Free Oscillation Analysis in the Coastal Area using Integrated Finite Difference Method

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Integrated finite difference method (IFDM) is used to solve one dimensional free oscillation problem in the coastal area. To evaluate the solution accuracy of IFDM in free oscillation analysis, two finite difference equations based on area discretization method and point discretization method are derived from the governing equations of free oscillation, respectively. The difference equations are transformed into a generalized eigenvalue problem, respectively. A numerical example is presented, for which the analytical solution is available, for comparing IFDM to conventional finite difference equation (CFDM), qualitatively. The eigenvalue matrices are solved by sub-space iteration method. The numerical results of the two methods are in good agreement with analytical ones, however, IFDM yields better solution than CFDM in lower modes because IFDM only includes first order differential operator in finite difference equation by Green's theorem. From these results, it is concluded that IFDM is useful for the free oscillation analysis in the coastal area.

Introduction

Since solutions of appropriate numerical methods such as conventional finite difference method and shooting method, are generally better than analytical ones in oceanographic works, numerical methods effectively have been applied to free oscillation problem (Ippen, 1966; Platzman, 1972; Tacker, 1977a, b). Such a case happens particularly in the problem with complex boundaries and bottom topography (Tacker, 1977a, b, Ippen, 1966).

Of those numerical methods, conventional finite difference method (iteration or shooting methods) based on point discretization method is widely used for the analysis of free oscillation in oceanographic field (Ippen, 1966; Platzman, 1972 & 1975; Tacker, 1977a, b). However, in finite different method, point discretization method can be worse than area discretization method because the latter is formulated by integration procedure.

Dey and Morrison (1979) successfully used integrated finite difference method (IFDM) to calculate potential distribution about a point source located in or on the surface of a half-surface with arbitrary two-dimensional conductivity distribution in which IFDM is based on area discretization procedure using Green's theorem.

This paper presents the applicability of IFDM to the free oscillation analysis in the coastal area with variable geometry and bottom topography. To verify the validity of the method, one dimensional free oscillation problem with open and closed boundary conditions is solved using IFDM and its resultant critically compared to those of CFDM and the analytical method.

Governing Equation

According to the shallow water wave theory, the governing equation for one dimensional free oscillation can be expressed in a domain D as

$$\nabla \cdot (gH\nabla\zeta) + \omega^2\zeta = 0 \tag{1}$$

where g is the gravitational acceleration, H is the depth, ζ the surface displacement, ω the natural frequency. Two kinds of boundary conditions on the boundary ∂D are given as

$$\frac{\partial\zeta}{\partial n} = 0 \tag{2}$$

$$\zeta = 0 \tag{3}$$

where n indicates the outward normal unit vector at boundaries. The solutions of equation (1) are composed of eigenvalues ω^2 and corresponding eigenfunctions ζ. Equation (1) and (2) are Neumann and Dirichlet type boundary conditions, respectively. We can select other types of boundary conditions according to free oscillation problem. In real world problem, generally, the former is considered as open type and the latter is closed type of boundary condition.

If we assume a constant water depth(H) through the domain, the following relations can be held:

$$c = \sqrt{gH} \tag{4}$$

$$\omega = \frac{2\pi}{T} \tag{5}$$

where c is the phase velocity and T the natural period. However, since H is not constant in real world, equation (4) cannot be directly used for the free oscillation analysis in a coastal area with general boundary and bottom topography. Therefore, numerical methods are pursued to solve the real world free oscillation problem with irregular bottom topography.

Finite Difference Method

We use the integrated finite difference method (IFDM) proposed by Dey and Morrison (1979) to solve the free oscillation mode of equation (1). For deriving an integrated finite difference equation in a domain D, equation (1) should be rewritten for a generalized one dimensional space,

$$\int_D [\nabla \cdot (gH\nabla\zeta) + \omega^2\zeta] dD = 0 \tag{6}$$

Using Green's theorem, equation (6) can be transformed by

$$\int_S gH \frac{\partial\zeta}{\partial n} ds + \int_D \omega^2\zeta \, dD = 0 \tag{7}$$

subject to boundary condition of equation (2) and (3). Where D is the domain of model and s is the boundary line of domain.

Equation (7) is simpler than equation (6) since the former includes first order derivative only, so that the numerical errors associated with finite difference approximation can be decreased.

