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A FUNCTIONAL CENTRAL LIMIT THEOREM FOR STRONGLY MIXING RANDOM MEASURES

TAE-SUNG KIM AND HYE-YOUNG SEO

1. Introduction

Let $\mathcal{B}^d$ denote the collection of Borel subsets of d-dimensional Euclidean space R^d. The space M of all nonnegative measures defined on $(R^d, \mathcal{B}^d)$ and finite on bounded sets will be equipped with the smallest σ -field $\mathcal{M}$ containing basic sets of the form $\{\mu \in M : \mu(A) \leq r\}$ for $A \in \mathcal{B}^d$, $0 \leq r < \infty$. A random measure X is a measurable map from a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ into $(M, \mathcal{M})$. The induced measure $P_X = P \circ X^{-1}$ on $(M, \mathcal{M})$ is the distribution of X. If X is a random measure and B is a Borel subset of R^d then $X(B)$ represents the random mass of the set B, i.e., a random variable. We refer the reader to Kallenberg(1983) for details. For the stationary random measure X define the K-renormalization of X to be the signed random measure X_K as

$$(1.1) \qquad X_K(B) = \frac{X(KB) - EX(KB)}{\sigma K^{\frac{d}{2}}}$$

where σ^2 is a constant which will be specified later(see (1.4) below) and let $X_K(t) = X_K(t_1, \dots, t_d)$ be defined by

$$(1.2) \qquad X_K(t) = X_K((0, t_1] \times \cdots \times (0, t_d]) \quad \text{for} \quad t \in [0, \infty)^d.$$

Let $\{X_K\}$ be a sequence of random measures on R^d. $\{X_K(B) : K \in N\}$ satisfies the central limit theorem if for any bounded $B \in \mathcal{B}^d$ $X_K(B)$ converges in distribution to $N(0, |B|)$ as $K \to \infty$ where $X_K(B)$ is defined in (1.1) and $|B|$ denotes the Lebesgue measure of B, and $\{X_K(t) : K \in N\}$ fulfills a functional central limit theorem if X_K converge weakly to the d-dimensional Wiener measure W. An important problem in the

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theory of random measures is to consider the limiting behaviors of the set functions in (1.1) and (1.2) as $K \to \infty$. Deo(1975) extended the concept of mixing random variables to random fields and proved a finite summability

$$0 < \sum_{j \in Z^d} |\mathrm{Cov}(X_0, X_j)| < \infty \tag{1.3}$$

under the appropriate mixing condition and Newman(1980) proved a central limit theorem for a stationary associated random field satisfying (1.3). Burton and Waymire(1985) extended this notion the random measure and proved the central limit theorem for associated random measure satisfying a simple summability condition

$$0 < \sigma^2 = \sum_{k \in Z^d} |\mathrm{Cov}(X(I_1), X(I_k))| < \infty, \tag{1.4}$$

where $I_k = (k-1, k]$ and obtained a scaling limit for a Poisson cluster random measure.

The main purpose of this paper is to investigate a central limit theorem and a functional central limit theorem for a sequence of strongly mixing random measures satisfying condition (1.4) by a tightness criterion of Bickel and Wichura(1971) and uniform integrability.

In Section 2, we define strongly mixing for the random measure and we also introduce some relationships among various limit theorems in Section 3. A functional central limit theorem for strongly mixing random measures is derived in Section 4 and finally this is applied to Poisson center cluster random measure in Section 5.

