X Control Charts under the Second Order Autoregressive Process[+]

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Abstract

When independent individual measurements are taken both S/c_4 and $\bar{R}/d_2$ are unbiased estimators of the process standard deviation. However, with dependent data $\bar{R}/d_2$ is not an unbiased estimator of the process standand deviation. On the other hand S/c_4 is an asymptotic unbiased estimator. If there exists correlation in the data, positive(negative) correlation tends to increase(decrease) the ARL. The effect of using $\bar{R}/d_2$ is greater than S/c_4 if the assumption of independence is invalid. Supplementary runs rule shortens the ARL of X control charts dramatically in the presence of correlation in the data.

1. Introduction

Control charts are used in the analysis and control of manufacturing process so as to produce satisfactory, adequate and economic quality. Samples are taken from a process and some appropriate statistics computed from the samples are plotted on a control chart in time order. There are two essential parts in all control charts: a target value and control limits. The control limits are set up so that the fluctuations within the control limits might be explained by "chance causes." However, if point falls outside the control limits, then it is an indication that an "assignable cause" of variation in the process has happened to change the process. Control charts are used to exhibit various kinds of deviations: a shift in the mean, a transient value, a trend, cyclical behavior, and increase in the variation. This paper deals with a shift in the mean.

Usually a process is monitored on the basis of similar subgroups. However,

+ 이 논문은 1993년도 학술진흥재단의 공모과제 연구비에 의하여 연구되었음.

production rate may be too slow to conveniently allow subgroup size greater than 1. Some examples are described in Roes, Does and Schurink(1933). In these cases, control charts for individual observations are used to monitor the process.

The run length is defined to be the number of samples to signal and the average run length(ARL) is its expected value. If the process is in control, then the ARL should be large because it is an indication of false alarm. On the other hand, if the process is not in control the ARL should be small because it is the number of samples required until out of control is found. Therefore, ARL is a good criterion for comparison of charts performance. A traditional assumption in quality control charts is that the observations are independent and identically distributed over time. However, it is common to have correlation in the manufacturing process, especially when automatic measurements are made on each manufactured item in production order.

The effects of autocorrelation have been studied for several types of control charts. Goldsmith and Witherfield(1961), Johnson and Bagshaw(1974), Bagshaw and Johnson(1975) have studied the effect of autocorrelation using time series model on CUSUM charts. Vasilopoulos and Stamboulis(1978) have proposed modification of $\bar{X}$ control chart limits for AR(1) process. VanBrackle(1992) has studied the effect of autocorrelation for EWMA and CUSUM control charts. Montgomery and Mastangelo(1991) have shown that the EWMA statistic provides the basis of an approximate procedure that can be useful for autocorrelated data. Statistical modeling and fitting of time series effects have also been proposed by Alwan and Roberts(1988). The effects of AR(1) and MA(1) processes on the retrospective X-charts, both with and without supplementary runs rule, have been studied using computers simulation by Maragah and Woodall(1992). It is pointed out in Alwan and Roberts(1988) that a few simple special cases of ARIMA models may serve as good approximations for many or even most practical applications. For instance EWMA chart is based on ARIMA(0, 1, 1). In this study, it is assumed that the underlying process is an AR(2) process. In this paper, my objective is to investigate the effects of autocorrelation on X charts in terms of average run length(ARL) using Markov chain approach and simulation method. To begin with, AR(p) process is explained in Section 2. In particular, AR(2) process is described in detail. In Section 3, the problem of estimating the process standard deviation is dealt with. Especially, $\bar{R}/d_2$ is to be compared with S/c_4. In Section 4, effects of the autoregressive parameters are to be investigated when using S/c_4 with large n. In Section 5, effect of using $\bar{R}/d_2$ is studied using simulation. Finally, in Section 6, supplementary runs rule is added to X charts when using S/c_4 to see how X control charts perform.

2. AR(P) process

Usually control chars are used for process control assuming that the observations are independent over time. However, serial autocorrelation is frequently not negligible in practice. It is assumed that autocorrelation is an inherent part of the process. Therefore, I want to consider a model that incorporates autocorrelation in the data, more specifically the second order autoregressive process. Limited study of the AR(1) process on X charts has been made by Baik(1991).

