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Least-Square Fitting of Intrinsic and Scattering Q Parameters

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ABSTRACT: Q estimates are made by direct measurements of energy loss per cycle from primary P and S waves, as a function of frequency. Assuming that intrinsic Q is frequency-independent and scattering Q is frequency-dependent over the frequencies of interest, the relative contributions of each, to a total observed Q, may be estimated. Test examples are produced by computing viscoelastic synthetic seismograms using a pseudospectral solution with inclusion of relaxation mechanisms (for intrinsic Q) and a fractal distribution of scatterers (for scattering Q). The composite theory implies that when the total Q for S-waves is smaller than that for P-waves (the usual situation), intrinsic Q is dominating; when it is larger, scattering Q is dominating. In the inverse problem, performed by a global least squares search, intrinsic Q_p and Q_s estimates are reliable and unique when their absolute values are sufficiently low that their effects are measurable in the data. Large Q_p and Q_s have no measurable effect and hence are not resolvable. Standard deviation of velocity (σ) and scatterer size (A) are less unique as they exhibit a tradeoff as predicted by Blair's equation. For the P-waves, intrinsic and scattering contributions are of approximately the same importance, for S-waves, the intrinsic contributions dominate.

INTRODUCTION

Usually, P- and S-wave velocities, density, and Poisson's ratio are the physical parameters derived from seismic data. Because quality factors Q_p and Q_s for P- and S-waves, respectively, are very sensitive to some parameters such as fluid viscosity,especially in reservoirs, they must be added to this list. Observed total Q values are a composite of absorption (intrinsic Q) and scattering (apparent Q).

For attenuation by absorption, the concept of a spectrum of relaxation mechanisms is used to implement the theory of viscoelasticity. A wave propagating in real material induces a non-instantaneous deformation, and not all of the energy can be recovered. The energy that is not dissipated is delivered in a finite time. This relaxation time may be a consequence of many processes such as grain boundary relaxation, thermoelasticity, diffusional motion of dislocations and point defects, etc. Some of them can be modelled with one mechanism and others using a spectrum of relaxation mechanisms.

In the viscoelastic constitutive relation two relaxation functions describe the dilatational and shear behavior for each relaxation mechanism in the medium. Solution of the two-dimensional wave propagation problem implies the introduction of three memory variables at each grid point, one for each dilatational relaxation mechanisms and two for each shear relaxation mechanism.

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Attenuation by scattering depends on how fast the lithological parameters change in space and how large these fluctuations are. Scattering refers to the interaction of seismic waves with spatial variations in material properties of the medium. These variations range in size from several seismic wavelengths to a small fraction of a wavelength. Scattering produces apparent attenuation with distance in a manner similar to intrinsic loss mechanisms (Frankel and Clayton, 1986; Crossley and Jensen, 1989; Wagner and Langston, 1993). Random media containing a small fluctuation varying randomly in space, superimposed on some average parameters, are generated by spatial autocorrelation functions, to simulate scattering Q effects.

Measurements of Q from seismic waves are of importance as indirect indicators of large scale Earth heterogeneity (by scattering) and both micro-scale physical rock processes (grain boundary relaxation, thermoelasticity, diffusional motion of dislocations, point defects) and the presence of fluids (by intrinsic absorption and dispersion).

Q measurement and Q_p/Q_s ratios are known to correlate with both tectonic (crustal and upper mantle) and near-surface soil and sediment conditions (Aki and Chouet, 1975; Phillips et al., 1988; Toksoz et al., 1988; Dougherty and Stephen, 1988; Clouser and Langston, 1991; Mayeda et al., 1992; Fehler et al., 1992).

Q measurements can be made via one of two fundamental approaches. Q effects, regardless of the specific Q mechanism or wave type, involve redistribution of energy from the primary propagating waves into the coda, either by scattering or dispersion (for scattering and intrinsic Q, respectively). Thus, in principle, equivalent Q estimates may be made either by considering the loss of energy from the primary waves, or the corresponding presence of energy in the coda.