2. Preliminaries and notations

Denote by $|A|$ the Lebesgue measure of $A \in \mathcal{B}^d$. For $x = (x_1, \ldots, x_d) \in R^d$, $y = (y_1, \ldots, y_d) \in R^d$, we let $\rho(x, y) = \max_{1 \le i \le d} |x_i - y_i|$. Let T be the closed interval $[0, T]$ and T^d the d-fold Cartesian product of T. Let C_d be the space of all continuous functions on T^d with the uniform metric and, as in Bickel and Wichura(1971), let us denote by D_d the Skorokhod function space on T^d. All properties of D_d that we need can be found in Bickel and Wichura(1971) (See, for example, [1] for details on the d-dimensional Skorokhod topology). A block in T^d is defined to be a set of the form $B = \Pi_{i=1}^{d}(s_i, t_i]$, $0 \le s_i \le t_i \le T$, $i = 1, \dots, d$. Disjoint blocks $B = \Pi_{i=1}^{d}(s_i, t_i]$ and $C = \Pi_{i=1}^{d}(s_i', t_i']$ are neighbors if they abut and have some $p \in 1, \dots, n$, $\Pi_{i \ne p}(s_i, t_i] = \Pi_{i \ne p}(s_i', t_i']$(for example, when $d = 3$ the blocks $(s_1, t_1] \times (s_2, t_2] \times (s_3, t_3]$ and $(t_1, t_1'] \times (s_2, t_2] \times (s_3, t_3]$ are neighbors $(s_1 < t_1 < t_1')$). For each i, $1 \le i \le d$, let

$$0 = a_1^{(i)} < b_1^{(i)} < a_2^{(i)} < b_2^{(i)} < \cdots < a_m^{(i)} < b_m^{(i)} = T$$

be real numbers. A collection of blocks in T^d is said to be strongly separated if it is of the form $\{\Pi_{i=1}^{d}(a_k^{(i)}, b_k^{(i)}], \quad 1 \le k \le m, 1 \le i \le d\}$, or if it is a subfamily of such a family of blocks. Let $W(t)$ be the d-dimensional Wiener measure on T^d. W on T^d is characterized by

(a) $P[W \in C_d] = 1$,

(b) If $B_1, B_2, \dots, B_k$ are pairwise disjoint blocks in T^d, then the increments $W(B_1), W(B_2), \dots, W(B_k)$ are independent normal random variables with means zero and variances $|B_1|, \dots, |B_k|$, respectively. If $B = \Pi_{i=1}^{d}(s_i, t_i]$, then

$$W(B) = \sum_{\epsilon_1 = 0,1} \cdots \sum_{\epsilon_d = 0,1} (-1)^{d - \sum \epsilon_i} W(s_1 + \epsilon_1(t_1 - s_1), \dots, s_d + \epsilon_d(t_d - s_d)).$$

A random measure X is stationary if for all bounded $B_1, B_2, \dots, B_k \in \mathcal{B}^d$ the distribution of $X((B_1 + \alpha), \dots, X(B_k + \alpha))$ is independent of $\alpha \in R^d$. All random measures discussed hence forth will be assumed to be stationary.

The usual condition imposed on a stochastic process defined on R^1 in order to prove a central limit theorem is a mixing condition, which ensures asymptotic independence of widely separated random variables. In an analogous fashion, Ivanoff(1982) has introduced the concept of strongly mixing for point process. Similarly, if X is a stationary random measure a strongly mixing condition may be imposed on $X(A)$ and $X(B)$, where $\rho(A, B) = \min_{x \in A, y \in B} \rho(x, y)$ is large. Suppose that X is a stationary random measure and that A is a set in $\mathcal{B}^d$. Let $\mathcal{F}(A)$ be the σ-field generated by the random variable $X(A')$, $A' \subseteq A$, $A' \in \mathcal{B}^d$. Denote the diameter of A by $d(A)$, where $d(A) = \sup_{x,y \in A} \rho(x, y)$.

Define $\alpha(r,d)$ by

$$(2.1)\quad \alpha(r,d) = \sup_{\substack{\rho(A_1,A_2)\ge r \\ d(A_1)\le d, d(A_2)\le d}} \ \sup_{\substack{U_1\in\mathcal{F}(A_1) \\ U_2\in\mathcal{F}(A_2)}} |P(U_1\cap U_2) - P(U_1)P(U_2)|.$$

Then X is said to be strongly mixing if $\alpha(Kr,Kd)\to 0$ as $K\to\infty$.

3. A classical scaling limit

THEOREM 3.1. *Let X be a stationary random measure and define $X_K(t)$ as in (1.2). Assume*

$$(3.1)\qquad 0<\sigma^2=\sum_{k\in Z^d}|Cov(X(I_1),X(I_k))|<\infty$$

where $I_k=(k-1,k]$, $k\in Z^d$. If $\{X_K(t)\}$ fulfills the functional central limit theorem, for each t, then $X_K(\cdot)$ satisfies the central limit theorem.

