■ 연구논문

Variation Stack-Up Analysis Using Monte Carlo Simulation for Manufacturing Process Control and Specification

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Abstract

In modern manufacturing, a product consists of many components created by different processes. Variations in the individual component dimensions and in the processes may result in unacceptable final assemblies. Thus, engineers have increased pressure to properly set tolerance specifications for individual components and to control manufacturing processes. When a proper variation stack-up analysis is not performed for all of the components in a functional system, all component parts can be within specifications, but the final assembly may not be functional. Thus, in order to improve the performance of the final assembly, a proper variation stack-up analysis is essential for specifying dimensional tolerances and process control.

This research provides a detailed case example of the use of variation stack-up analysis using a Monte Carlo simulation method to improve the defect rate of a complex process, which is the commutator brush track undercut process of an armature assembly of a small motor. Variations in individual component dimensions and process mean shifts cause high defect rate. Since some dimensional characteristics have non-normal distributions and the stack-up function is non-linear, the Monte Carlo simulation method is used.

1. Introduction

In modern manufacturing, a product consists of many components created by different processes. The proper assembly of these components depends on the dimensional variations of each component from their nominal values. If manufacturing processes produce a component that is outside its tolerance specification range, the part will not fit into the rest of the assembly, and becomes scrap. But the problem is even more complex. Even if the individual components are within their tolerance specification ranges, component variations can be compounded during assembly such that the final assembly product is outside its performance requirement.

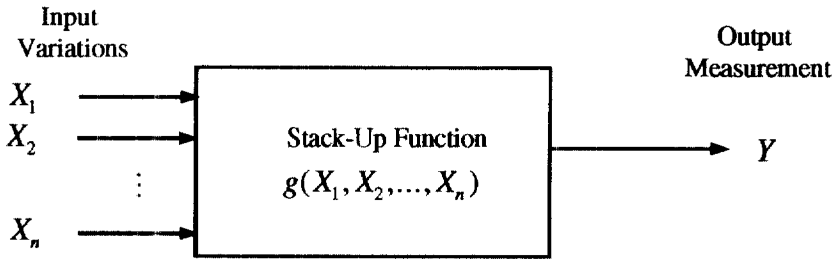
The tolerance of each dimensional characteristic is usually given as a range within which the dimension may deviate from its nominal value. It is usually set by design engineers using their experience, by draftsman, or as a part of default routine of a CAD system. The worst situation is where a proper stack-up analysis of dimensional tolerances has not been performed for all of the components in a functional system. When this is not done, all component parts can be within specifications, but the final assembly may not be functional. Thus, in order to improve the performance of the final assembly, a proper stack-up analysis is essential for specifying dimensional tolerances of each component.

The variation stack-up analysis determines the relationship between an assembly and its components and computes the expected variation of the assembly based on the components tolerances and variations in manufacturing system. The three key elements that are necessary for the variation stack-up analysis of the assembly are shown in ⟨Figure 1.1⟩. The three key elements are identified as follows:

(1) Input variations have an effect on the performance of a finished assembly product. This includes all the variations from individual component tolerances and variations due to random factors existing in the manufacturing system. These variations are specified with their means, standard deviations, and their statistical distribution types.

(2) Output measurement is a performance characteristic of the final assembly product. It will be expressed as a functional relationship to the dimensional characteristics of each component.

(3) Stack-up function relates the input variations or independent variables and the output measurement or dependent variable.



⟨ Figure 1.1 ⟩ Three key elements for the variation stack-up analysis

The Monte Carlo simulation method is the most recent technique developed for variation stack-up analysis [1, 5, 10, 16]. It is particularly useful when: (1) a large number of component dimensional characteristics affects the assembly dimension, (2) the component dimensional characteristics are known to have a probability distribution which is not normal, (3) the relationship between the component dimensions and the assembly dimension is not linear, (4) the assembly dimension may be physically impossible to measure, such as a dimension hidden in an assembly or an internal clearance.

The simulation analysis estimates the effects of variations of component dimensional characteristics on the assembly dimension by using a computer model of individual component characteristics and the assembly process. The model is first created to express the assembly as a function of the component dimensional characteristics. All anticipated characteristics and process variations are identified and presented by statistical distributions. The relationships between the component dimensions and the final assembly are expressed in the model equations. The distribution of each characteristic or each process step is then defined in terms of the statistical distribution type, mean, and variation. The model may be a simple or complex combination of individual component dimensional characteristics.

The magnitude of variation is determined by software using a Monte Carlo technique. Each simulated characteristic value is randomly chosen from a modeled distribution. In a Monte Carlo simulation, the computer uses a pseudo random number generator to draw values from each characteristic distribution and combines them using the model equation to obtain a simulated value of the assembly. This process is repeated many times and simulates what would happen if many assemblies were made at random from characteristics described in the model. The simulated assembly is then summarized in a histogram and used to evaluate tolerances.

Recently, commercial PC-based packages enabling the simulation of tolerances have become available. Pugh (1988) briefly describes a software package called PRISM which allows the user to simulate processes. A more comprehensive package, Variation Simulation Analysis software, is also available (1990). This software uses statistical simulation techniques to predict the amount of variation that can occur in an assembly due to specified design tolerances and manufacturing or assembly variation. Additionally, it can determine the locations of the predicted variation, the contributing factors, and their percentage of contribution. This package includes a useful graphics preprocessor for interfacing with CAD packages and interactive simulation model building as well as a simulation language. A package, called GA-2000, by John Deere and Co. is also available (1991).

There are two applications of the Monte Carlo simulation method to actual tolerancing problems. Doydum and Perreira (1991) presented an analytical method using the Monte Carlo simulation for selecting the dimensions and tolerances of mating parts and precision of assembly equipment where the mating parts possessed simple geometrics such as line and circle. Another application of the Monte Carlo simulation method is to the circuit tolerance analysis problem which, although it has the attraction of insensitivity to the number of toleranced components, is computationally expensive. Soin and Rankin (1985) have used variance prediction techniques to improve simulation efficiency.

A sensitivity analysis can help engineers evaluate the contribution of each input variation to the final assembly performance variability. By ranking the contribution of each input variable, the engineers can know which input variations are critical to the variation of the output measurement. Therefore, the engineers can invest corrective action efforts most profitably to reduce the variability of the final assembly. The analysis is performed separately from random simulations, and varies each input variation to its high, low, and median values, one at a time, while holding all other input variations at their median values. It then notes the effect, if any, on the output measurement. It can be called the High-Low-Median (HLM) analysis.

For each input variation, the range R_Y which is total amount of output measurement changed when the input variation is changed from high to low to median can be calculated:

$$R_Y = Y_{max} - Y_{min}$$

where Y_{max} is the maximum value that the output measurement reached and Y_{min} is the minimum value that the output measurement reached.