The finite difference approximation for equation (1) and (7) satisfying boundary conditon (2) and (3) can be derived as follows:

conventional finite difference approximation

$$gH_i\left(\frac{\zeta_{i+1} - 2\zeta_i + \zeta_{i-1}}{\Delta x^2}\right) + \left(\frac{H_{i+1} - H_i}{\Delta x}\right)\left(\frac{\zeta_{i+1} - \zeta_i}{\Delta x}\right) + \omega^2\zeta_i = 0 \tag{8}$$

integrated finite difference approximation

$$g\bar{H}\left(\frac{\zeta_{i+1} - \zeta_i}{\Delta x}\right) + \omega^2\Delta x\zeta_i = 0 \tag{9}$$

where the subscript i is a node number, $\bar{H}$ is the averaged depth of grid space, Δx is the grid size. As the grid size decreases, depth of the numerical model approaches to real world depth.

Equations (8) and (9) can be transformed into a generalized eigenvalue equation. A variety of numerical methods for the solution of generalized algebraic eigenvalue problems are available (Bathe, 1982; Ryu, 1984). Among them the subspace iteration method is suitable for the partial eigensolution of large scale problems such as equations (8) and (9) (Ryu, 1984).

Numerical Example

To examine the applicability of IFDM, we adopt one dimensional oscillation model, for which an analytical solution is available. It is a domain of continental shelf type with length scale of 50km where the water depth increases lineary maximun depth of 200m like Fig. 1.

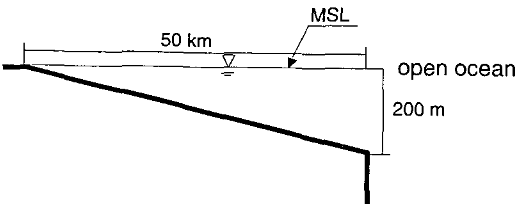

Fig. 1. Domain and boundary of numerical model.

The analytical solution of the model can be derived as Bessel function of order zero (Lim, 1992), i.e.

$$\zeta(x) = Jo(k_n) \qquad (10)$$
$$k_n = 2\omega_n\sqrt{xL/gH}$$

where Jo is the Bessel function, L the total length of domain, x the horizontal coordinate value and n the mode number. The lowest eigenmode of equation (10) based on boundary conditions of equations (2) and (3) can be solved as

$$Jo(k_n) = 0 \qquad (11)$$
$$k_{1,2,3\ldots} = 2.40482,\ 5.52007,\ 8.65372$$

To compute numerical solutions of the example using equations (8) and (9), the model domain is discretized as 101 nodes and 100 grids. Therefore, the size of matrices and consequently the number of eigensolutions must be 100 since Dirichlet type boundary condition of equation (3) is included.

Using the finite difference superposition procedure, equation (7) and (8) for the whole domain and boundary can be written as the generalized eigenvalue equation as follows:

$$Ax = \lambda Bx \qquad (12)$$

where λ is the eigenvalue that equals ω^2. The matrices A and B in equation (12) satisfy the positive-definite condition since it contains Derichlet type boundary condition of equation (3).

Analytical and numerical solutions for a few lowest modes are presented in Table 1 and Fig. 2. Numerical solutions of natural frequencies are shown to have reasonable accuracy compared with the analytical ones as shown in Table 1. Corresponding numerical eigenvectors associated with the same eigenvalues show a good agreement with analytical ones in Fig. 2. The eigenfunctions of CFDM and of analytic method in higher mode is a little difference comparing ones of IFDM. Solutions of IFDM show better than ones of CFDM in the lowest modes 1, 2 and 3 which are the most important modes in free oscillation. Therefore, the accuracy of the IFDM is verified qualitatively, so that IFDM is very useful for the free oscillation analysis in a coastal area.

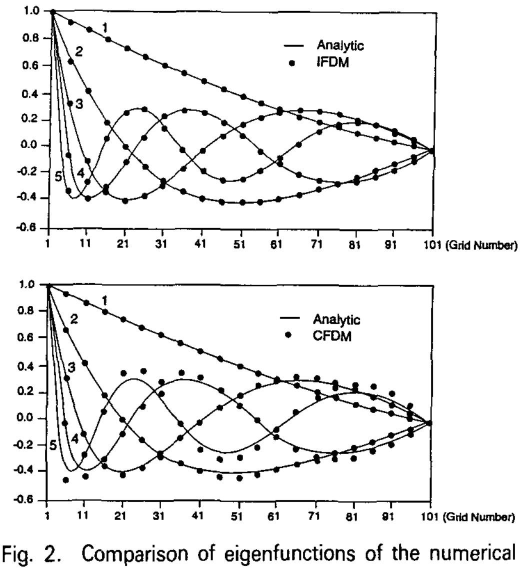

Fig. 2. Comparison of eigenfunctions of the numerical and the analytical methods where 1, 2, 3, 4 and 5 are free oscillation mode number.

