)p_0(\mu_0 \mid D_0)d\mu_0$$
$$= \frac{1}{[2\pi(\sigma_0^2+\sigma_0'^2)]^{1/2}} \exp\left[-\frac{(z_0-\mu_0')^2}{2(\sigma_0^2+\sigma_0'^2)}\right]$$

and

$$\int_{-\infty}^{\infty} f_1(z_1 \mid \mu_1, D_1)p_1(\mu_1 \mid D_1)d\mu_1$$

$$= \frac{1}{[2\pi(\sigma_1^2+\sigma_1'^2)]^{1/2}} \exp\left[-\frac{(z_1-\mu_1')^2}{2(\sigma_1^2+\sigma_1'^2)}\right],$$

ETC can be rewritten by

$$ETC = C_r\gamma\phi\left(\frac{(\omega-\mu_1')}{(\sigma_1^2+\sigma_1'^2)^{1/2}}\right)+(1-\gamma)C_a\left[1-\phi\left(\frac{(\omega-\mu_0')}{(\sigma_0^2+\sigma_0'^2)^{1/2}}\right)\right]+C_s, \qquad 5$$

where $\phi(\cdot)$ is the standard normal distribution function.

In case of $\sigma_0^2+\sigma_0'^2=\sigma_1^2+\sigma_1'^2=\sigma'^2$, it can be easily shown that ETC has its minimum value at

$$\omega = \frac{\mu_1'+\mu_0'}{2}+\frac{\sigma'^2}{\mu_1'-\mu_0'}\ln[(1-\gamma)C_a/\gamma C_r].$$

When $\sigma_0^2+\sigma_0'^2\neq\sigma_1^2+\sigma_1'^2$, the optimal cutoff value can be obtained as follows. Let $A=\sigma_1''^2-\sigma_0''^2$, $B=\mu_1'\sigma_0''^2-\mu_0'\sigma_1''^2$, $C=\mu_0'^2\sigma_1''^2-\mu_1'^2\sigma_0''^2-2\sigma_0''^2\sigma_1''^2\ln[(1-\gamma)C_a\sigma_1''/\gamma C_r\sigma_0'']$, where $\sigma_1''^2=\sigma_1^2+\sigma_1'^2$ and $\sigma_0''^2=\sigma_0^2+\sigma_0'^2$.

Then

$$\frac{\partial ETC}{\partial\omega} = \frac{\gamma C_r}{\sigma_1''}\phi\left(\frac{\omega-\mu_1'}{\sigma_1''}\right)-\frac{(1-\gamma)C_a}{\sigma_0''}\phi\left(\frac{\omega-\mu_0'}{\sigma_0''}\right)$$

$$= \frac{\gamma C_r}{\sigma_1''}\phi\left(\frac{\omega-\mu_1'}{\sigma_1''}\right)\left[\exp\left\{\frac{1}{2\sigma_1''^2\sigma_0''^2}(A\omega^2+2B\omega+C)\right\}-1\right].$$

$\partial ETC/\partial\omega=0$ yields the quadratic equation $A\omega^2+2B\omega+C=0$. When the discriminant $D=B^2-AC$ is strictly positive, it has two distinct real roots, and $\partial^2 ETC/\partial\omega^2>0$ at $\omega=(-B+\sqrt{B^2-AC})/A$ and $\partial^2 ETC/\partial\omega^2<0$ at $\omega=(-B-\sqrt{B^2-AC})/A$. Therefore, in the case of $\sigma_1''^2\neq\sigma_0''^2$ with $D>0$, the optimal cutoff value is $\omega^*=(-B+\sqrt{B^2-AC})/A$ and the expected total cost per item is $\gamma C_r\phi[(\omega^*-\mu_1')/\sigma_1'']+(1-\gamma)C_a[1-\phi[(\omega^*-\mu_0')/\sigma_0'']]+C_s$. Note that if $D\leq 0$ then $\partial/\partial\omega ETC\geq 0$ for $\sigma_1''^2>\sigma_0''^2$ and $\partial/\partial\omega ETC\leq 0$ for $\sigma_1''^2<\sigma_0''^2$, and therefore ω^* is $-\infty$ or $+\infty$, which means the optimal screening procedure is to implement the one which gives the minimum expected total cost per item among the following three procedures

PROCEDURE I : Accept all items without inspection.
PROCEDURE II : Accept any item satisfying $X \geq \omega^*$, where

$$\omega = \begin{cases} \dfrac{\mu_1' + \mu_0'}{2} + \dfrac{\sigma''^2}{\mu_1' - \mu_0'} \ln[(1-\gamma)C_a / \gamma C_r], & \text{if} \quad \sigma_1''^2 = \sigma_0''^2 = \sigma''^2 \\ (-B + \sqrt{B^2 - AC}) / A, & \text{if} \quad \sigma_1''^2 \neq \sigma_0''^2, \ D>0 \end{cases} \tag{6}$$

PROCEDURE III : Reject all items without inspection.