From this range calculation, an approximation of the output measurement variance, σ_Y^2 due to an input variation is determined. The variance is a statistic that defines the amount of spread that a group of data has. Since the group of data generated from the HLM analysis is only two values, output measurement high and low, the variance is approximated. For HLM purposes, simulation assumes that the output measurement behaves normally, and that the range between output measurement high and low is equal to six standard deviations, or:

$$R_Y = 6\sigma_Y$$

$$\sigma_Y^2 = \frac{R_Y^2}{36}$$

where σ_Y^2 is the variance of the output measurement due to an input variation.

HLM analysis calculates σ_Y^2 in this manner for all n input variations and their effect on all output measurements. Then, the results from each output are summed to give the main effect variance:

$$\sigma_{HLM}^2 = \sigma_{Y_1}^2 + \sigma_{Y_2}^2 + \cdots + \sigma_{Y_n}^2$$

The percentage which each input variation contributes to the overall HLM variation of the output measurement is:

$$\% \text{ Contribution} = \left(\frac{\sigma_{Y_i}^2}{\sigma_{HLM}^2} \right) \times 100$$

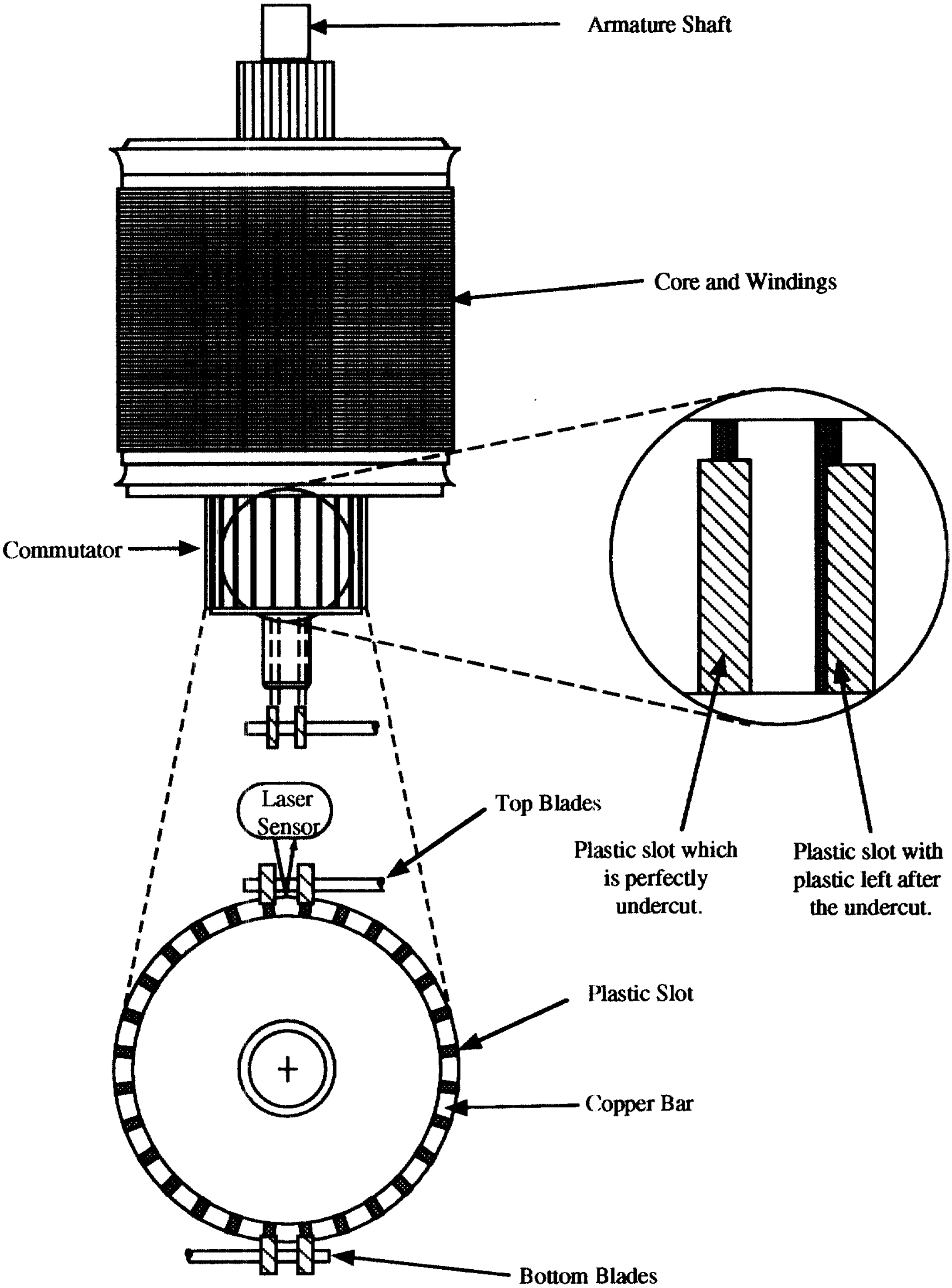
This process ranks the input variations in order of contribution to the variation of the output measurement based on the HLM analysis and determines the major contributors.

2. Case Example : Commutator Brush Track Undercut Process

The case study was conducted on the commutator brush track undercut process of an armature assembly of a small motor. Dimensional variations in the components of the armature assembly and process variations in the undercut machine resulted in unacceptable final assemblies. The study will (1) identify the causes of high reject rate at this process, (2) evaluate the performance of the armature assembly by performing the proper variation stack-up analysis using the Monte Carlo simulation method, and (3) improve the quality of the armature assembly by changing specifications of component dimensional characteristics, or specifying process variations based on the simulation results.

2.1 Product and Process Descriptions

An armature assembly in ⟨Figure 2.1⟩ which is the rotating element of a small motor includes a shaft, a core and windings, and a commutator. The armature winding is suitably placed in slots in the core and the coil ends are terminated to appropriate commutator bars. Current flow in the armature winding produces a magnetic field which reacts with the field flux to produce torque. The commutator forms a mechanical switch with four brushes which ensures that the



〈 Figure 2.1 〉 An armature assembly of a small motor and the commutator brush track undercut process machine.

current flow through the armature winding is in the proper direction at any instant. Successful operation of the motor depends, to a large extent, upon proper operation of the commutator and upon the choice and location of the brushes [13, 17].

Commutator segments are made of copper, and each copper bar is insulated by plastic slots between the bars. The brushes, which are made of graphite and other materials, are rubbed off through contact with the commutator surface and through arcing between the brushes and the commutator bars. Since the plastic is harder than the brush materials, the plastic is undercut.

As seen in 〈Figure 2.1〉, the undercut machine consists of four cutting blades and an optical laser sensor detecting the plastic slot to be cut first. The undercut machine mills out 28 plastic slots on the brush track. The plastic slots are located through the use of the optical laser sensor, and four cutting blades then index into the part, cutting four diametrically opposed slots at the depth indicated on the process illustration. After the cut is completed, the cycle repeats 6 more times until 28 plastic slots have been cut.

2.2 Problem Statement

The undercut process does not produce armatures with no plastic left on the commutator brush track surface 100% of the time. The remaining plastic after the undercut process causes the excess brush wear and the brush material smears on the commutator surface, deteriorating the performance of starting motors. Thus, this process is critical to the performance of the small motor. In order to rework the worst armatures, 100% inspection after the undercut process is necessary.

