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An Optimization Approach to the Wind-driven Ocean Circulation Model

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It has been demonstrated for the finite-difference ocean circulation model that the problem of uncertain forcing and input data can be tackled with an optimization techniques. The uncertainty problem in interesting flow properties is exploring a finite difference ocean circulation model due to the uncertainty in the driving boundary conditions. The mathematical procedure is based upon optimization method by the conjugate gradient method using the simulated data and a simple barotropic model. An example for the ocean circulation model is discussed in which wind forcing and the steady-state circulation are determined from a simulated stream function.

Introduction

We study the problem of "inadequate" input data for the finite difference ocean circulation models. Inadequate in this context means unequally distributed, with gaps and points or regions of poor accuracy or both. The attribution of the data to the grid points of models is traditionally done by interpolation, filtering, objective mapping etc., which leads to uneven accuracy at individual grid points. But the finite difference ocean circulation models do not normally distinguish between more or less reliable input data, and poor quality data might have a significant impact on the solution.

An approach taken here is looking for an "optimal solution" or "the best solution". We write the model equations as inequalities instead of equalities as a reflection of the varying accuracy of the data. Also unequal weights might be attributed to individual inequalities. The resulting system can no longer be solved by an exact sense; only an 'Optimal' can be founded. Optimal in this context means a solution that optimizes an objective quantity. This quantity was also called an "objective function" or "cost function".

The purpose of this paper is to show, by a simple example, that the methods associated with the names "inverse method", "objective mapping", "optimal estimation", etc., can be combined with the enormous content of information intrinsic to the finite difference model representation of ocean dynamics and evolution. The solution is found by optimizing the objective function while the inequalities act as constraints. Therefore, we explore the solution of conventional finite difference ocean circulation models by optimization methods designed specifically to determine the uncertainties of the solution and to permit reasonably easy exploration of observational strategies.

The study of the steady wind-driven ocean circulation is generally held to have begun with the linear calculations of Stommel (1948) and Munk (1950). Nonlinear effects in the boundary currents or jets were investigated by Charney (1955), Morgan (1956), Carrier and Robinson (1962), Veronis (1964, 1966) and other authors. Harrison and Stalos (1982) studied the simplest nonlinear wind-driven ocean circulation system, the barotropic vorticity equation driven by steady zonal winds and with linear bottom friction, previously studied most intensively by Veronis (1966).

We will examine a simple model describing the wind-driven circulation in a square-basin ocean. The forcing is left uncertain in a small part of the basin and solutions are calculated that minimize (maximize) the sum of the stream function squared ψ^2 that can be a measure of the potential energy. The solutions are discussed and the impact of a number of additional constraints on them is shown.

Ocean Circulation Model

In this study, we will demonstrate the linear and nonlinear programming technique with a finite difference models of the wind-driven ocean circulation in a square basin of uniform depth D and side lengths L on a β-plane. These models were adopted deliberately for their simplicity. Thus, without much additional effort, nonlinear effects can be included and oceanographically interesting problems can be tackled (Harrison and Stalos, 1982). We have restricted ourselves to the linear case where the solutions themselves is put on the new solution technique and the impact of the constraints used on the solutions.

The underlying equation is the time independent vorticity equation which has been used by Veronis (1966a, b). The transformed equation in the limit of very weak flow as follows.

$$\frac{\partial\psi}{\partial x} + \varepsilon\nabla^2\psi = -\sin\pi x \sin\pi y \tag{1}$$

$$\varepsilon = \frac{k}{\beta l} \tag{2}$$

where ψ is the transport stream function. Ekman number (nondimensional friction parameter) ε describes the width of the western boundary current, k is the friction parameter, β is the Coriolis effect, and $L=\pi l$ is the horizontal dimension of the basin.

The boundary conditions for the stream function are $\psi=0$ at x=0.1 and y=0.1. ε was set to 0.2 so that ψ_{max} is at x=0.25 and most of the basin is in Sverdrup-balance.