For Procedure II, the proportion of conforming items in the population after screening is given by

$$\delta = \gamma[1-\phi(\frac{\omega^*-\mu_1'}{\sigma_1''})]/\{1-\phi(\frac{\omega^*-\mu_0'}{\sigma_0'})+\gamma[\phi(\frac{\omega^*-\mu_0'}{\sigma_0''})-\phi(\frac{\omega^*-\mu_1'}{\sigma_1''})]\}.$$

Case where (σ_0^2, σ_1^2) are unknown

In case of unknown σ_0^2, σ_1^2 with known μ_0, μ_1, the inverse-gamma prior distributions for unknown parameters are considered. The inverse-gamma density function can be written as

$$k(\sigma_i^2) = \frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)}(\sigma_i^2)^{-(\alpha_i - 1)} \exp\{-\beta_i / \sigma_i^2\}, \qquad \sigma_i^2 > 0,\ \alpha_i > 0,\ \beta_i > 0.$$

Hence, the posterior density function of σ_i^2, $q_i(\sigma_i^2 \mid D_i)$, $i=0, 1$ can be obtained as follows.
Since

$$\begin{aligned} q_0(\sigma_0^2 \mid D_0) &\propto \frac{1}{[\sigma_0^2(2\pi)]^{n/2}} \exp[-\frac{\sum_{i=1}^{n}(X_{0i}-\mu_0)^2}{2\sigma_0^2}] \frac{\beta_0^{\alpha_0}}{\Gamma(\alpha_0)}(\sigma_0^2)^{-(\alpha_0-1)} \exp\{-\beta_0/\sigma_0\} \\ &\propto (\sigma_0^2)^{-(\alpha_0 - 1 + n/2)} \exp\{-(\sigma_0^2)^{-1}[\sum_{i=1}^{n}(X_{0i}-\mu_0)^2/2 + \beta_0]\} \\ &\propto (\sigma_0^2)^{-(\alpha_0' - 1)} \exp\{-(\beta_0'/\sigma_0^2)\}, \end{aligned}$$

where $\alpha_0' = (\alpha_0 + n/2)$ and $\beta_0' = [\sum_{i=1}^{n}(X_{0i}-\mu_0)^2/2 + \beta_0]$, the posterior density function of σ_0^2, $q_0(\sigma_0^2 \mid D_0)$, is the inverse gamma with $\alpha_0' = (\alpha_0 + n/2)$ and $\beta_0' = [\sum_{i=1}^{n}(X_{0i}-\mu_0)^2/2 + \beta_0]$. Similarly, that of σ_1^2, $q_1(\sigma_1^2 \mid D_1)$, is also inverse gamma with $\alpha_1' = (\alpha_1 + m/2)$ and $\beta_1' = [\sum_{j=1}^{m}(X_{1j}-\mu_1)^2/2 + \beta_1]$. Then, the expected total cost per item, ETC, is

$$ETC = C_r\gamma\int_{-\infty}^{\omega}\int_{0}^{\infty} f_1(z_1 \mid \sigma_1^2, D_1)\, q_1(\sigma_1^2 \mid D_1)\, d\sigma_1^2 dz_1$$
$$+ (1-\gamma)C_a\{1-\int_{-\infty}^{\omega}\int_{0}^{\infty} f_0(z_0 \mid \sigma_0^2, D_0)\, p_0(\sigma_0^2 \mid D_0)\, d\sigma_0^2 dz_0\} + C_s$$
$$= C_r\gamma\int_{-\infty}^{\omega} \frac{\beta_1'^{\alpha_1'}\Gamma(\alpha_1'+1/2)}{(2\pi)^{1/2}\Gamma(\alpha_1')}[(z_1-\mu_1)^2/2+\beta_1']^{-(\alpha_1'+1/2)} dz_1$$
$$+ (1-\gamma)C_a\{1-\int_{-\infty}^{\omega} \frac{\beta_0'^{\alpha_0'}\Gamma(\alpha_0'+1/2)}{(2\pi)^{1/2}\Gamma(\alpha_0')}[(z_0-\mu_0)^2/2+\beta_0']^{-(\alpha_0'+1/2)} dz_0\}$$
$$+C_s.$$