The manufacturing engineers investigated the process to find major causes of the position variation of the plastic slot and resulting in plastic left after the undercut operation. However, it was difficult to find the major contributors, because many dimensional characteristics of the components affect the final result and their stacking variation is not linear. Moreover, some distributions of the dimensional characteristics are not normal. Therefore, the Monte Carlo simulation method for the variation stack-up analysis will be attempted.

3. Simulation Model for Variation Stack-Up Analysis

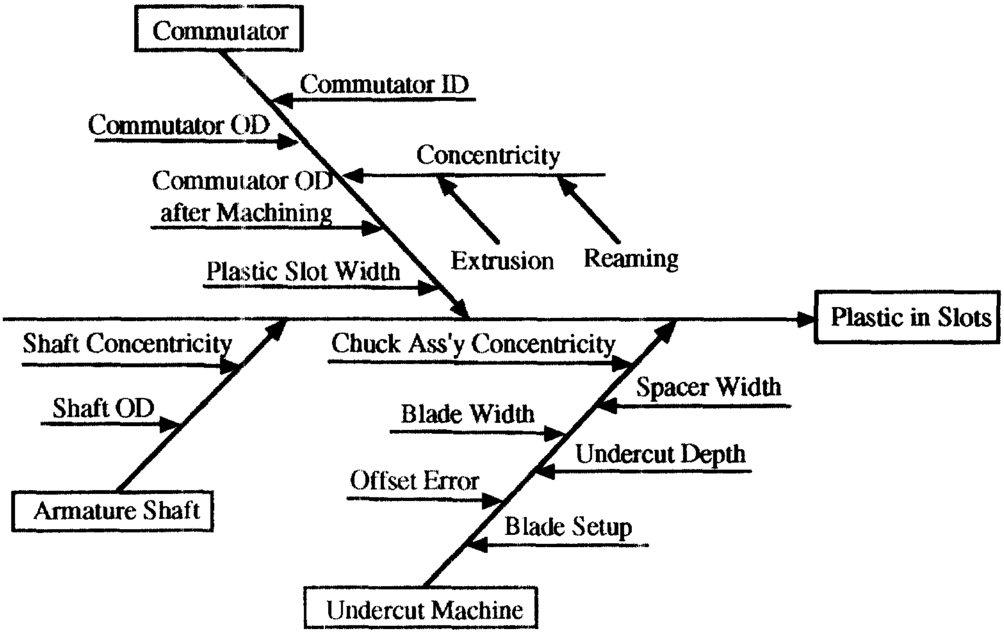
3.1 Input Variations

The input variations in this simulation model are dimensional characteristics of each component of the armature assembly and process variations in the undercut process. These input variations will determine the output measurement. The cross-functional team of design and manufacturing engineers was organized to investigate all the input variations which affect the output measurement. In order to clearly illustrate the input variations, a cause and effect diagram is used.

A cause and effect diagram for the commutator brush track undercut process is shown in 〈Figure 3.1〉. All the variations which may affect the amount of plastic left after the commutator brush track undercut process were listed. There are three categories: commutator, armature shaft, and undercut machine.

In the commutator and the armature shaft, all dimensional characteristics are listed, which affect the amount of plastic left after the undercut process. The commutator concentricity due to extrusion and reaming process and the armature shaft concentricity are critical to the plastic slot position variation in the undercut process. These concentricities cause the plastic slot position variation when the undercut machine rotates the armature assembly around the armature shaft center in the undercut operation. The plastic slot position variation due to these concentricities will be discussed later.

In the undercut machine, all variations which exist during the operation are listed. Some of the variations under the undercut machine are not specified in the engineering drawing. This diagram was developed by the cross-functional team of design and manufacturing engineers.



〈 Figure 3.1 〉 A cause and effect diagram of the amount of plastic left after the undercut process.

All dimensional characteristics listed in the cause and effect diagram will be specified in a simulation model in terms of statistical distribution type, mean, and standard deviation. First, they are specified based on the design specifications to facilitate understanding of how the product was designed. Since the process capability of 1.33 is required at the design stage, $\pm 4\sigma$ tolerance limits are used for each characteristic. Mean values of individual characteristics are assumed to be equal to the nominals of their design specifications. 〈Table 3.1〉 shows design specifications of all dimensional characteristics listed in the cause and effect diagram. All characteristics except concentricities are assumed to be normally distributed. The concentricities of the commutator, the armature shaft, and the undercut machine chuck assembly have Rayleigh distributions, which will be discussed later.

〈 Table 3.1 〉 Design specifications of the input variations.

Input Variation		Assumed Distribution	Design Specification* [mm]
X_1	Commutator ID	Normal	11.270 ± 0.100
X_2	Commutator OD	Normal	28.600 ± 0.120
X_3	Comm OD after Machining	Normal	28.200 ± 0.120
X_4	Plastic Slot Width	Normal	0.850 ± 0.060
X_5	Comm Shell Concentricity	Rayleigh	None
X_6	Molded Comm Concentricity	Rayleigh	$0 + 0.127$
X_7	Shaft Concentricity	Rayleigh	None
X_8	Shaft OD	Normal	11.820 ± 0.025
X_9	Chuck Concentricity	Rayleigh	None
X_{10}	Spacer Width	Normal	2.186 ± 0.025
X_{11}	Blade Width	Normal	1.016 ± 0.025
X_{12}	Undercut Depth	Normal	0.750 ± 0.200
X_{13}	Offset Error	Normal	None
X_{14}	Blade Setup Error	Normal	None

* Each specification is disguised for product confidentiality.

Note that some characteristics do not have the design specifications in 〈Table 3.1〉. Actual manufacturing data of these characteristics are collected in order to obtain the estimates of the means and the standard deviations. 〈Table 3.2〉 lists the characteristics and their means and standard deviations.

〈 Table 3.2 〉 Some input variations specified by actual manufacturing data.

Input Variation		μ [mm]	σ [mm]	Source
X_5	Comm Shell Concentricity	0.058	0.030	64 samples
X_7	Shaft Concentricity	0.022	0.011	64 samples
X_9	Chuck Concentricity	0.016	0.009	Engineering Knowledge
X_{13}	Offset Error	0.048	0.036	24 samples
X_{14}	Blade Setup Error	0.042	0.024	24 samples

The commutator shell concentricity which is generated in the extrusion process does not have a design specification. The armature shaft concentricity, which is due to cold heading and centerless grinding, is not specified in the engineering drawing. Thus, for each concentricity, 64 samples of size 5 are collected and the total indicator reading (TIR) data of the samples are measured to estimate the mean and standard deviation. The standard deviation is a part-to-part variation because no mean shift was detected. The chuck concentricity of the undercut machine is not specified in the design and cannot be measured easily. Thus, we are forced to estimate its mean and standard deviation from the engineering knowledge.