Suppose that the observation $X'(t)$ at time t is

$$X'(t) = X(t) + \delta\sigma_X,$$

where

$$X(t) = \phi_1 X(t-1) + \phi_2 X(t-2) + \cdots + \phi_p X(t-p) + a(t).$$

The random shock $a(t)$ is an uncorrelated random variable with $E(a(t))=0$ and $\mathrm{Var}(a(t))=\sigma_a^2$. Properties of an AR$(p)$ process are described in Box and Jenkins(1976). It is assumed that $X(t)$ is a stationary process. Then the parameters $\phi_1, \phi_2, \cdots, \phi_p$ satisfy some specified conditions. For $p=2$, the parameters ϕ_1 and ϕ_2 satisfy

$$\begin{aligned} &\phi_1 + \phi_1 < 1 \\ &\phi_2 - \phi_1 < 1 \\ &-1 < \phi_2 < 1\,. \end{aligned}$$

The stationary region for the parameters ϕ_1 and ϕ_2 is shown is Figure 1. The variance of AR(p) process is known to be

$$\sigma_X^2 = \sigma_a^2 \,/\, (1-\rho(1)\,\phi_1 - \rho(2)\,\phi_2 - \cdots - \rho(p)\,\phi_p)\,,$$

wher $\rho(i)$, $i=1, 2, \cdots, p$ is the autocorrelation between $X(t)$ and $X(t+i)$. Then the variance of the AR(2) process is

$$\sigma_X^2 = \frac{1-\phi_2}{1+\phi_2}\;\frac{\sigma_a^2}{(1-\phi_2)^2 - \phi_1^2}\,.$$

Note that as ϕ_1 or(and) ϕ_2 approaches the boundary of the stationary region in

Figure 1 the process variance increases. Note also that for AR(1) process

$$\sigma_X^2 = \sigma_a^2 / (1 - \phi_1^2) .$$

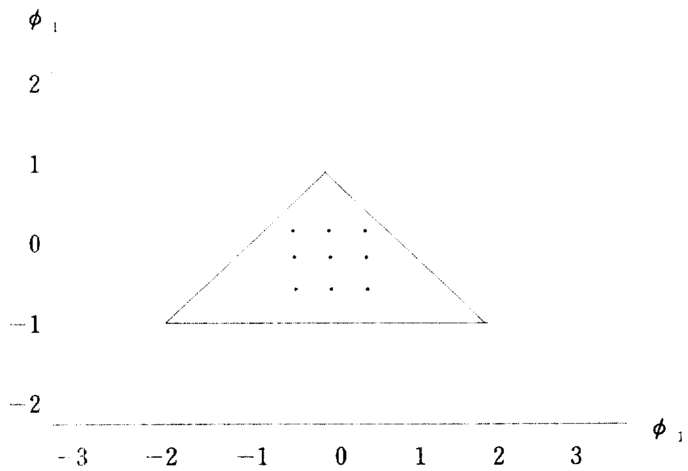

Figure 1. Stationary region of an AR(2) process

3. Estimation of the Process Standard Daviation

If the process standard deviation σ_X is known, then it can be used to set the control limits. The traditional control limits would be $\mu_0 \pm 3\sigma_X$ where μ_0 is the target value. However, the process stsndard deviation σ_X is not usually known. Hence, it needs to be estimated from previous observations. There are a couple of ways to estimate the process standard deviation σ_X .

According to Ryan(1989), the most commonly used procedure to estimate the process standard deviation σ_X when the smple size is only 1 is to create ranges by taking differences of successive observations, and dropping the sign of the difference when it is negative. Let $R(t)$ be the range of observation $X'(t+1)$ and $X'(t)$; $R(t) = | X'(t+1) - X'(t) |$. If the process mean is in control($\delta = 0$), then the average of the moving range of size 2 is used to estimate σ_X; $\hat{\sigma}_X = \overline{R}/d_2$ where d_2 is a correction factor that makes $\overline{R}/d_2$ an unbiased estmator of the process standard deviation σ_X if the sequential observations are independent. Ryan(1989) gives values of d_2 for different sample sizes. For example, $d_2 = 2/\pi^{1/2}$ for a moving range of size 2.