The differential equation (1) is transformed into a finite difference equation by approximating the derivatives as centered differences (Roache, 1982). The grid size was chosen to be equal for x and y, $\Delta x = \Delta y = 1/n$ leading to n^2 grid points in the interior of the basin. ε and Δx cannot be chosen independently as at least one, and preferably three grid points in the x-direction have to lie in the western boundary current. The finite difference representation of equation (1) is thus a collection of linear algebraic equations as follows

$$A(\psi) = 0 = \frac{\partial\psi}{\partial x} + \varepsilon\nabla^2\psi \tag{3}$$

The linear problem was solved by Stommel (1948). Uncertainties in the forcing are introduced by prescribing bound constraints for $A(\psi)$. Thus, equation (3) can be rewritten as a set of range inequality constraints

$$LB_k \leq A(\psi) \equiv C_k \leq UB_k \tag{4}$$

where the lower bound LB_k and upper bound UB_k describe the forcing by the wind stress curl. Note that LB_k and UB_k directly describe not the deviations from the steady state but the uncertainty in the forcing. Ideally, LB_k and UB_k are equal. C_k denotes the k^{th} constraint (Eq.(4)) transformed into an equality constraint by the addition of a bounded slack variable. An advantage of the formulation (4) is that the uncertainty in the forcing would normally be dependent on the position.

Optimization Procedure

Interesting properties of the ocean circulation that can be modeled with a nonlinear barotropic models are western boundary current transports, potential or kinetic energies, etc. We define a number of diagnostic objective functions F_i, that describe interesting features of the flow. The following ones have been used by Schröter and Wunsch (1986)

i) $F_1 = \psi_{max}$: F_1 is to be interpreted as the western boundary current transport and is linear in ψ.

ii) $F_2=\int\psi^2 da$: $F_2 \propto$ potential energy and is nonlinear in ψ; da is the area differential.

iii) $F_3=\int(\nabla\psi)^2 da$:$F_3 \propto$ kinetic energy and is nonlinear in ψ.

These properties will be called "objective function". We will look for solutions that drive one of the quantities F_i to its extreme minimum or maximum value. Thus, the mathematical procedure to find the extreme values of objective functions F_i subject to the constraints. That is, we will solve the problems as follows

$$\text{Minimize } F(\psi) \text{ subject to } LB_k \leq A(\psi) \leq UB_k \tag{5}$$

For F we choose

$$F=\sum_{k=1}^{n}(\psi_k)^2 \tag{6}$$

which is an expression of the potential energy of the flow. Many other definitions F_i would serve the purpose equally well. If the number of "active" constraints is equal to the number of unknowns, the definition of F_i is irrelevant.

The optimization is performed as the minimization of an unconstrained augmented Lagrangian function L,

$$L= \pm F-\sum_{k=1}^{n}\lambda_k C_k+\rho C^T C \tag{7}$$

where, C is the n×1 vector with the elements C_k, λ_k is the corresponding Lagrangian multiplier that describes the sensitivity of the objective function F to variation in C, and ρ is a penalty parameter that describes the weight of small violations of the constraints C_k (Luenberger, 1984). The sign of F in equation (7) is positive in the case of a minimization of F and negative in the case of its maximization.

The minimization of L is regarded as successful when

i) the norm of the projected Hessian of L at the solution is very small ($< 10^{-8}$),

ii) the RMS violations of the constraints $\frac{1}{n}(C^T C)^{1/2}$ is relatively small ($< 10^{-3}$).

The fully posed problem (equation (1)) was first solved this way; here the solution is fully constrained as well as the maximum of F is also its minimum, as the feasible region for the solution ψ is one point in region. This solution agrees well with Stommel's analytic solution of equation (1) (Fig. 1; Stommel, 1948). The ratio of ψ to Stommel's analytical solution at the grid points varies between 0.985 and 0.989. Fig. 1 shows the normalized stream function for the fully posed problem.

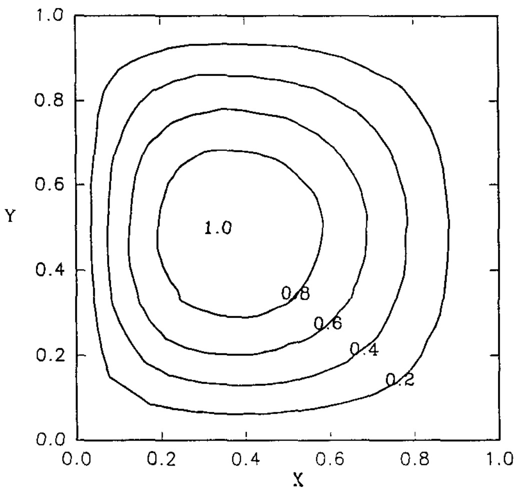

Fig. 1. Contour of the normalized stream function ψ/ψ_{max} for the fully posed problem (Stommel, 1948).