The optical laser sensor has errors in detecting the first plastic slot to be undercut. This is called the offset error and is estimated from the 24 samples of size 5 undercut armatures. Even if the optical laser sensor works perfectly, the blade setup is a critical factor which may affect the total amount of plastic after the undercut operation. Manufacturing engineers believe that the final machining process may knock some plastic left on the trailing edge after the undercut process. Thus, they bias the means of offset and blade setup such that the bias can increase the probability of plastic being left on the trailing edge.

3.2 Output Measurement

The manufacturing engineers believe that the more plastic left after the undercut process, the greater the amount of smearing that occurs on the commutator surface, and that this affects the speed performance of the starting motor. In the final test of the starting motor, most of the motors which are rejected due to speed failure show the severe smearing on the commutator surface.

In order to use the total amount of plastic left after the undercut process as an output measurement in the simulation model, we investigate the relationship between the total amount of plastic left and the amount of smearing. We define the dependent variable in the regression model as follows:

$$y = \begin{cases} 1 & \text{no smear} \\ 2 & \text{little smear} \\ 3 & \text{moderate smear} \\ 4 & \text{much smear} \end{cases}$$

Then the regression model is

$$y = 0.512x + 0.764, \qquad R^2 = 0.856$$

where x is the total amount in mm of plastic left after the undercut process. Therefore, we can conclude that the more plastic left after the undercut process, the greater will be the amount of smearing on the commutator surface.

Next, we investigate the relationship between the total amount of plastic left and performance speed in rpm under a 2 *Ft / Lb* load. The regression equation is

$$y = -71.654x + 286.403, \qquad R^2 = 0.713$$

where x is the total amount in mm of plastic left after the undercut process. Therefore, we can conclude that the total amount of plastic left after the undercut process affects the performance speed of the final product. As the total amount of plastic left increases, the performance speed reduces at a 2 *Ft / Lb* load.

Consequently, we select the total amount of plastic left after the undercut process as an output measurement in the simulation model.

3.3 Stack-Up Function

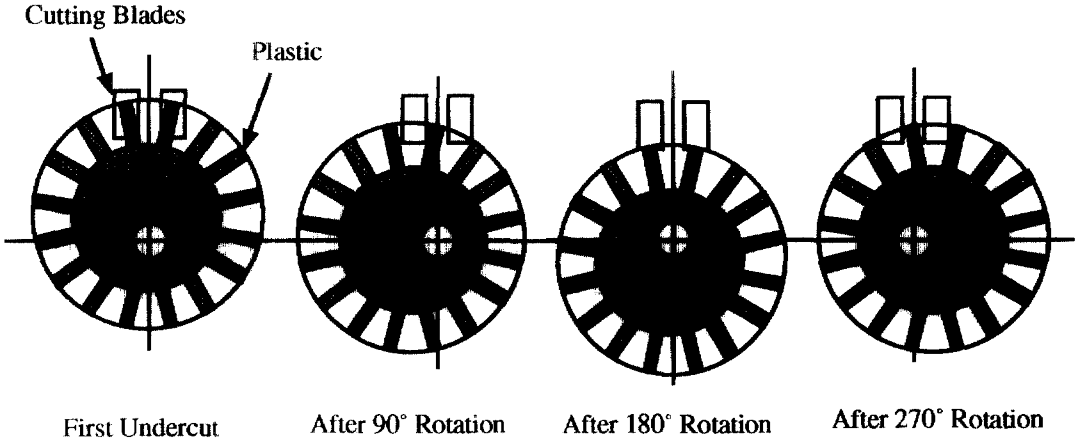
The stack-up function which relates the input variations to the output measurement is non-linear in this example because many sine and cosine functions are involved in the rotation of the armature assembly during the undercut operation. A geometrical model of plastic slot position variation due to commutator concentricity will be discussed.

3.3.1 Effect of Commutator Concentricity on Plastic Slot Position Variation

The mathematical equation of plastic slot position variation due to commutator concentricity can be developed from two different examples in ⟨Figure 3.2⟩ and ⟨Figure 3.3⟩. For demonstration purposes only, two cutting blades are shown in these figures. In both examples, the commutator is assumed not to be concentric to the rotating axis of the armature shaft because of the commutator concentricity

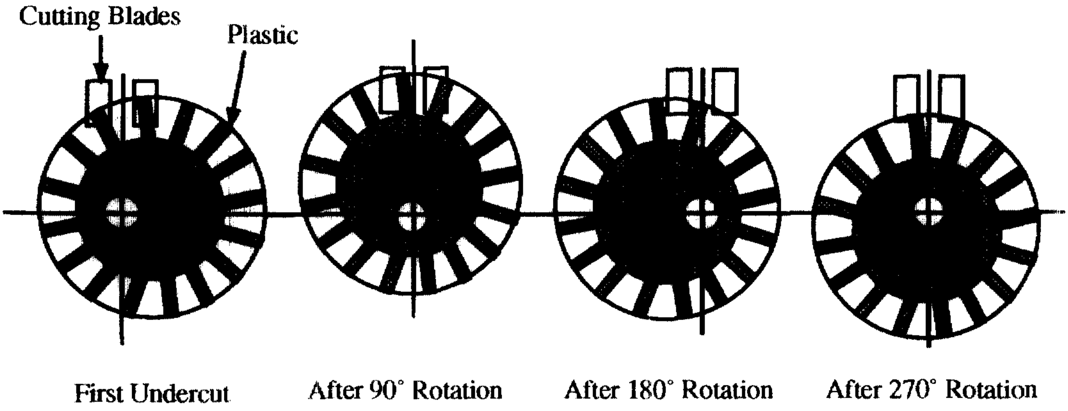
The undercut machine chucks the armature shaft and rotates it in counterclockwise. An optical laser sensor determines the first plastic slot to be undercut.

In 〈Figure 3.2〉, the commutator center is vertically in line with the armature shaft center. If the optical laser sensor correctly works in detecting the first plastic slots to be cut, then in the first cut, two cutting blades undercut plastic such that no plastic remains on the surface of both plastic slots. If the commutator rotates 90° counterclockwise after the correct first cut, the cutting blades undercut some copper rather than plastic because of the commutator concentricity in this exaggerated example. After 180° rotation counterclockwise, the blades undercut plastic correctly because both centers of commutator and armature shaft are vertically in line with each other. After 270° rotation, the blades undercut some copper rather than plastic in the opposite direction to the case of 90° rotation.



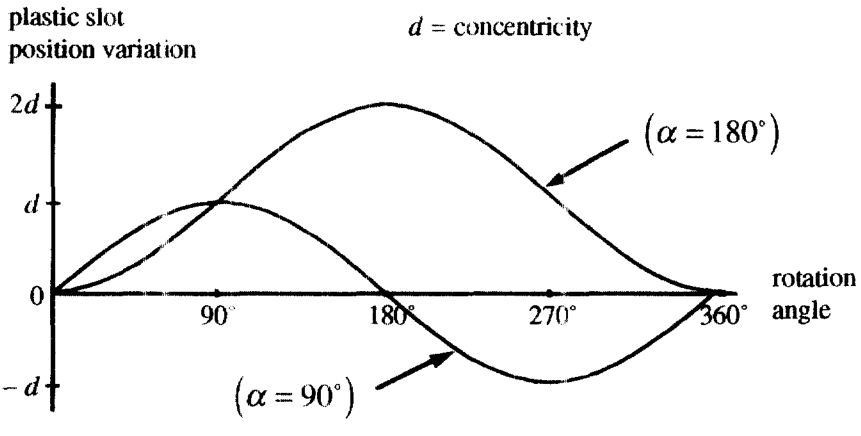
〈 Figure 3.2 〉 Plastic slot position variation due to commutator concentricity-centers of commutator and armature shaft are vertically in line with each other.