Tabel 1. Eigenvalues (ω, sec^{-1}) of numerical model

Mode	Finite Difference Method		Analytic
	IFDM	CFDM	Method
1	0.001064	0.001062	0.001064
2	0.002443	0.002439	0.002443
3	0.003825	0.003823	0.003831
4	0.005208	0.005200	0.005220
5	0.006552	0.006595	0.006610

Summary And Conclusion

For the verification of validity and applicability of the IFDM proposed by Dey and Morrison (1976), one dimensional free oscillation problem in a coastal area with a variable bottom topography is solved. Two finite difference equations are derived from the governing equation using area discretization method and point finite difference method (conventional finite difference method (CFDM)). Green's theorem, very useful to improve the numerical accuracy of integral equation, is introduced in IFDM. The respective difference equations are transformed into a generalized algebraic eigenvalue problem. A numerical model of one dimensional free oscillation is solved, for which the analytical solution is available. The results prove that the solutions obtained by IFDM are better than those by CFDM. Therefore, IFDM is a very reasonable numerical scheme for the application to free oscillation problem.

It is concluded that the usability of integrated finite different method for the practical analysis of free oscillation with general boundary and bottom topography in a coastal area can be applied.

References

Bathe, K. J. 1982. Finite Element Procedures in Engineering Analysis. Englewood Cliffs, 537.

Dey, A. and Morrison, H. F. 1979. Resistivity Modelling for Arbitrarily Shaped Two Dimensional structures, Geophysical Prospecting, 27, 106-136.

Defant, A. 1960. Physical Oceanography. Vol. 2, Pergamon Press,

Horikawa, K. 1978. Coastal Engineering, An Introduction to Engineering. University of Tokyo Press, Tokyo.

Ippen, A. T. 1966. Estuary and Coastline Hydrodynamics. New York: McGraw-Hill.

Lim, K. S. 1992. Lecture note of "Atmosphere and Ocean Interaction" at Dept. of Oceanography in 1992. Nat'l. Fisheries Univ. of Pusan.

Ryu, Y. S. 1984. A study on Nonlinear Structure and Design Sensitivity Analysis Methods, Ph.D. Thesis, The Univ. of Iowa.

Tacker, W. C. 1977a. Irregular grid finite difference technique: Simulation of oscillations in shallow circular basins. J. Physical Oceanogr. 7, 284-292.

Tacker, W. C. 1977b. Comparison of Finite Element and Finite Difference Schemes, J. Physical Oceanogr. 8, 676-679.

Received July 28, 1994
Accepted November 5, 1994

적분차분법을 이용한 연안역에서의 해수고유진동해석

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Dey and Morrison (1979)이 육상의 전기탐사문제의 해결에 성공적으로 적용한 적분차분법(integral finite different method)의 해양에서의 응용성을 연구해보기 위해, 해석해가 존재하는 연안역의 해수고유진동 문제를 도출하여 기존의 고유진동문제에 적용하여 보았다. 그 응용성의 평가는 기존 해양에 널리 적용되는 기존차분법(conventional finite different method)으로 구한 수치결과와 적분차분법으로 구한 결과를 해석해와의 비교검증을 통하여 실시되었다. 그 결과 적분차분법으로 구한 고유치와 고유함수값이 기존차분법으로 구한 결과보다 좋은것으로 나타났다. 이러한 결과는 적분차분법의 경우 원래의 기본방정식에 Green's theorem을 적용함으로써, 기본방정식에 존재하는 2계 미분연산자가 1계 미분연산자로 해석적으로 처리되었기 때문으로 사료된다. 따라서 적분차분법을 이용하여 해수고유진동문제를 비롯한 다른 유사문제를 풀 경우 기본차분법보다 좋은 결과가 나올 것으로 사료된다. 또한 미분방정식의 수치해를 구할 경우 적분법이 차분법보다 좋은해를 줄 수 있다는 것을 증명한것으로 사료된다.