The other approach to the estimation of the process standard deviation is to use the sample standard devation. Ryan(1989) suggests taking a sample of at least 50 observations, and calculating the sample variance $S^2 = \sum (X'(t) - \overline{X'})^2/(n-1)$ in order to use S/c_4 as an estimator of the process standard deviation where $\overline{X'}$ is the average of $X'(1)$, $X'(2)$, $\cdots$, $X'(n)$ and c_4 is a correction factor that makes S/c_4 an unbiased estimator of σ_X if the sequential observations are independent. Values of c_4 for different sample sizes are also given in Ryan(1989). For instance, $c_4 = 0.9949$ for $n=50$. Note that c_4 is always less than 1.

Both $\overline{R}/d_2$ and S/c_4 are unbiased estimatiors of the process standard deviation σ_X for independent data. Hence, Wadsworth, Stephens and Godfrey(1986, p. 194) propose using any one of the above two estimators. However, Cryer and Ryan (1990) showed that if the observations are independent, then the variance of $\overline{R}/d_2$ is at least 60% greater than that of S/c_4. A number of other possible estimators of the standard deviation have been discussed by Roes, Does and Schurink(1993). But attention will be restricted to the above two most commonly used estimatiors

Cryer and Ryan(1990) have also shown that even if the data are correlated S/c_4 is still an asymptotic unbiased estimator of σ_X. However $\overline{R}/d_2$ is a biased estimator of σ_X. More specifically, if we let $r(k) = \mathrm{Cov}(X'(t), X'(t+k))$, then for a general AR($p$) process

$$X'(t) - X'(t-1) \sim N(0, 2(r(0) - r(1))).$$

Therefore, $E(R(t)/d_2) = (1 - \rho(1))^{1/2}\sigma_X$.
Hence, if the average of moving ranges of size 2 is used

$$E(\overline{R}/d_2) = (1 - \rho(1))^{1/2}\sigma_X$$

If the underlying process is an AR(2) process then

$$E(\overline{R}/d_2) = (1 - \phi_1/(1-\phi_2))^{1/2}\sigma_X$$

since $\rho(1) = \phi_1/(1-\phi_2)$ for AR(2) process. Note that for AR(1) process

$$E(\overline{R}/d_2) = (1 - \phi_1)^{1/2}\sigma_X$$

since $\rho(1) = \phi_1$ for AR(1) process. Hence, if the process mean is in control then S/c_4 should be used to estimate the process standard deviation σ_X.

4. X Control charts with $3\sigma_X$ (Markov chain approach)

It may be interesting to consider how X control charts perform as the correlation structure changes in terms of the autoregressive parameters. It is assumed that the correlation structure has been indentified and the control limits has been set at $\mu_0 \pm 3\sigma_X$. This would be the case when using S/c_4 as an estimator of the process standard deviation with large n assuming that the process is in control.

Since the underlying process can be modeled with AR process Markov chain approach can be used to evaluate the ARL of X charts. For a simple AR(1) process, if we want to know where the process is at time $t+1$, then we need to know where the process was at time t. At each time the process could be in any state within the control limits. In other words, in Figure 2, states k_1 and j_1 at times t and $t+1$ could be any states within the control limits. The total number of states for AR(1) process is assumed to be ν. That is, the whole interval within the control limits is discretized into ν discrete intervals. Then, for any given state, say, k_1 at time t it is probabilistically determined what the next state will be at time $t+1$.