In 〈Figure 3.3〉, The commutator center is horizontally in line with the armature shaft center. This case shows an entirely different result in terms of the amount of plastic left after the undercut operation. If the commutator rotates 90° counterclockwise after the correct initial cut, the cutting blades are supposed to undercut plastic slots 5 and 6. But in this exaggerated example, the cutting blades undercut plastic slots 6 and 7 rather than plastic slot 5 and 6. As the rotation continues to 180°, the plastic slot position variation increases and the undercut worsens. At 180° rotation, the plastic slot position variation has maximum value and the undercut is worst. After 180° rotation, the plastic slot position variation decreases and the undercut gets better.



〈 Figure 3.3 〉 Plastic slot position variation due to commutator concentricity-centers of commutator and armature shaft are horizontally in line with each other.

The relationships between plastic position variation and rotation angle in both examples are summarized in 〈Figure 3.4〉. It shows different position variation patterns by varying the rotation angle. If a uniform random variable, α, is introduced for determining the direction in which the commutator center points, then 〈Figure 3.2〉 and 〈Figure 3.3〉 are the same at $\alpha = 90°$ and $\alpha = 180°$ respectively.



〈 Figure 3.4 〉 Plastic slot position variation versus rotation angle in both examples.

From these examples, a general equation of the plastic slot position variation due to commutator concentricity can be developed. The equation covers all cases from $\alpha = -180°$ to $\alpha = 180°$.

$$\text{position variation} = d\,\{\cos(\text{rotation angle} + \alpha) - \cos\alpha\}$$

where α is a uniformly distributed random variable used to determine the direction in which the commutator center will point.

Commutator concentricity d is another random variable which is Rayleigh distributed. The magnitude of the concentricity can be generally expressed as:

$$d = \sqrt{x^2 + y^2}$$

where x and y are positioning errors in x and y directions, respectively. These errors are independent and, from practical experience, the distributions are almost normal. If the positioning errors x and y are assumed independent and normally distributed random variables with an equal standard deviation σ and a zero mean value, then the density distribution of the magnitude of the commutator concentricity, d is given

$$f(x) = \begin{cases} \dfrac{d}{\sigma^2} \exp\left(-\dfrac{d^2}{2\sigma^2}\right), & d \geq 0,\ \sigma > 0 \\ 0 & \text{elsewhere} \end{cases}$$

which is the density of a Rayleigh distribution [Davenport, 1970]. The commutator concentricity is a summation of four concentricities generated by different processes: (1) at the extrusion process of commutator shell, the anchor configuration and profile of the extruded commutator shell, (2) the molded commutator concentricity after the reaming process, (3) the armature shaft concentricity due to cold heading and centerless grinding, and (4) the undercut machine chuck concentricity.

3.3.2 Total Amount of Plastic Left

The total amount of plastic left in the final armature assembly can be expressed in terms of geometrical variations in ⟨Figure 3.5⟩. If a plastic slot is cut by blade #1 or #3, then the dimensions $\overline{EF}$ and $\overline{GH}$ in ⟨Figure 3.5⟩ are expressed (see Table 3.1):

$$\overline{EF} = \frac{\overline{EH} - X_4}{2} - \frac{X_{pos} - \{(X_3 \sin\theta - X_{11}) - X_{10}\} + X_{13} - X_{14}}{\cos\theta}$$

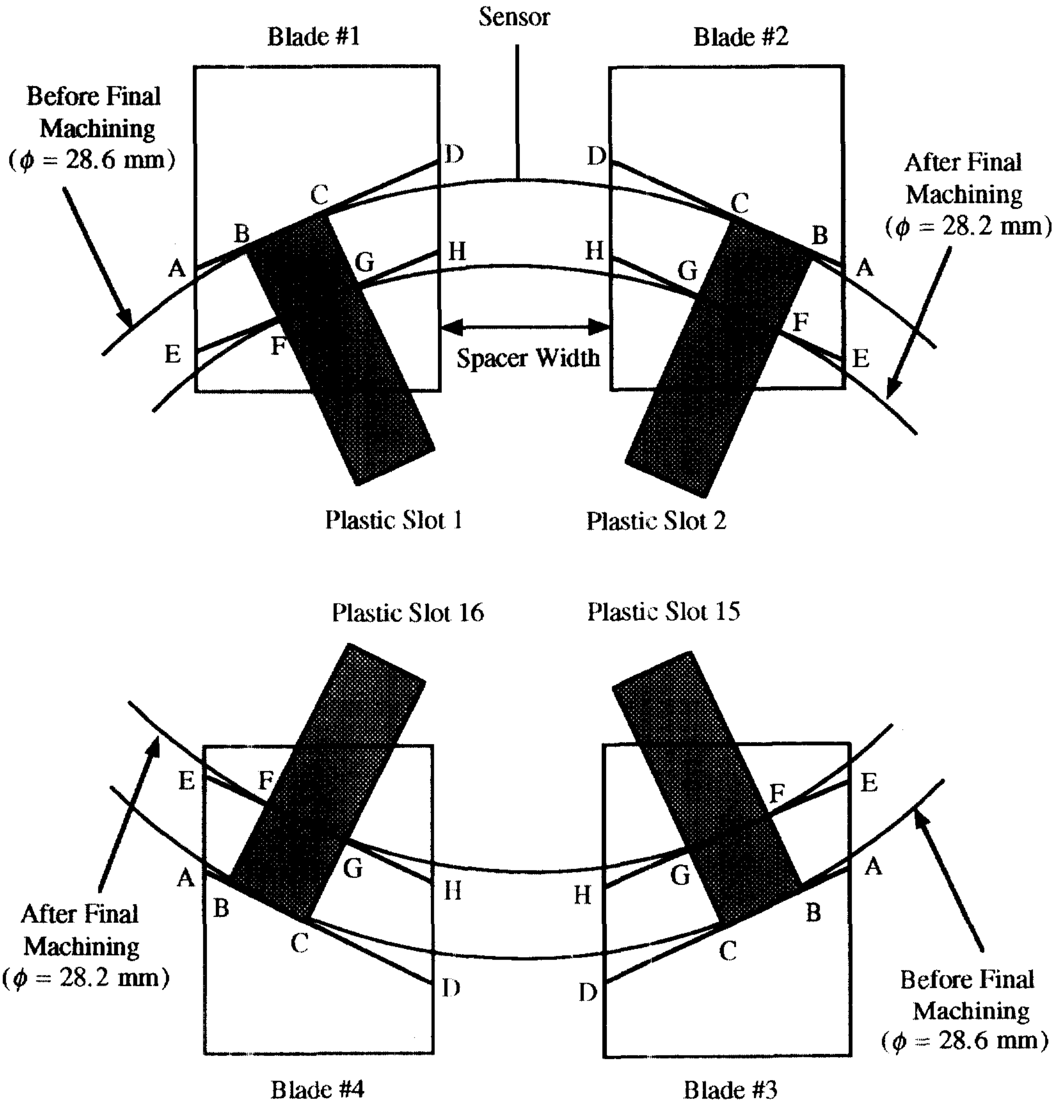
$$\overline{GH} = \frac{\overline{EH} - X_4}{2} + \frac{X_{pos} - \{(X_3 \sin\theta - X_{11}) - X_{10}\} + X_{13} - X_{14}}{\cos\theta}$$

where X_{pos} = Plastic slot position variation due to the commutator concentricity,