Results and Discussion

Following the approach outlined the upper and lower bounds of the constraints (5) was varied in the Sverdrup-region. We assume that the curl of the wind stress is uncertain by a factor of 3 in this area ($LB_k=3(-\sin \pi x_i \cdot \sin \psi y_j)$; $UB_k=1/3(-\sin \psi x_i \cdot \sin \psi y_j)$). Fig. 2 and Fig. 3 show the stream functions resulting from the maximization and minimization of the objective function F, respectively. In all Figures ψ differs much more from the fully posed solution (Fig. 1) than the minimized ψ. This stems from the fact that for the maximization the constraints C_k approach their lower bounds LB_k(maximum of modulus of forcing) while for the minimization the constraints become active at UB_k(minimum of modulus of forcing) and LB_k differs more from the fully posed forcing than UB_k. The replacement of F (equation (6)) by a linear function

$$F_{lin}=\sum_{k=1}^{n}\psi_k,\ \psi_k\geq 0 \tag{8}$$

led to the same solutions for ψ as the range constraints become active at their lower (upper) bound. We can thus choose the objective function for numerical convenience. For the minimization of (7) a modified Newton algorithm was used and the quadratic F produced better convergence than the linear one. If F_{lin} is chosen as the objective function, a Simplex algorithm could be used for solving our problem but the inclusion of nonlinear terms in the vorticity equation (1), which we anticipate in a next step of our examination would not be possible.

A modified Newton algorithm can be used to minimize the objective function according to the following procedure:

i) Initial guess is chosen to minimize objective function,
ii) Calculate the value of the objective function,
iii) Calculate the gradient of the objective function,
iv) Use an optimization procedure (conjugate gradient method; Luenberger, 1984) with the information on the value and gradient of the parameters and return to step 2. Continue iterations until objection is at its minimum values.

Five optimization runs were made with bounds on ψ and one run with velocity constraints. A number of model runs were made for different positions of the line constraining ψ. A summary of the result is given in Table 1 for linear case. Two parameters were chosen to represent the 'closeness' of the the solution to the fully posed ψ (Run 1; Fig. 1). First the kinetic energy integrated over the basin (in arbitrary units), and second the objective function. With the exception of Run 4 and 5 with maximized F, both parameters give the same order of 'closeness' to the fully posed ψ. As the line of constraints on ψ approaches the region of uncertain forcing (Run 2, Fig. 2) the solutions become closer to the fully posed one, both for maximization and minimization (Run 3, 4 and 5; Fig. 3). A line in the east-west direction has little influence (Run 6), as the changes in ψ due to the uncertain forcing are more significant in the east-west than in the north-south direction. A more severe bounding of ψ (Run 7) has a significant impact on the solution. Fig. 4 shows the imposition of velocity constraints (Run 8) at two points in the western boundary current for both u and v-components results in solution very close to the fully posed ψ (Table 1).

Apparently, not only the imposed constraints and bounds but also the dynamics of the barotropic velocity equation constrain the flow. This fact is not unexpected and can be proved. The corresponding C_k in equation (5) becomes equal to the forcing in the fully posed problem and thus removes the uncertainty in the forcing at those points completely. This holds for the maximization as well as the minimization of the objective function F.

Table 1. The results of the optimization procedure to the finite difference ocean circulation model

Run	Description	Kinetic energy (arbitrary units)	Objective function
		Minimum(Maximum)	
1	Fully posed (Stommel's sol.)	1.00(1.00)	1.00(1.00)
2	Forcing in factor	0.79(1.78)	0.79(1.79)
3	ψ known along x=3/n to 10%	0.83(1.26)	0.81(1.29)
4	ψ known along x=5/n to 10%	0.87(1.20)	0.85(1.22)
5	ψ known along x=7/n to 10%	0.89(1.21)	0.88(1.18)
6	ψ known along x=4/n to 10%	0.81(1.43)	0.79(1.44)
7	ψ known along x=5/n to 5%	0.92(1.11)	0.90(1.13)
8	Four velocity constraints	0.99(1.01)	0.99(1.01)

NB.: The percentage indicates an accuracy of % for the bounds on ψ imposed along a line crossing the basin.