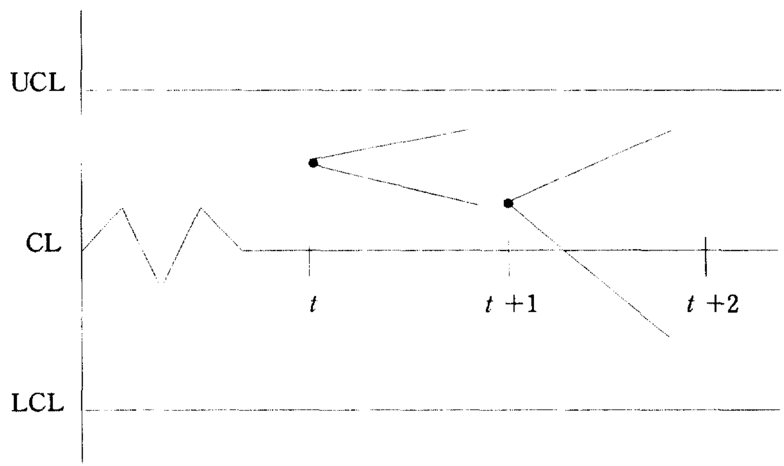

Figure 2. Markov chain representation

For an AR(p) process, if we want to know where the process is at time $t+1$, then we need to know where the process was at times $t+p-1$, $t+p-2$, $\cdots$, and t. At each time the process could be in any state. Let $k_1, k_2, \cdots, k_p$ be a sequence of states that the process is in at times $t+p-1$, $t+p-2$, $\cdots$, and t. Let $j_1, j_2, \cdots, j_p$ be a sequence of states that the process is in at times $t+p-2$, $t+p-3$, $\cdots$, and $t+1$. In this case, the transitional probability for an AR(p) process is

$$
\begin{aligned}
&P((k_1, k_2, \cdots, k_p) \to (j_1, j_2, \cdots, j_p)) \\
&= P(X'(t-p+2) \in I(j_1), X'(t-p+3) \in I(j_2), \cdots, X'(t+1) \in I(j_p) \mid \\
&\quad X'(t-p+1) \in I(k_1), X'(t-p+2) \in I(k_2), \cdots, X'(t) \in I(k_p)) \\
&\doteqdot P(X'(t+1) \in I(j_p) \mid X'(t-p+1) \in I(k_1), X'(t-p+2) \in I(k_2), \cdots, \\
&\quad X'(t) \in I(k_p)), \quad \text{if} \quad k_2 = j_1, k_3 = j_2, \cdots, k_p = j_{p-1} \\
&= P(\phi_1(i(k_p) - \delta\sigma_X) + \phi_2(i(k_{p-1}) - \delta\sigma_X) + \cdots + \phi_p(i(k_1) - \delta\sigma_X) + \\
&\quad a(t) + \delta\sigma_X \in I(j_p)),
\end{aligned}
$$

where $I(j_m)$ is the interval for state j_m and $i(j_m)$ is the middle point of the interval $I(j_m)$. Then the ARL vector becomes

$$
\text{ARL} = (I-Q)^{-1}\,\underline{1}\,,
$$

where Q is a ν^p by ν^p matrix whose elements can be obtained from the previous transitional probability and $\underline{1}$ is a ν^p by 1 column vector of 1's. Note that the total number of transient states for a general AR(p) process is ν^p.

Let ARLν be the ARL when there are ν^p transient states. Similar to the procedure of Brook and Evans(1972), an extrapolation to an infinite number of transient states is based on fitting the following formula by least squares:

$$
\text{ARL}\nu = \text{asymptotic ARL} + A/\nu^2 + B/\nu^4.
$$

As p increases, the total number of transient states that is needed for relatively accurate asymptotic value of ARL increases exponentially. A greater number of transient states is needed for an accurate asymptotic value of ARL near the boundary of the stationary region.

As an illustration, suppose that $p=2$. Then the transitional probability becomes

$$
\begin{aligned}
&P((k_1, k_2) \to (j_1, j_2)) \\
&\doteqdot P(X'(t+1) \in I(j_2) \mid X'(t-1) \in I(k_1), X'(t) \in I(k_2)), \quad \text{if } j_1 = k_2 \\
&= P(\phi_1(i(k_2) - \delta\sigma_X) + \phi_2(i(k_1) - \delta\sigma_X) + a(t) + \delta\sigma_X \in I(j_2)).
\end{aligned}
$$

Here Q is a ν^2 by ν^2 matrix. For simplicity, it is assumed that the process starts at a fixed point. In this case, if we let $X'(-1) = \mu_0 + \delta\sigma_X$ and $X'(0) = \mu_0 + \delta\sigma_X$ at times -1 and 0 respectively and if j_m is such that $X'(-1) \in I(j_m)$ and $X'(0) \in I(j_m)$, then the $((j_m - 1)\nu + j_m)^{th}$ element of the ARL vector is my desired ARL for a given deviation δ with starting state of j_m. Even with an AR(2) process, the total number of transient states that is needed for an accurate asymptotic value of ARL increases very rapidly as ν increases. In this paper it is found that relatively accurate value of ARL can be obtained with ν = 9, 12, 15, 18 and 21 transient states. An extrapolation to an infinite number of transient states is based on those ARL's. The computation is done assuming that the random shock $a(t)$ for the unit time series is $a(t) \sim N(0, 1)$.