$\theta = \frac{360/n}{2}$,

n = Number of plastic slots in the commutator.

If $\overline{EF}$ has a negative value, then it is the amount of plastic left after the final machining process on the trailing edge. If $\overline{GH}$ has a negative value, then it is the amount of plastic left after the final machining process on the leading edge.



〈 Figure 3.5 〉 A cross-sectional view of commutator undercutting with four cutting blades.

If a plastic slot is cut by blade #2 or #4, then the dimensions $\overline{EF}$ and $\overline{GH}$ in ⟨Figure 3.5⟩ are expressed:

$$\overline{EF} = \frac{\overline{EH} - X_4}{2} + \frac{X_{pos} + \{(X_3 \sin\theta - X_{11}) - X_{10}\} + X_{13} - X_{14}}{\cos\theta}$$

$$\overline{GH} = \frac{\overline{EH} - X_4}{2} - \frac{X_{pos} + \{(X_3 \sin\theta - X_{11}) - X_{10}\} + X_{13} - X_{14}}{\cos\theta}$$

If $\overline{EF}$ has a negative value, then it is the amount of plastic left after the final machining process on the leading edge. If $\overline{GH}$ has a negative value, then it is the amount of plastic left after the final machining process on the trailing edge.

Total amount of plastic left in the final armature assembly is the summation of negative values of $\overline{EF}$ and $\overline{GH}$ for all the plastic slots.

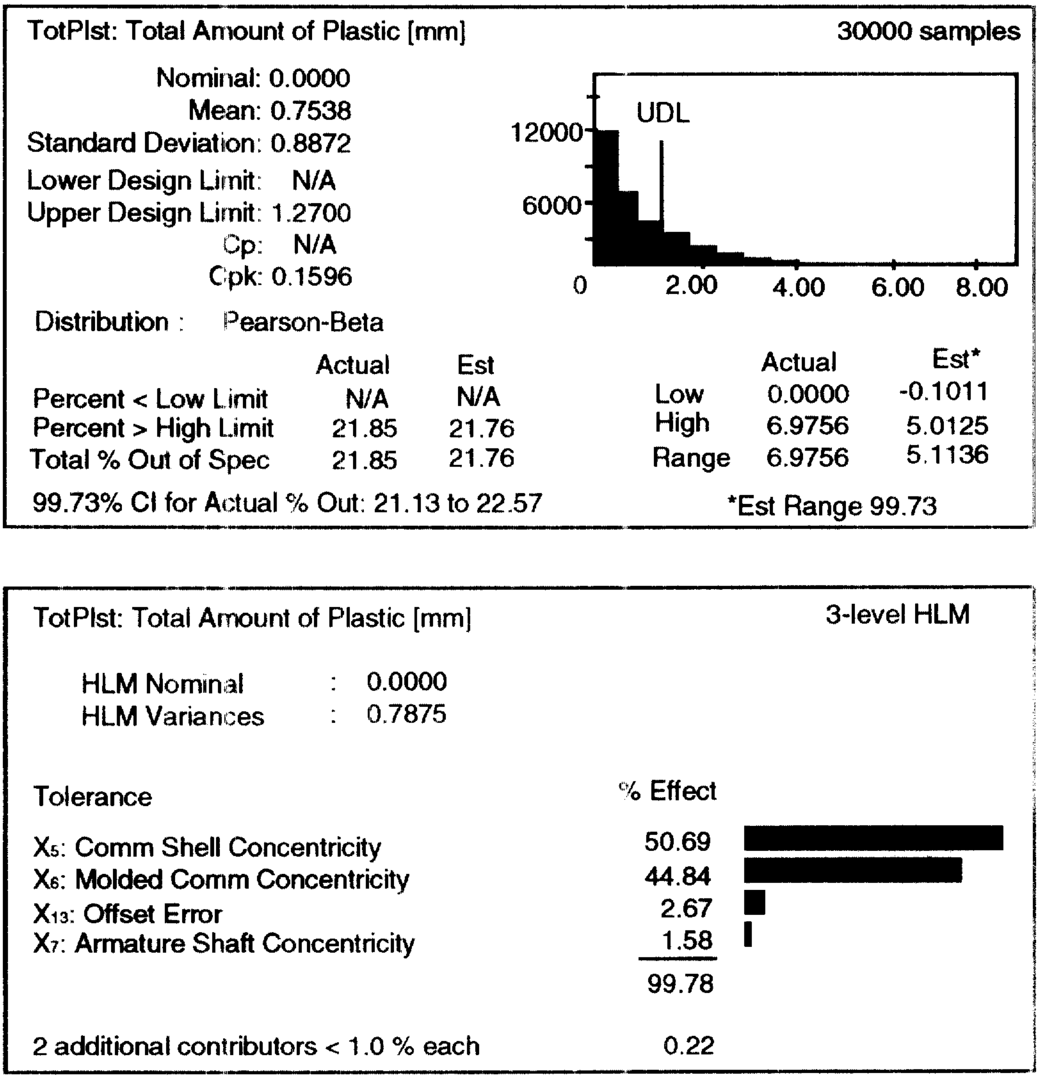
4. Monte Carlo Simulation Results

4.1 Under the Current Production Situation

The VSA® simulation software package (1990) is used to statistically analyze the total amount of plastic left in the final armature assembly. In order to understand how the product is actually being manufactured, 30,000 simulation runs are performed, in which each input variation is specified based on the actual manufacturing data.

⟨Figure 4.1⟩ is the VSA simulation results and shows that the current manufacturing situation cannot produce plastic free armatures 100% of the time. An average of 0.7538 mm of the total amount of plastic will be left in the final armature assembly. If we set the upper design limit to 1.27 mm which is the maximum allowed total amount of plastic before the reliability of the motor is affected according to field return data, then 21.85% of the armatures manufactured have more than 1.27 mm of the total amount of plastic in the final armature assembly. Among the 30,000 samples, the greatest amount of plastic left is 6.9756 mm. The 99.73% confidence interval of the reject rate is 21.13 to 22.57%. The process capability of the undercut process is 0.1596 which means the this process is not capable of producing a good armature assembly which can lie within the design specification limits.