Recent developments in deterministic numerical modeling of seismic waves now allow computation of synthetic seismograms that include apparent Q due to multiple scattering (e.g., Frankel and Clayton, 1986; Levander, 1990; Gibson and Levander, 1988) and intrinsic Q due to viscoelasticity (e.g., Carcione *et al.*, 1988a, b). Thus, intrinsic and scattering Q effects may be synthesized, separately or together, and used as the basis of illustration and evaluation of techniques for Q estimation from field data. Below we use a pseudospectral implementation of the 2-D viscoelastic wave equation; the results are fairly complete in that they automatically contain all multiple scattering, diffractions, converted waves, and intrinsic attenuation and dispersion of all P- and S-waves present.

Our objectives in this paper are to illustrate both synthesis of data for, and estimation of, parameters that produce Q effects. Modeling includes computation of 2-D elastic seismograms that contain intrinsic and scattering Q effects together. Estimation of total apparent Q and separation of intrinsic and scattering Q effects is based on frequency behavior for both P- and S-waves.

SIMULATION AND ESTIMATION OF INTRINSIC AND SCATTERING Q EFFECTS

A prerequisite for deterministic numerical simulation of seismic waves is specification of the model parameters in a way that can be explicitly input into the model. For intrinsic Q, we follow the viscoelastic formulation of Carcione *et al.* (1988b), in which Q behavior is defined through the superposition of relaxation mechanisms, via relaxation times. For scattering, we follow the approach of Frankel and Clayton (1986) and Crossley and Jensen (1989) which gives a fractal distribution of velocity (and/or density) perturbations from a (typically spatial varying) background velocity. Intrinsic and scattering Q contributions are separable in observed apparent Q as they have predictably different frequency-dependent behaviors.

For construction of synthetic examples we use the survey geometry in Fig. 1. An incident plane (P or S) wave propagates through a medium with defined scattering and/or attenuating properties and is recorded at a line of equally spaced receivers. (This geometry is used for convenience only, and is not an inherent limitation). The 2-D synthesis produces two-component (vertical and horizontal) displacement responses. We use only the vertical when analyzing P-waves, and the horizontal when analyzing S-waves, as these correspond to the directions of dominant particle motion. This is justified as only relative amplitudes are used in Q estimation. When the wavefront is not perpendicular to the line of receivers (which is the usual case for field data) the apparent slowness is corrected using the angle of incidence and the phase velocity. Parameters for a viscoelastic model with self-similar scattering are shown in Fig. 2 to illustrate the combination of intrinsic Q and scattering Q effects.

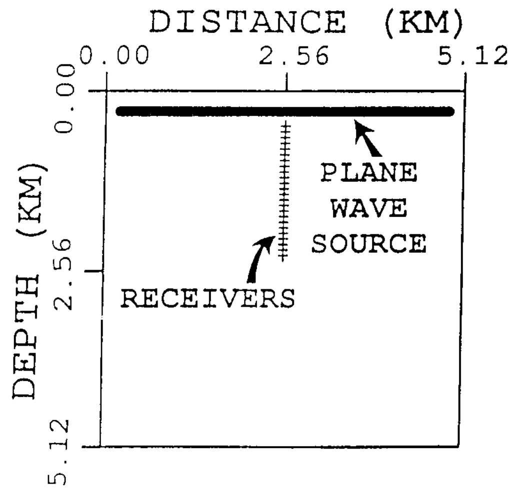

Fig 1. Model geometry for synthetic examples. The receiver array has 46 elements of which the center 36 are used. Each pair of traces provides separate Q estimates.

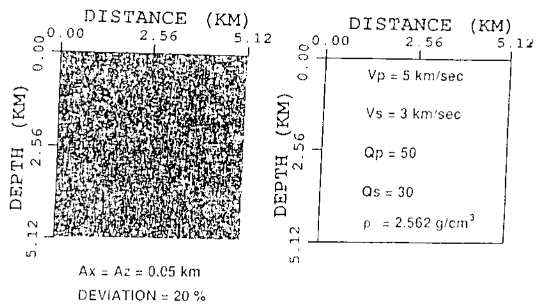

Fig. 2. Model which is combined with scattering and viscoelasticity.