Proof. First we consider the special case that $A\in\mathcal{B}^d$ is bounded and open. Then $A=\cup_1^\infty B_i$ for countable collection $\{B_i\}_1^\infty$ of disjoint blocks. From the assumption of a functional central limit theorem it follows that as $K\to\infty$, $X_K(\cup_1^n B_i)\xrightarrow{\mathcal{D}}N(0,|\cup_1^n B_i|)$ for all n. But as $n\to\infty$, $N(0,|\bigcup_{i=1}^n B_i|)\xrightarrow{\mathcal{D}}N(0,|A|)$ and from (3,1) it is easy to see that as $n\to\infty$, $X_K(\bigcup_1^n B_i)\xrightarrow{P}X_K(A)$ uniformly in K. ($\xrightarrow{P}$ denotes convergence in probability.) Thus, according to Theorem 4.2 of Billingsley [2] $X_K(A)\xrightarrow{\mathcal{D}}N(0,|A|)$ as $K\to\infty$.

For general bounded sets $A\in\mathcal{B}^d$, the above argument may be repeated, approximating A with open sets.

DEFINITION 3.2. (Burton and Kim, (1988)) If X is a stationary random measure we say that X satisfies a classical scaling limit if X lies in the domain of attraction of Gaussian white noise for the scaling parameter $K^{\frac{d}{2}}$, i.e., for all disjoint rectangles (products of finite intervals) $B_1,B_2,\ldots,B_n$, $(X_K(B_1),\ldots,X_K(B_n))$ converges in distribution (as $K\to\infty$) to a multivariate normal with mean vector 0 and diagonal covariance matrix whose diagonal terms are $|B_1|,\ldots,|B_n|$ where $|B_i|$ is the Lebesgue measure of B_i.

THEOREM 3.3. *Let X be a stationary random measure and define $X_K(t)$ as in* (1.2). *Assume that the random measure X satisfies* (3.1) *and $\{X_K(t)\}$ fulfills the functional central limit theorem, for each t. Then X satisfies the classical scaling limit.*

Proof. From Theorem 3.1, it follows that for $A \in \mathcal{B}^d$, A bounded, $X_K(A) \xrightarrow{\mathcal{D}} N(0,|A|)$. It remains to be proven that for $B_1,\ldots,B_j \in \mathcal{B}^d$, B_i bounded, $i = 1,\ldots,j$, $B_i \cap B_h = \emptyset$, $i \neq h$, $(X_K(B_1),\ldots,X_K(B_j))$ converges in distribution (as $K \to \infty$) to a multivariate normal with mean vector 0 and diagonal covariance matrix whose diagonal terms are $|B_1|,\ldots,|B_j|$, where $|B_i|$ is the Lebesgue measure of B_i. This statement is proven using techniques similar to those used to prove Theorem 3.1. The sets $\{B_i\}_1^j$ may be approximated by disjoint sets $\{A_i\}_1^j$, each of which is a finite union of disjoint blocks.

THEOREM 3.4. *Let X be a stationary random measure satisfying condition* (3.1). *Define $X_K(A)$ as in* (1.1). *Assume*

(i) *for any bounded $A \in \mathcal{B}^d$, $X_K(A) \xrightarrow{\mathcal{D}} N(0,|A|)$,*

(ii) *X is strongly mixing.*

Then X satisfies the classical scaling limit.

Proof. It is sufficient to show that if $A_1,\ldots,A_j \in \mathcal{B}^d$, A_i is bounded, $i,h = 1,\ldots,j$, $A_i \cap A_h = \emptyset$, $i \neq h$, then