In order to determine which input variations have contributed the most variation of total amount of plastic left in the final armature assembly, a HLM analysis is conducted. This analysis ranks input variations according to their contribution percentage. The major contributors which, in total, have more than 90% effect on the variation of the total amount of plastic are the commutator shell and the molded commutator concentricities. Thus, the plastic slot position variation during the undercut operation is the most important factor to the total amount of plastic left. Note that the commutator concentricity which has the greatest contribution is not specified in the design stage and not controlled during the extrusion process.



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TotPlst: Total Amount of Plastic [mm]                              30000 samples

              Nominal: 0.0000
                 Mean: 0.7538
   Standard Deviation: 0.8872
   Lower Design Limit:  N/A
   Upper Design Limit: 1.2700
                   Cp:  N/A
                  Cpk: 0.1596

 Distribution :    Pearson-Beta

                          Actual     Est                   Actual      Est*
 Percent < Low Limit       N/A       N/A          Low      0.0000    -0.1011
 Percent > High Limit     21.85     21.76         High     6.9756     5.0125
 Total % Out of Spec      21.85     21.76         Range    6.9756     5.1136

 99.73% CI for Actual % Out: 21.13 to 22.57              *Est Range 99.73
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```
TotPlst: Total Amount of Plastic [mm]                              3-level HLM

     HLM Nominal        :  0.0000
     HLM Variances      :  0.7875

 Tolerance                                        % Effect

 X5: Comm Shell Concentricity                      50.69
 X6: Molded Comm Concentricity                     44.84
 X13: Offset Error                                  2.67
 X7: Armature Shaft Concentricity                   1.58
                                                   -----
                                                   99.78

 2 additional contributors < 1.0 % each             0.22
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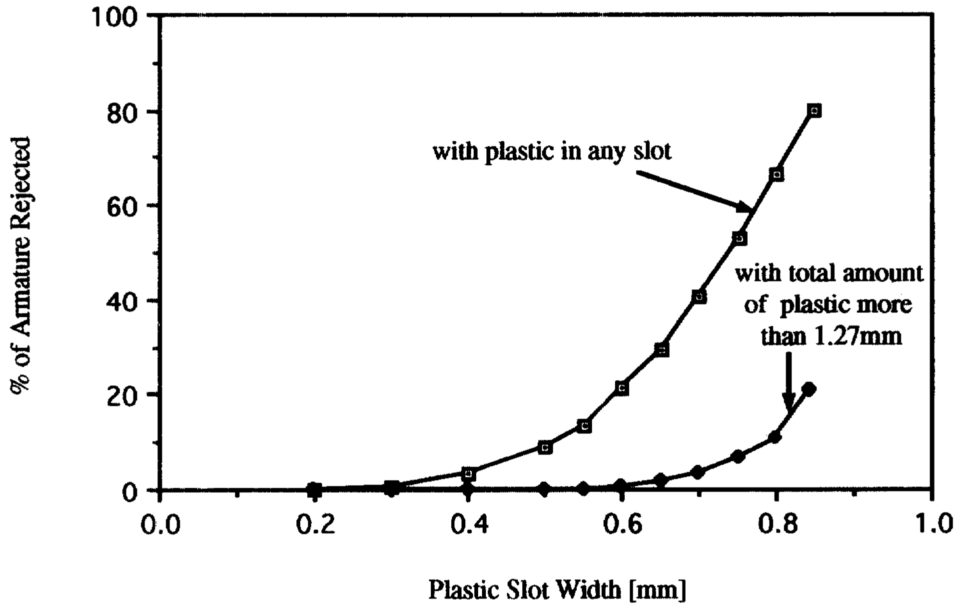
〈 Figure 4 1 〉 VSA simulation result on total amount of plastic under the current production situation.

4.2 Effect of Reducing Plastic Slot Width

This simulation model can be used to predict the effect of reducing the plastic slot width before actual components are manufactured. 〈Figure 4.2〉 shows the effect of changing the plastic slot width. Currently, the nominal of the plastic slot width is 0.85 mm, and, according to the simulation, 21.85% of armatures have more than 1.27 mm of accumulated plastic after the undercut process. By reducing the plastic slot width to 0.55 mm, we can expect no armature to have more than 1.27 mm of total amount of plastic left in the final armature assembly. If our goal is plastic free armature after the undercut process, then plastic slot width should be reduced to 0.20 mm.



〈 Figure 4.2 〉 Effect of reducing the plastic slot width on total amount of plastic left

4.3 Changing Specifications of Major Contributors

Since the current manufacturing situation cannot produce the good final armature assembly which can satisfy the functional requirements, some improvements must be done. As seen the effect of reducing the plastic slot width on total amount of plastic left in the final armature assembly, we can reduce the plastic slot width to 0.620 mm with the same variation. The commutator concentricity which is the major contributor in the HLM analysis cannot be improved easily. Thus, by improving the reaming operation, we can reduce the molded commutator concentricity which is the second major contributor. Also, the manufacturing engineers changed the shaft machining process to improve the shaft concentricity. The offset error is the third important factor, but it comes from the laser sensor and cannot be improved. The blade setup error is primarily from the process mean shift variation which can be improved by changing the setup procedure. ⟨Table 4.1⟩ summarizes the changes of design specifications and production situation.

⟨ Table 4.1 ⟩ Changes of design specifications and production situation.