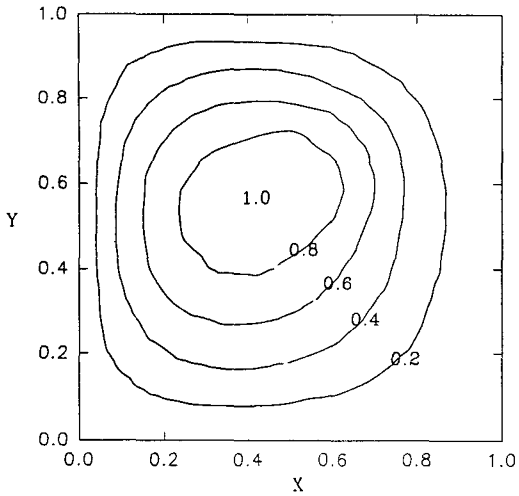

Fig. 2. Contour of the normalized stream function ψ/ψ_{max}. The objective function is maximized.

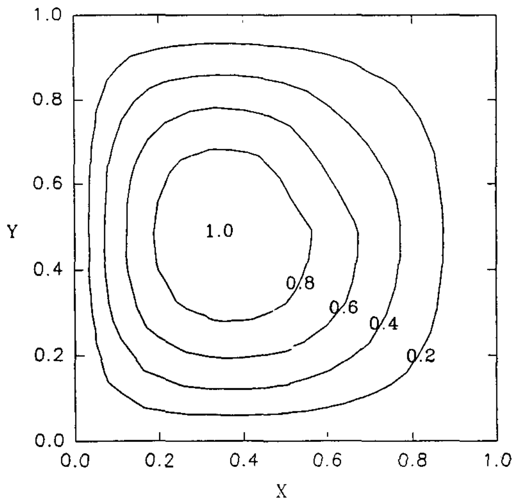

Fig. 3. Contour of the normalized stream function ψ/ψ_{max} with the additional position constraint. The objective function is minimized.

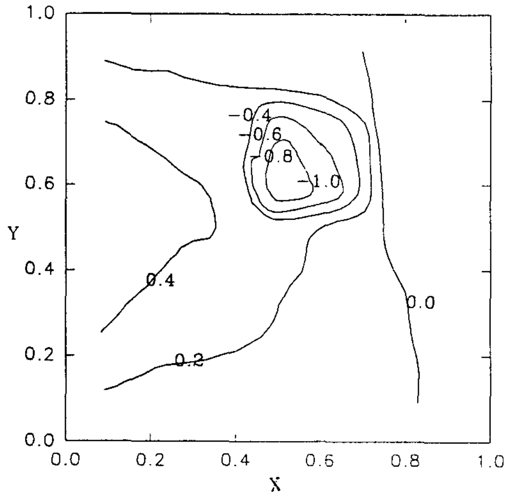

Fig. 4. Contour of the normalized vorticity function ∇^2/∇^2_{max} corresponding to Fig. 2.

Summary and Perspectives

It has been demonstrated for the finite-difference ocean circulation model that the problem of uncertain forcing and input data can be tackled with an optimization techniques. The uncertainty problem in interesting flow properties is exploring a finite difference ocean circulation model due to the uncertainty in the driving boundary conditions. The mathematical procedure is based upon optimization method by the conjugate gradient method using the simulated data and a simple barotropic model. An example for the ocean circulation model is discussed in which the wind forcing and the steady-state circulation are determined from simulated stream function observations.

The next steps from where we have reached will be i) introduction of different objective functions and other kinds of constraints, and ii) application of the method to the general ocean circulation model. (i) and (ii) will increase our knowledge of the optimization techniques and the application of the method to the real problems, respectively.

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An Optimization Approach to the Wind-driven Ocean Circulation Model

해수순환모델에 대한 최적화 방법

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최적해(optimal solution)를 구하는 최적화기법이 정방해양의 해수순환을 기술하는 간단한 유한차분모델에 대한 부정확한 외력 및 입력 자료 문제에 적용되어짐을 보였다. 수학적 절차는 추정된 자료 및 간단한 순압성 해수순환모델(wind-driven ocean circulation model)을 이용하여 공액경사법에 의한 최적화 방법에 기초하였다. 해는 부정확성 작용에 제약조건을 적용한 목적함수(objective function)를 최적화함으로써 발견되어지며, 이에 대한 부가제약조건(additional constraints) 수의 영향을 살펴보았다.