Q estimates in this paper are made from in-situ measurements of energy loss per cycle from direct arrival P and S waves (Aki and Richards, 1980), as a function of frequency using

$$1/Q(\omega) = -\triangle E(\omega)/2\pi E(\omega) \quad (1)$$

where $-\triangle E(\omega)/2\pi$ is energy loss per cycle and $E(\omega)$ is peak energy. Assuming that intrinsic Q (Q_{in}) and scattering Q (Q_{sc}) have an additive relationship (Mayeda *et al.*, 1992), the relative contributions of each, to a total observed Q (Q_t), may be estimated using

$$1/Q_t = 1/Q_{in} + 1/Q_{sc} \quad (2)$$

Test examples are produced by computing visco-elastic synthetic seismograms using a pseudospectral solution with inclusion of relaxation mechanisms for intrinsic Q (Carcione *et al.*, 1988b) and a fractal distribution of scatterers for scattering Q (Frankel and Clayton, 1986).

$$Q_p(\omega) = Re(M_1^c + M_2^c)/Im(M_1^c + M_2^c) \quad (3.a)$$

$$Q_s(\omega) = Re(M_2^c)/Im(M_2^c) \quad (3.b)$$

where complex moduli M_1^c, M_2^c are defined in terms of stress relaxation time (τ_ε) and strain relaxation time (τ_σ);

$$M_v^c(\omega) = M_v[1 - L_v + \sum(1 + i\omega\tau_{\varepsilon l})/(1 + i\omega\tau_{\sigma l})] \quad (3.c)$$

where Lv is the number of relaxation mechanisms.

In the inverse problem, intrinsic Q is assumed to be frequency-independent, and scattering Q frequency-dependent over the frequencies of interest. Estimation of scattering Q as a function of frequency, the medium properties of average scatterer size (A) and the velocity deviation (σ) associated with the scatterers, and intrinsic Q values, is performed by a global least square search.

The key relationship for scattering Q is (Blair, 1990; Tang and Burns, 1992)

$$1/Q(K,D) = \beta\sigma^2\gamma(D)^{1-D}(KA)^D/[1 + (KA)\gamma(D)]^{1-D} \quad (4)$$

where K is wave number ($=\omega/V$), D is the spatial dimension ($=1$, 2 or 3), σ is the standard deviation of the medium fluctuations as a percentage of the unperturbed model values, A is the dominant scatterer size, and β and γ are coefficients that depend on scatterer shape and D, respectively. In field data, intrinsic or scattering Q may dominate, but it is necessary to estimate both because, as if only one of the two are assumed to be present, the estimates will be biased. Figs. 3 and 4 show synthetic examples for both P- and S-waves. The model combines the intrinsic Q parameters in the right side of Fig. 2 ($Q_p = 50$, $Q_s = 30$) with the scattering Q parameters in the left side of Fig. 2 ($A = 0.05$ km, $\sigma_p = 20\%$, $\sigma_s = 12\%$). The resulting seismograms (Fig. 3), as expected, show effects that the primary waves are attenuated, as are the scattered waves, and the latter include both coherent and incoherent waves and converted waves.

For the concurrent estimation of Q_{in} and the scattering parameters, without a prior knowledge, we convert the

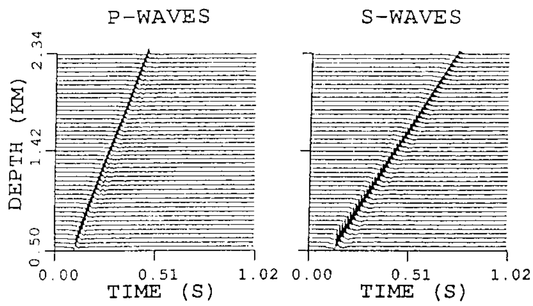

Fig. 3. Synthetic P-Wave (left) and S-wave (right) seismograms for a model with both scattering Q (the left side of Fig. 2) and intrinsic Q (the right side of Fig. 2). Q estimates from these seismograms are in Fig. 4.