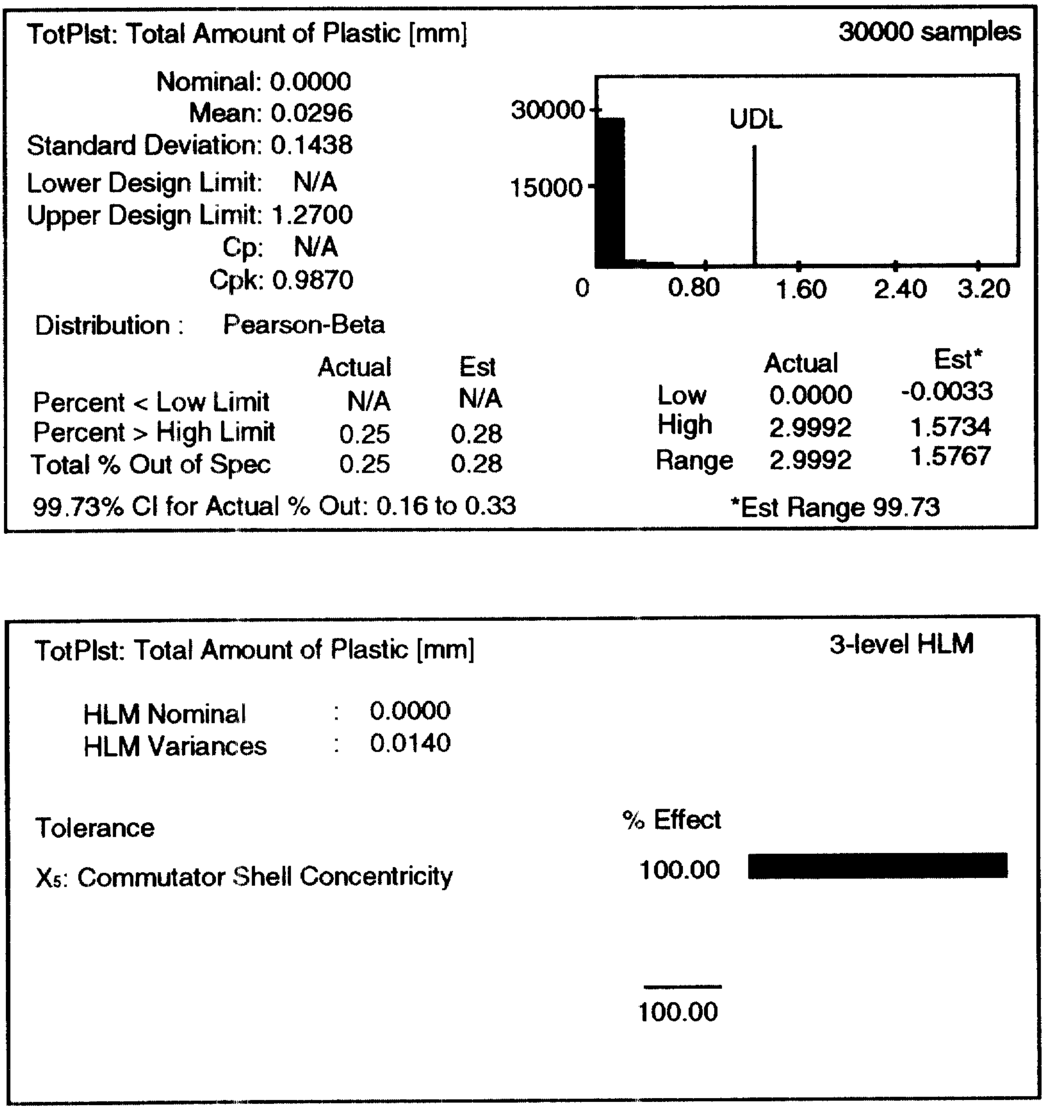
	Before		New	
Characteristic	μ [mm]	σ [mm]	μ [mm]	σ [mm]
Plastic Slot Width	0.850	0.020	0.620	0.020
Molded Comm Concentricity	0.053	0.028	0.027	0.014
Shaft Concentricity	0.022	0.011	0.019	0.010
Blade Setup Error	0.002	0.024	0.000	0.017

⟨Figure 4.3⟩ is VSA simulation results after improving the design specifications and production situation. An average of 0.0296 mm of total amount of plastic will be left in the final armature assembly, and 0.25% of the armatures manufactured will have more than 1.27 mm of total amount of plastic left in the final armature assembly. Among the 30,000 samples, the greatest amount of plastic left is 2.9992 mm. The 99.73% confidence interval of the reject rate is 0.16 to 0.33%. The process capability of the undercut process is 0.9870, which means that this process is still not capable enough to produce good armature assemblies that can lie within the design specification limits 100% of the time, but it is much better than before.

HLM analysis shows that the major contributor is the commutator shell concentricity and that it has 100% effect on the variation of total amount of plastic left in the final armature assembly. Thus, we can conclude that without improving the commutator shell concentricity, we cannot produce good armature assemblies that have less than 1.27 mm of the total amount of plastic left. If the mean of the commutator shell concentricity can be reduced to 0.021 mm from 0.058 mm, any armature assembly will have less than 1.27 mm of the total amount of plastic left in the final armature assembly 100% of the time.

5. Conclusions

This research is concerned with solving variation problems in existing manufacturing processes which have nonlinear variation stacking functions, non-



TotPlst: Total Amount of Plastic [mm]   30000 samples

Nominal: 0.0000
Mean: 0.0296
Standard Deviation: 0.1438
Lower Design Limit: N/A
Upper Design Limit: 1.2700
Cp: N/A
Cpk: 0.9870

Distribution : Pearson-Beta

	Actual	Est
Percent < Low Limit	N/A	N/A
Percent > High Limit	0.25	0.28
Total % Out of Spec	0.25	0.28

	Actual	Est*
Low	0.0000	-0.0033
High	2.9992	1.5734
Range	2.9992	1.5767

99.73% CI for Actual % Out: 0.16 to 0.33

*Est Range 99.73

TotPlst: Total Amount of Plastic [mm]   3-level HLM

HLM Nominal : 0.0000
HLM Variances : 0.0140

Tolerance	% Effect
X_5: Commutator Shell Concentricity	100.00
	100.00

〈 Figure 4.3 〉 VSA simulation result on total accumulated plastic after changing design specifications and the production situation.

normal distributions of dimensional characteristics. A variation stack-up analysis using a Monte Carlo simulation method can be used to solve the variation problem. As an example of the variation problem, a case study of the commutator brush track undercut process of a small motor was discussed.

This problem has 14 dimensional characteristics as input variations and total amount of plastic left in the final armature assembly as an output measurement.

Regression analysis shows that the total amount of plastic left is highly correlated to the amount of smearing and, finally, to the performance speed of the motor.

We have developed the geometrical model of position variation due to the commutator concentricity which occurs during the manufacturing processes. The magnitude of the commutator concentricity has a Rayleigh distribution which is unsymmetrical and left skewed. The total amount of plastic left in the final armature assembly is expressed in terms of this geometrical variations.

In order to understand how the product is produced, we simulate based on the current production situation using actual manufacturing data. According to the simulation result, 21.85% of armatures have more than 1.27 mm of accumulated plastic after the undercut process under the current production situation. In order to improve the quality of armatures, it is necessary to change the design specification and to improve the process. From the HLM analysis, the major contributors are the commutator shell and molded commutator concentricities.

Since the commutator shell concentricity, the first major contributor, is generated in the extrusion process, and the extrusion process can not easily be improved, manufacturing engineers decided to improve the reaming process which generates the molded commutator concentricity, the second major contributor. In addition to the improvement of the reaming process, the plastic slot was reduced to 0.62 mm, thus reducing the total amount of plastic left. However, the process improvement and design specification change cannot produce a plastic free armature 100% of the time. The extrusion process needs to be investigated further to reduce commutator shell concentricity, but this may be a very expensive operation.

The HLM analysis used in the case study has a limitation. It assumes that only main effects of tolerances are significant and interactive effects between the tolerances are not present. For example, when tolerance T_i is varied alone, it will cause the output measurement to vary by amount a_i ; when tolerance T_j is varied alone, it will cause the output measurement to vary by amount a_j ; when both tolerances T_i and T_j are varied, the output variation will be equal to $a_i + a_j$ This assumption, however, does not always hold true, because of interactive effects. Regression analysis [2, 6, 7, 8, 14] or experimental design [9, 11] can be used to determine the major contributors. By constructing full-factorial experiments, we can assess the significance of interactive affects in addition to main affects. This provision allows for groups of contributors to be studied as a unit alongside any combination of individual and/or grouped input variable sets.

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