Table 1 shows the ARL for some values of ϕ_1 and ϕ_2 within the stationary region in Figure 1, and for δ = 0.0, 0.5, 1, 3. Values of ϕ_1 and ϕ_2 are arbitrarily chosen to show the effect of each parameter on the control chart. They are −0.4, 0 and 0.4 respectively. According to Table 1, for a given value of ϕ_2, the ARL is smaller when $\phi_1 < 0$ than when $\phi_1 > 0$. For instance, suppose that $\phi_2 = 0$ and $\mu = \mu_0 + 0.5\sigma_X$. Then the ARL for $\phi_1 = -0.4$ is 158.033 while the ARL for $\phi_1 = 0.4$ is 167.701. The same phenomenon holds when $\phi_1 = 0$ and $\phi_2 \neq 0$. For instance, the ARL when $\phi_1 = 0$, $\phi_2 = -0.4$ and $\mu = \mu_0 + 0.5\sigma_X$ is 158.646 while the ARL when $\phi_1 = 0$, $\phi_2 = 0.4$ and $\mu = \mu_0 + 0.5\sigma_X$ is 168.048.

〈 Table 1 〉 ARL of X charts of an AR(2) process with

$$UCL = \mu_0 + 3\sigma_X$$
$$CL = \mu_0$$
$$LCL = \mu_0 - 3\sigma_X$$

ϕ_1	ϕ_2 \ δ	0	0.5	1	3
−0.40	0.40	531.065	219.015	67.650	1.886
0.00	0.40	384.947	168.048	51.552	2.239
0.40	0.40	531.142	267.862	98.703	3.564
−0.40	0.00	384.223	158.033	44.287	1.786
0.00	0.00	370.398	155.224	43.895	2.000
0.40	0.00	384.223	167.701	51.032	2.424
−0.40	−0.40	385.312	159.495	44.711	1.713
0.00	−0.40	384.947	158.646	44.563	1.862
0.40	−0.40	385.286	162.161	46.731	2.095

Next, it may also be interesting to know the effect of ϕ_2 together with the effect of ϕ_1. It is known that the effect of ϕ_1 alone is to increase the ARL whether the process is in control or not with the effect of positive ϕ_1 being larger than that of negative ϕ_1. But now if ϕ_2 is also positive then the ARL gets even greater. However, the effect of ϕ_1 is not great when $\phi_2 < 0$. The behavior of ARL can also be explained in terms of correlations:

$\rho(1) = \phi_1 / (1 - \phi_2)$
$\rho(2) = \phi_1 + \phi_1^2 / (1 - \phi_2)$

That is, positive correlations tend to increase the ARL while negative correlations tend to decrease the ARL. Finally, the ARL increases as ϕ_1 or(and) ϕ_2 approaches the boundary the stationary region.

5. Effects of $\overline{R}/d_2$ and S/c_4 (Simulation)

In Section 4, it is assumed that the control limits have been set up using S/c_4 with large n in order to estimate the process standard deviation. However, when individual measurements are taken the usual textbook approach to obtaining the control limits for an X control chart is to estimate the process standard deviation σ_X in terms of moving ranges of consecutive observations. Therefore, it may be interesting to see how X control charts perform when using $\overline{R}/d_2$ as an estimator of σ_X.

〈Table 2〉 Average run length and its standard error of an AR(2) process when S/c_4 is used with large n (5,000 simulations)