$$(X_K(A_1),\ldots,X_K(A_j)) \xrightarrow{\mathcal{D}} (M(A_1),\ldots,M(A_j))$$

where $M(A_1),\ldots,M(A_j)$ are independent and $M(A_i) \sim N(0,|A_i|)$ $i = 1,\ldots,j$. If this can be proven when $A_1,\ldots,A_j$ are disjoint blocks, the proof of Theorem 3.3 shows that the desired result follows. Therefore, assume that $A_1,\ldots,A_j$ are disjoint blocks. Let $(A_{1p},\ldots,A_{jp})$ be blocks such that $A_{ip} \subset A_i$, $|A_i - A_{ip}| < \frac{1}{p}$, $p = 1,2,\ldots$, $\rho(A_{ip},A_{hp}) > 0$, $i,h = 1,\ldots,j$, $i \neq h$. Let $r_p = \min_{i\neq h}\rho(A_{ip},A_{hp})$ and $d = d(\bigcup_{i=1}^j A_i)$. Then by induction it follows easily that for $H_i \in \mathcal{B}^d$, $i = 1,2,\ldots,j$,

$$\begin{aligned}|P(X_K(A_{1p}) \in H_1,\ldots,X_K(A_{jp}) \in H_j) - \Pi_{i=1}^j P(X_K(A_{ip}) \in H_i)|\\ < j\alpha(Kr_p,Kd).\end{aligned}$$

Therefore, since $\alpha(Kr_p, Kd) \to 0$ as $K \to \infty$ and $X_K(A_{ip}) \xrightarrow{\mathcal{D}} N(0, |A_{ip}|)$ then $(X_K(A_{1p}), \ldots, X_K(A_{jp})) \xrightarrow{\mathcal{D}} (M(A_{1p}), \ldots, M(A_{jp}))$ where $M(A_{1p}), \ldots, M(A_{jp})$ are independent random variables and $M(A_{ip}) \sim N(0, |A_{ip}|)$. It is also easily seen that as $p \to \infty$,

$$(M(A_{1p}), \ldots, M(A_{jp})) \xrightarrow{\mathcal{D}} (M(A_1), \ldots, M(A_j))$$

where $M(A_1), \ldots, M(A_j)$ are the independent normal random variables with mean 0 and variances $|A_1|, \ldots, |A_j|$, $|\ |$ being the d-dimensional Lebesgue measure on T^d. In addition,

$$(X_K(A_{1p}), \ldots, X_K(A_{jp})) \xrightarrow{P} (X_K(A_1), \ldots, X_K(A_j))$$

as $p \to \infty$, uniformly in K, since

$$\begin{aligned}
&P\{\rho((X_K(A_{1p}), \ldots, X_K(A_{jp})), (X_K(A_1), \ldots, X_K(A_j))) > \epsilon\} \\
&\leq \sum_1^j P\{X_K(A_i - A_{ip}) > \epsilon\} \\
&\leq \sum_1^j \frac{1}{\epsilon^2} E[X_K^2(A_i - A_{ip})] \\
&= \sum_1^j \frac{1}{\sigma^2 \epsilon^2 K^d} \mathrm{Var}[X(K(A_i - A_{ip}))] \\
&= \sum_1^j \frac{1}{\sigma^2 \epsilon^2 K^d} \mathrm{Cov}(X(K(A_i - A_{ip})), X(K(A_i - A_{ip}))) \\
&\leq \sum_1^j \frac{1}{\sigma^2 \epsilon^2 K^d} \sum_{s \in \Lambda} \sum_{t \in \Lambda} |\mathrm{Cov}(I_s, I_t)| \\
&\leq \sum_1^j \frac{mK^d |A_i - A_{ip}|}{\sigma^2 \epsilon^2 K^d} \sum_{t \in Z^d} |\mathrm{Cov}(X(I_1), X(I_t))| \\
&\leq \frac{mj}{\epsilon^2 p}
\end{aligned}$$

according to (3.1), stationarity and boundedness of A_i where, if Λ is the one that the total number of unit cubes is smallest among $\cup_r I_r$ covering $KA_i - KA_{ip}$, then there exists a positive number m such that the total number of unit cubes in Λ is not larger than $mK^d|A_i - A_{ip}|$. Therefore, by Theorem 4.2 of Billingsley ([2]),

$$(X_K(A_1), \ldots, X_K(A_j)) \xrightarrow{\mathcal{D}} (M(A_1), \ldots, M(A_j)),$$

where $M(A_1), \ldots, M(A_j)$ are independent normal random variables with means zero and variances $|A_1|, ..., |A_j|$, $|\ |$ being the d-dimensional Lebesgue measure on T^d.