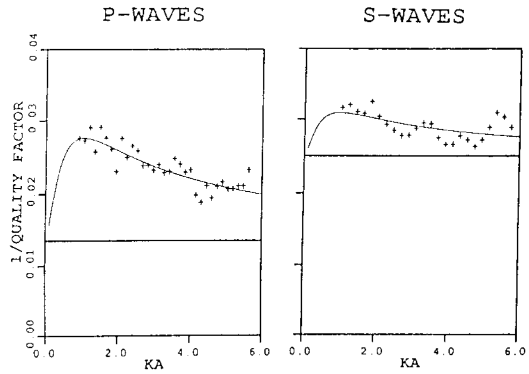

Fig. 4. Separation of scattering Q and intrinsic Q contributions in the same data (P-waves, left; S-waves, right). In each, the heavy horizontal line is the intrinsic Q estimate and the light curved line is the scattering Q estimate. All are estimated concurrently by least squares fitting to the data points. The corresponding correlation distance and % velocity deviation in the scatterers are obtained from the curve fitting coefficients.

ω values, at which Q measurements are made, to wavenumbers (K) using $K = \omega/V$; this requires knowing the average velocity V, so observed apparent velocities (V_a) must be converted to medium propagation velocities by correcting for the propagation angle using Snell's law

$$V = Va \ Sin\ \theta \quad (5)$$

The parameter fitting procedure is used to loop over sets of discrete values of Q_p, Q_s, A, $\beta\sigma_p^2$, and $\beta\sigma_s^2$ in a global search to find the combination that produces the best least squares fit to equation (2), where Q_{sc} has the form given in equation (4); this global search is feasible as the number of independent parameters is small. We further restrict the peak attenuation to be at $KA \approx 1$ for $D = 2$ (after Frankel and Clayton, 1984; Tang and Burns,

Table 1. Results of inversion of the synthetic data in Fig. 4.

Parameter	Estimated Value	Correct Value
Intrinsic Q_p	74	50
Intrinsic Q_s	40	30
A(m)	50	50
σ_p(%)	22.4	20.0
σ_s(%)	13.4	12.0
KA	0.98	$\approx$ 1.0

1992; Wagner and Langston, 1992; Roth and Korn, 1993) and KA$\approx$1.5 for D=3 (after Tang and Burns 1992). Assuming isotropic scattering, we set $\beta=1$, which allows to be estimated from $\beta\sigma^2$. Other constraints applied are $A_s = A_p$ (as the scatterer distribution and shape should be the same for both P- and S-waves), $\gamma_s=\gamma_p$ (as γ depends only on the scattering dimension D), and $\sigma_s=(V_s/V_p)\ \sigma_p$ (which says that V_p and V_s velocity fluctuations vary together in a fixed ratio). None of these constraints (assumptions) are necessary, but they are physically reasonable, and reduce the valid solution space. The step sizes used in searching the solution space, for synthetic data, were 2.0 for Q_p, 2.0 for Q_s, 0.01 km for A, 0.005 $(km/s)^2$ for σ^2 and 0.1 for γ. The best fit results (Table 1) indicate that scatterer correlation distance (A) is the most accurately estimated parameter, followed by the velocity deviations (σ_p and σ_s), and finally, the intrinsic Q values (Q_p and Q_s). This is encouraging, considering the indeterminate nature of the scattering process; other realizations of the medium with the same statistical, but different actual, properties would produce different errors in the fitting. The estimation errors could be decreased by considering more data trace pairs, but the amount of data used here is realistic. Because of the additive relation (2), any error in estimation in the position of the Q_{in} baseline gives a complementary error in the Q_{sc} parameter values, especially σ. Here, the estimated intrinsic Q values are high corresponding to less attenuation and the estimated σ values are high (corresponding to more scattering, and hence more attenuation).