ϕ_1	ϕ_2 \ δ	0	0.5	1	3
−.40	.40	537.902	212.261	67.640	1.822
		(7.583)	(2.942)	(.918)	(.016)
.00	.40	379.466	168.511	50.022	2.124
		(5.303)	(2.325)	(.703)	(.026)
.40	.40	522.933	265.795	96.908	3.617
		(7.452)	(3.727)	(1.318)	(.073)
−.40	.00	390.640	159.431	44.607	1.794
		(5.398)	(2.243)	(.614)	(.015)
.00	.00	369.429	158.198	42.672	2.023
		(5.195)	(2.242)	(.597)	(.020)
.40	.00	385.193	171.647	48.754	2.378
		(5.563)	(2.415)	(.674)	(.030)
−.40	−.40	392.064	160.595	44.738	1.794
		(5.503)	(2.226)	(.617)	(.014)
.00	−.40	385.198	159.385	44.707	1.890
		(5.462)	(2.219)	(.619)	(.016)
.40	−.40	385.996	165.562	45.889	2.048
		(5.471)	(2.385)	(.636)	(.021)

Tables 2 and 3 show simulation results when using S/c_4 and $\bar{R}/d_2$ as estimators of the process standard deviation σ_X respectively. Note that Table 2 is the simulation results of Section 4(Table 1). Therefore, Table 2 should be similar to Table 1. In Table 3, the control limits are in fact assumed to be

$$\mu_0 \pm 3(1-\phi_1/(1-\phi_2))^{1/2}\sigma_X$$

since $E(\bar{R}/d_2) = (1-\rho(1))^{1/2}$ and $\rho(1) = \phi_1/(1-\phi_2)$ for an AR(2) process. Note that the control limits for an AR(1) process are

$$\mu_0 \pm 3(1-\phi_1)^{1/2}\sigma_X.$$

Therefore, the effect of positive ϕ_1 for an AR(1) process is to decrease the ARL while the effect of negative ϕ_1 is to increase the ARL compared to the ARL that can be obtained with S/c_4 as an estimator of σ_X as seen from the simulation results of Tables 2 and 3.

〈 Table 3 〉 Average run length and its standard error of an AR(2) process when $\bar{R}/d_2$ is used (5,000 simulations)

ϕ_1	ϕ_2 \ δ	0	0.5	1	3
−.40	.40	10746.050	3035.464	629.566	6.947
		(105.535)	(43.103)	(8.754)	(.090)
.00	.40	379.466	168.511	50.022	2.124
		(5.303)	(2.325)	(.703)	(.026)
.40	.40	23.790	19.052	10.497	1.068
		(.315)	(.267)	(.170)	(.006)
−.40	.00	2704.328	892.577	187.532	3.021
		(38.310)	(12.975)	(2.693)	(.034)
.00	.00	369.429	158.198	42.672	2.023
		(5.195)	(2.242)	(.597)	(.020)
.40	.00	53.236	31.874	13.229	1.447
		(.730)	(.446)	(.185)	(.014)
−.40	−.40	1543.624	526.957	123.874	2.554
		(21.922)	(7.355)	(1.734)	(.024)
.00	−.40	385.198	159.385	44.707	1.890
		(5.462)	(2.219)	(.619)	(.016)
.40	−.40	96.438	46.729	17.309	1.507
		(1.345)	(.641)	(.237)	(.013)

Since $0 < 1-\phi_2 < 2$ for stationary AR(2) process, $\phi_1/(1-\phi_2) > 0$ if and only if $\phi_1 > 0$. That is, effect of positive (negative) ϕ_1 is to decrease (increase) the ARL whether $\phi_2 > 0$ or not compared to the ARL that can be obtained with S/c_4 as an estimator of σ_X. For instance, the ARL when $\phi_1=0.4$, $\phi_2=0$, $\delta=0.5$ in Table 2 is 171.647 while the ARL in Table 3 is 31.874. Now if ϕ_2 is also positive then the ARL in Table 3 gets even decreased. For instance, if $\phi_1=0.4$, $\phi_2=0.4$, $\delta=0.5$, then the ARL is 19.052. However, if $\phi_1/(1-\phi_2)<0$ then the ARL increases compared to the ARL that can be obtained when using S/c_4. For instance, the ARL when $\phi_1=-0.4$, $\phi_2=0$, $\delta=0.5$ in Table 3 is 892.577 while the ARL in Table 2 is 159.531. Note that the ARL when $\phi_1=-0.4$, $\phi_2=-0.4$ and $\delta=0.5$ in Table 3 is 526.957 while the ARL when $\phi_1=-0.4$, $\phi_2=0.4$, $\delta=0.5$ is 3035.464. In other words, if $\phi_1 < 0$ and $\phi_2 > 0$ then the ARL's are a lot larger than would be expected when using S/c_4 as an estimator of σ_X.