4. A functional central limit theorem

THEOREM 4.1. *Let X be a stationary strongly mixing random measure and satisfy* (3.1). *Assume that for $A \in \mathcal{B}^d$, A bounded, $|A| > 1$, there exist constants $C < \infty$ and $\delta > 0$ such that*

$$E(|X(A) - EX(A)|^{2+\delta}) \leq C(\sigma^2|A|)^{\frac{1+\delta}{2}} \tag{4.1}$$

Then $\{X_K(t) : t \in T^d\}$ fulfills the functional central limit theorem.

Proof. As defined in (1.1) for a block $B \subset T^d$

$$X_K(B) = \frac{X(KB) - EX(KB)}{\sigma K^{\frac{d}{2}}} \tag{4.2}$$

and if $B = \Pi_{i=1}^d (s_i, t_i]$, then $KB = \Pi_{i=1}^d (Ks_i, Kt_i]$, $s_i, t_i \in T$. From (4.1) and Schwarz inequality it follows that for neighboring blocks B and F and $\delta > 0$

$$\begin{aligned}
&P[\min(|X_K(B)|, |X_K(F)|) \geq \lambda] \\
&\quad \leq \lambda^{-(2+\delta)} E(|X_K(B)||X_K(F)|)^{\frac{2+\delta}{2}} \\
&\quad \leq \lambda^{-(2+\delta)} (E|X_K(B)|^{2+\delta} E|X_K(F)|^{2+\delta})^{\frac{1}{2}} \\
&\quad \leq \lambda^{-(2+\delta)} (\sigma^2 K^d)^{-(1+\frac{\delta}{2})} (C(\sigma^2 K^d |B|)^{(1+\frac{\delta}{2})} C(\sigma^2 K^d |F|)^{(1+\frac{\delta}{2})})^{\frac{1}{2}} \\
&\quad = \lambda^{-(2+\delta)} C(|B||F|)^{\frac{1}{2}+\frac{\delta}{4}} \\
&\quad \leq \lambda^{-(2+\delta)} C((|B| + |F|)^2)^{\frac{1}{2}+\frac{\delta}{4}} \\
&\quad = \lambda^{-(2+\delta)} C|B \cup F|^{1+\frac{\delta}{2}}.
\end{aligned}$$

Thus by Theorem 3 of Bickel and Wichura (1971) the sequence $\{X_K\}$ is tight. It should be noted that Bickel and Wichura ([1]) assumed that $X_K(\cdot)$ vanishes along the lower boundary of T^d

$$\cup_{1\le p\le d}[0,T]\times\cdots\times[0,T]\times\{0\}\times[0,T]\times\cdots\times[0,T],$$

(0 is in the pth position). But by (4.1), $P(X(A)=0)=1$ if $|A|=0$, so a version of X_K exists which is zero along the lower boundary. Suppose X is the limit in distribution of a subsequence. It remains to show that X must be distributed as W. X must be continuous with probability one, since $X_K(\cdot)$ has jumps of at most size $(\sigma K^{d/2})^{-1}$. From (4.2) and a condition (3.1) it is easily seen that $EX_K(t)=0$ and $E\{X_K^2(t)\}\longrightarrow |t|$, where $|t|=(t_1\times\cdots\times t_d)$. By condition (4.1), for K large enough,

$$E(|X_K(t)|^{2+\delta})\le(\sigma K^{\frac{d}{2}})^{-(2+\delta)}C(\sigma^2K^d|t|)^{1+\frac{\delta}{2}}=C(|t|)^{1+\frac{\delta}{2}}. \tag{4.3}$$

and so $\{X_K^2(t)\}$ and $\{X_K(t)\}$ are uniformly integrable. This implies that $E(X(t))=0$ and $E(X^2(t))=|t|$ according to Theorem 5.4 of Billingsley ([2]).