CONCLUSION

Scattering effects in seismic data can be simulated by numerical solution of the wave equation (by the pseudospectral method) by including a fractal distribution of velocity (and/or density) perturbations from a background velocity. The resulting seismograms for the fractal model, in which the deviations are self similar at all scales, clearly show the scattered as well as the direct waves and include both coherent and incoherent waves and converted waves. The resulting seismograms from the model combined with intrinsic and scattering Q effects, as expected, show that the primary waves are attenuated, as are the scattered waves.

For inversion, Q estimation is performed by considering the loss of energy from the primary waves. After computation of 2-D elastic seismograms that contain scattering effects alone and intrinsic Q effects alone comparison of the numerical estimates and the analytic predictions indicate that equation (1) is useful for scattering Q estimation as well as for intrinsic Q estimation. The separation of scattering and intrinsic Q is based on the additive relation (2) with assumption that intrinsic Q is independent of frequency in the range of frequencies considered and that scattering Q is frequency dependent with the maximum attenuation occurring at wavelengths near the dominant scatterer size in the medium. Estimation of scattering Q as a function of frequency, and the medium properties including average scatterer size and velocity deviation of the medium is performed using equation (4) by a least-squares method.

The best fit results from synthetic seismic data after combining intrinsic Q and scattering Q effects show that model parameters including the scatterer correlation distance (A), the velocity deviations (σ_p and σ_s), and the intrinsic Q values (Q_p and Q_s) are relatively accurately estimated but with different solution uniqueness. Intrinsic Q_p and Q_s estimates are reliable and unique when their absolute values are sufficiently low that their effects are measurable in the data. Large Q_p and Q_s have no measurable effect and hence are not resolvable. σ and A are less unique as they exhibit a trade-off as predicted by Blair's (1990) equation.

The estimation errors could be decreased by considering more data, especially with more low frequencies. Because of the additive relation (2), any error in estimation of intrinsic Q value gives a complementary error in estimation of scattering Q value. Comparison of the numerical estimates and input data indicate that this procedure is a viable technique for parameter estimation of field data.

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最小自乘法에 依한 固有 Q와 散亂 Q의 測定

康益凡, G.A. McMechan, 閔庚德

요 약: Quality factor Q 값은 처음 到着한 P波의 週期當 에너지 損失을 周波數의 函數로 直接 測定할 수 있다. 이때 關心의 對象이 되는 周波數帶 (주로 1-100 Hz)內에서 固有 Q는 周波數와 無關하고, 散亂 Q는 周波數와 密接한 關係가 있다는 假定下에 固有 Q값과 散亂 Q값의 全體 Q값에 對한 相對的인 比率을 計算할 수 있다. 이에 對한 檢證은 彈性波가 粘彈性이고 不均質한 媒質을 通過할 때의 合成彈性波 記錄紙를 만들고 固有 Q에 對해서는 緩和機具(relaxation mechanism)가, 散亂 Q에 對해서는 散亂(scatter)에 對한 fractal 分布가 包含되는 pseudospectral 解를 利用하여 實施될 수 있다. 대체로 S波의 全體 Q값이 P波의 全體 Q값보다 작을 때는 固有 Q값이 散亂 Q값보다 더 크며, S波의 全體 Q값이 P波의 全體 Q값보다 클 때는 固有 Q값이 散亂 Q값보다 더 작다는 것이 定說로 되어있다. 逆으로, 全體 Q값은 合成彈性波 記錄紙로 부터 最小自乘法을 利用하여 求할 수 있다. 이때 假定된 Q값의 絶對값이 充分히 작아야만 P波와 S波의 固有 Q값(Q_p와 Q_s)의 假定은 信憑性이 높고 또한 唯一한 값을 가질 수 있다. 散亂 Q값으로 부터 決定할 수 있는 媒質의 速度와 散亂의 크기에 對한 標準偏差는 Blair의 數式에서 豫測할 수 있듯이 서로 相互補完 關係에 있기 때문에 여러가지의 값을 가질 수 있다. 本 硏究結果에 依하면, P波에 있어서는 固有 Q와 散亂 Q가 모두 重要한 要素로 作用하며, S波에 있어서는 固有 Q가 散亂 Q보다 더 重要한 要素로 作用한다.