6. Supplementary runs rule (Simulation)

Supplementary runs rules are frequently used in control charts. Commonly used rules are given by the Western Electric Handbook(1956), Wheeler(1983), and Champ and Woodall(1987). In the Section, it is assumed that a signal is given (1) if one observation is outside 3-sigma limits, (2) if two out of there consecutive observations fall between the same 2-sigma and 3-sigma limits, (3) if four out of five consecutive observations fall between the same 1-sigma and 3-sigma limits, or (4) if eight consecutive observations are on the same side of the center line. Here, sigma refers to the process standard deviation σ_X.

Simulation results are shown in Table 4. First, the results show that even for independent observations supplementary runs rule decrease the ARL dramatically. For instance, the ARL with the above supplementary runs rules when the process is in control is 92.044 while the ARL without the runs rules is 369.193 in Table 2. Simulation results in Table 4 for independent observations are similar to the exact results in Champ and Woodall(1987). Now if ϕ_1 is positive(assuming that $\phi_2=0$) then the effect of supplementary runs rules becomes greater. For example, when $\phi_1=0.4$ and $\phi_2=0$ the ARL when the process is in control is 33.258. However, if ϕ_2 is also positive, say, $\phi_2=0.4$ then the effect of supplementary runs rules decreases the ARL further down to 17.971. On the other hand, if the process is out of control then it takes less number of observations before control charts signal with supplementary runs rules. Even with negative values of ϕ_1 or(and) ϕ_2 the ARL with supplementary runs rules is much smaller than the ARL without supplementary runs rule in Table 2.

〈 Table 4 〉 Average run length and its standard error of an AR(2) process when S/c_4 as an estimator of σ_X and with supplementary runs rules. (5,000 simulations)

ϕ_1	ϕ_2 \ δ	0	0.5	1	3
−.40	.40	113.178	27.773	8.748	1.673
		(1.564)	(.324)	(.068)	(.011)
.00	.40	43.611	22.274	9.856	1.812
		(.566)	(.270)	(.103)	(.014)
.40	.40	17.971	15.306	10.119	1.958
		(.185)	(.164)	(.111)	(.017)
−.40	.00	189.259	36.254	9.202	1.701
		(2.654)	(.455)	(.083)	(.011)
.00	.00	92.044	27.654	9.193	1.792
		(1.253)	(.341)	(.089)	(.013)
.40	.00	33.258	19.483	9.337	1.946
		(.411)	(.226)	(.090)	(.016)
−.40	−.40	298.726	45.151	9.407	1.722
		(4.160)	(.588)	(.081)	(.011)
.00	−.40	193.411	36.200	9.078	1.779
		(2.629)	(.469)	(.077)	(.012)
.40	−.40	89.183	25.260	8.874	1.859
		(1.196)	(.301)	(.079)	(.014)

7. Conclusions

It has been shown that with large number of observations S/c_4 is better than $\bar{R}/d_2$ as an estimator of the process stanard deviation σ_X with or without correlation in the data.

For an AR(1) process, effect of ϕ_1 is to increase the ARL whether the process is in control or not with the effect of positive ϕ_1 being larger than that of negative ϕ_1. With additional positive value of ϕ_2 the ARL becomes even greater.

The effect of using $\bar{R}/d_2$ as an estimator of the process standard deviation instead of S/c_4 is enormous when dealing with correlated data. Especially, if ϕ_1 and ϕ_2 are positive then the ARL's that can be obtained using $\bar{R}/d_2$ are a lot smaller than would be expected when using S/c_4. However, if $\phi_1 < 0$ and $\phi_2 > 0$

the ARL's are a lot larger with $\bar{R}/d_2$ than with S/c_4.

Supplementary runs rules cause the ARL to decrease. The decrease in ARL is noticeable when ϕ_1 and ϕ_2 are both positive. However, even for independent data the decrease in the ARL is great with supplementary runs rules.

In this paper the effect of correlation in the data has been investigated assuming that the underlying process can be modeled in terms of a time series, especially the first and second order autoregressive process. For any higher order of autoregressive process the properties of control charts can be investigated using the same Markov chain approach or simulation method that have been used in this paper.

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