Finally let $B_1,B_2,\ldots,B_m\subset T^d$ be strongly separated blocks and let $r=\min_{1\le i\ne j\le m}\rho(B_i,B_j)$. Because $B_1,\ldots,B_m$ are strongly separated, $r>0$, and let $d=d(\bigcup_{i=1}^m B_i)$. It follows trivially that $\rho(KB_i,KB_j)\ge Kr$ for $i\ne j$. Also, if $I\subset\{1,\ldots,m\}$, $d(\bigcup_{i\in I}KB_i)\le Kd$. Thus, for all K and $i\ne j$, by the strong mixing condition it follows that if $H_1,\ldots,H_m$ are arbitrary linear Borel sets, if $i\ne j$

$$\begin{aligned}&|P(X_K(B_i)\in H_i,X_K(B_j)\in H_j)-P(X_K(B_i)\in H_i)P(X_K(B_j)\in H_j|\\&\quad\le\alpha(Kr,Kd).\end{aligned} \tag{4.4}$$

By induction, it is easily seen that

$$\begin{aligned}&|P(X_K(B_1)\in H_1,\ldots,X_K(B_m)\in H_m)-\Pi_{i=1}^m P(X_K(B_i)\in H_i)|\\&\quad\le m\alpha(Kr,Kd)\longrightarrow 0\quad\text{as}\quad K\longrightarrow\infty.\end{aligned} \tag{4.5}$$

Thus, X must have independent increments and so every subsequence $\{X_{K'}\}$ of $\{X_K\}$ has a further subsequence $\{X_{K''}\}$ which converges weakly to the Wiener measure W on T^d. Therefore $\{X_K\}$ fulfills the functional central limit theorem.

5. Poisson cluster random measures

We apply Theorems 3.4 and 4.1 to Poisson center cluster random measures. These cluster random measures have been used as models of infinite divisibility and self-similarity ([5],[12]) as well as models of natural phenomena such as storm systems and galaxies ([11]). These are constructed as follows. Let U be a stationary Poisson point process with parameter ρ. Let $V = \{V_x | x \in R^d\}$ be a collection of i.i.d. random measures with $E[V_x(R^d)] = \xi < \infty$. Then we say that X is a cluster random measure with centers U and members V if

$$X(B) = \sum_{x:U(x)>0} V_x(B - x) \tag{5.1}$$

for each bounded Borel set B. We denote X by $[U, V]$.

LEMMA 5.1. *Let $X = [U, V]$ as above with V an i.i.d. random measure such that $E[V(R^d)^2] = \eta < \infty$. Then*

$$\sum_{k \in Z^d} |Cov(X(I_1), X(I_k))| = \rho\eta < \infty. \tag{5.2}$$

where $I_k = (k - 1, k]$.

Proof. See the proof of Theorem 5.3 of Burtion and Wayimire([4]).

LEMMA 5.2. *Let $X = [U, V]$ as above. Let B be a rectangular box in R^d and $0 \le \delta \le 2$. If $E[(V_x(R^d))^{2+\delta}] < \infty$ then*

$$E[|X(B) - EX(B)|^{2+\delta}] \le |B|^{\frac{1+\delta}{2}} E[(V(R^d))^{2+\delta}]. \tag{5.3}$$

Proof. See the proof of Theorem 3.1 of Burton and Kim([3]).

THEOREM 5.3. *Let $X = [U, V]$ as above with V an i.i.d. random measure such that $E[V_x(R^d)^2] = \eta < \infty$. If X is strongly mixing then X satisfies a classical limit with parameter $\rho\eta$.*

Proof. According to (5.2) and Theorem 3.4 the proof of Theorem 5.3 is complete.

THEOREM 5.4. *Let $X = [U, V]$ as above. Assume*
(i) *X is strongly mixing,*
(ii) *$E[(V_x(R^d))^{2+\delta}] < \infty$ for $0 < \delta \leq 2$.*
Then $\{X_K\}$ fulfills the functional central limit theorem.

Proof. According to (5.2), (5.3) and Theorem 4.1 the proof of Theorem 5.4 is complete.

REMARK. (1) Ivanoff ([8]) has shown that if the V was a point process with factorial moment density functions up to order 4 then a stationary Poisson cluster process satisfies the functional central limit theorem.

(2) Burton and Kim ([3]) have shown that if $E[(V_x(R^d))^{2+\delta}] < \infty$ then the stationary Poisson cluster random measure X fulfills the functional central limit theorem by the fact that Poisson cluster random measure X is associated.

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Department of Mathematics
Won Kwang University
Iri City 570-749, Korea