By differentiating ETC with respect to ω, an optimal cutoff value can be obtained by solving the equation

$$\frac{[(\omega-\mu_1)^2/2+\beta_1']^{-(\alpha_1'+1/2)}}{[(\omega-\mu_0)^2/2+\beta_0']^{-(\alpha_0'+1/2)}} = \frac{(1-\gamma)C_a}{C_r\gamma}\,\frac{\beta_0'^{\alpha_0'}\Gamma(\alpha_0'+1/2)\Gamma(\alpha_1')}{\beta_1'^{\alpha_1'}\Gamma(\alpha_1'+1/2)\Gamma(\alpha_0')}. \qquad (7)$$

It is difficult to solve Equation (7) analytically for different values of α_0' and α_1'. A numerical search such as regula falsi can be used to find the roots of it. However, Equation (7) obviously has a finite number of roots. If α_0' and α_1' are equal, Equation (7) becomes the quadratic equation $A'\omega^2-2B'\omega+C'=0$ with $A'=(1/\beta_1'-K/\beta_0')$, $B=(\mu_1/\beta_1'-K\mu_0/\beta_0')$, and $C=2(1-K)+(\mu_1^2/\beta_1'-K\mu_0^2/\beta_0')$ where $K=(1-\gamma)C_a\sqrt{\beta_1'}/(\gamma C_r\sqrt{\beta_0'})$. Hence, when $\alpha_0'=\alpha_1'$ and $1/\beta_1'=K/\beta_0'$, it can be easily shown that $\omega^*=(\mu_1+\mu_0)/2+[\beta_0'\beta_1'(1-\beta_0'\beta_1')]/(\beta_0'\mu_1-\beta_1'\mu_0)$. When $\alpha_0'=\alpha_1'$ and $1/\beta_1' \neq K/\beta_0'$ with discriminant $D'=B'^2-A'C'>0$, we can find the optimal cutoff value minimizing ETC, from two distinct roots of $A'\omega^2-2B'\omega+C'=0$.

4. A Numerical Example

Consider a transistor that is incorporated into an equipment. It is known that early failure of the transistor that brings about the failure of the equipment is costly. Since the lifetime of a transistor cannot be measured until it fails, it is proposed to screen the transistor by measuring its noise which characterizes the failure of transistor in use. It is known that the transistor's noise is normally distributed and that the transistor with lower noise has longer product life. Therefore, we use a screening procedure to accept any item satisfying $X \leq \omega^*$

where ω^* is an optimal cutoff value to be determined. The data (James (1985)) given in Table 1 show transistors' noise (rms) of ten conforming and ten nonconforming items. The costs due to type I and type II misclassification errors are $C_r = 1.0$ and $C_a = 4.0$, respectively, and the screening inspection cost is $C_s = 0.05$. The proportion of good transistors before screening inspection is 70%.

Then, when μ_0 and μ_1 are unknown and $\sigma_0^2 = \sigma_1^2 = 9.0$, if the parameters of normal prior distributions are given by $(\xi_0, \xi_1, \tau_0^2, \tau_1^2) = (15, 10, 3, 3)$, an optimal cutoff value $\omega^* = 12.2615$ from (6). Hence the optimal screening procedure is to accept all items for which $X \le 12.2615$ and $ETC = 0.3614$, which is less than $(1-\gamma) C_a$ or C_r. In this case, the proportion of conforming items after screening is $\delta = 92.35\%$.

In case of unknown σ_0^2, σ_1^2, if $\alpha_0 = \alpha_1 = 9.0$, $\beta_0 = 3.0$, $\beta_1 = 1.0$ and $(\mu_0, \mu_1) = (10.0, 15.0)$, then, from (8), an optimal cutoff value is $\omega^* = 12.3762$. Therefore, the optimal screening procedure is accept any item that $X \le 12.3762$.

〈 Table 1 〉 Raw data : transistor's noise (rms)

T=0	15.5	12.3	18.4	15.9	15.0	17.5	15.4	14.8	12.9	16.9
T=1	7.9	5.8	4.2	9.4	11.0	7.9	10.4	9.2	9.6	15.4

5. Concluding Remarks

When the prior informations are available, optimum screening procedures with some parameters unknown are obtained for the normal model where the distribution of continuous screening variable given the dichotomous performance variable is normal. The normal prior distributions for unknown (μ_0, μ_1) and the inverse gamma prior distributions for unknown (σ_0^2, σ_1^2) are considered. In case of unknown (μ_0, μ_1), a closed-form solution is obtained which is similar to that of all parameters known case. For the case where (σ_0^2, σ_1^2) are unknown, it is difficult to find optimal cutoff value analytically except when two shape parameters of inverse gamma prior distributions are equal. These optimum screening procedures may be extended to the logistic model and to the case with all parameters unknown.

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