大韓土木學會論文集
第14卷 第5號 · 1994年 9月
pp. 1129~1136

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물리 모형을 토대로한 호우 예측

Heavy Rainfall Prediction by the Physically Based Model

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Abstract

A point heavy rainfall process is physically modeled. It uses meteorological variables at the ground level as its inputs. The components of the model are parameterized based on well established observations and the previous studies of cloud physics. Particular emphasis is placed on the efficiency of accretion. So we adopt the modified skew-symmetric model for hydrometeor size distribution function that is suitable for the heavy rain cloud. The dominant parameters included in the model are estimated by the optimization technique. The rainfall intensity is predicted by the model with the medium values of estimated parameters.

요 지

지점 강우 과정을 물리적으로 모형화하였다. 모형의 입력 변수는 지상의 기상 변수이다. 모형의 성분은 관측치와 구름 물리학의 선행 연구를 토대로 매개 변수화되었다. 특별히 강조되는 것은 집적 효율을 평가하는 것이다. 수정 비대칭 모형의 수운적 크기 분포 함수를 적용한 결과 그 모형은 호우에 적합하였다. 모형에 포함된 주요 매개변수는 최적화 기법에 의하여 평가하였다. 강우 강도는 평가된 매개변수의 중앙값을 사용하여 예측하였다.

1. Introduction

Cloud or precipitation phenomenon is described by dynamics and thermodynamics of air, microphysical processes of cloud system, microphysical processes involved in the cloud formation and the production of precipitation. Marson (1957) emphasized that the phenomenon can be understood through interaction of the processes. Hydrometeorologists have been trying to formulate the process of water vapor condensation, precipitation mechanisms, and subcloud evaporation of falling hydrometeors. Srivastava (1967) proposed one dimensional cloud model. Recently, Georgakakos and Bras (1984a) modeled precipitation processes. They took into account major components of cloud physics leading to precipitation. This approach has a number of advantages: linear equation of cloud moisture content, the inputs to the model are observed at ground level, scales comparable to the size and time response of river basins. Neverthe-

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less it is not sure that the model can be applied for heavy rainfall prediction. Lee *et al.* (1992a, b) investigated whether the model is suitable to the prediction of heavy rainfall for Chonju Weather Service Station (CWSS). The results of their study show that for total rainfall the deviation between the calculated and the observed rainfall is small, but for the rainfall intensity the difference is quite big. They proved that the dominant factors in the proposed model are the hydrometeor size distribution (HSD), updraft velocity of air, and mean hydrometeor diameter.

For heavy rainfall prediction the precipitation model proposed by Georgakakos and Bras (1984a) must be modified so that it may be equipped with the suitable HSD function. For this purpose, the physically based conceptual model is described in chapter 2. In chapter 3, we analyze characteristics of modified skew-symmetric HSD(MSSHSD). Parameters in model equations are estimated in chapter 4. In chapter 5, the model is evaluated by numerical analysis using the meteorological data of CWSS.

2. Model structure

For modeling of precipitation processes, we consider cloud column over a unit area on the weather station. The storm cloud system is regarded as a reservoir of condensed water. Fig. 1 schematizes the concept with model variables of interest.

In the Fig. 1, O_b and O_t are outflow moisture, I is inflow moisture and X is water mass content of control volume. The amounts of O_t and O_b will depend on the size of hydrometeors and the resultant force of inertia due to updraft wind velocity and their weight. The distribution of hydrometeors in size is represented by a function N(D), where D is diameter of hydrometeor. Since the hydrometeor is evaporated during the trip to ground surface, a portion of O_b is transformed to precipitation R.

A mass conservation equation of the defined model above would be

$$\frac{dX(t)}{dt} = I(t) - O_t(t) - O_b(t) \qquad (1)$$

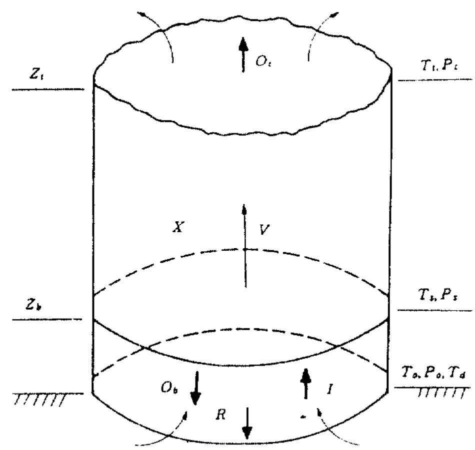

Fig. 1. Schematic representation of control volume at time t

where I(t) is the input mass rate due to the condensation of water vapor, X(t) is the mass of liquid water equivalent in the control volume at time t, and $O_t(t)$ and $O_b(t)$ on control surface are previously defined. The moisture-laden inflowing air that rises through control volume will supply water mass to it. The equation of input mass rate is given by

$$I = \rho_w v \, \Delta W \, dA \qquad (2)$$

where ρ_w is the vertically averaged density of moist air through the control volume, v is the vertically averaged updraft velocity of inflowing air, and ΔW is the change in specific humidity in the inflowing air between bottom and top of control volume with the unit area measure dA. The updraft wind velocity is assumed to vary linearly with height from the bottom of the cloud column, reaching maximum at the elevation of the average preassure between top and bottom. This velocity at the top and bottom of control volume is equal to a portion β of its v. Convection can arise from buoyant or mechanical forces. However, in our physically based model mechanical action is neglected and it is assumed that vertical velocity of a convective element in our control volume obeys a law of the following type which Sulakvelize (1969) suggested.

$$v=\varepsilon_1\cdot[c_p\cdot(T_m-T_{s'})]^{0.5} \tag{3}$$

where ε_1 is a constant parameter, c_p is the specific heat of dry air under constant pressure. T_m is the cloud temperature at a certain level and $T_{s'}$ is the corresponding ambient air temperature.

Input mass rate I is the function of temperature T_o, pressure p_o and dew-point temperature T_d at the ground level, pressure at top of cloud p_t and the velocity v. The storm cloud is vigorously developed by the strong updraft. In that case p_t is low. However, the value of p_t also depends on the past history of the storm. After the storm persiststed for several hours, the opposite phenomena could be occured. Assuming that this p_t is the function of v, it is parameterized as follows (Georgakakos and Bras, 1984b):

$$\frac{p_t-p_l}{\varepsilon_2-p_l}=\frac{1}{1+\varepsilon_3\cdot v} \tag{4}$$

where p_l is the lowest value among the pressures that can be estimated, ε_2 and ε_3 are constant parameters.

The output mass rate per unit area to the action of the updraft v_β at the top of the cloud is given by

$$O_t=\int_0^{D'}\frac{\Pi}{6}\rho_w D^3 N(D)(v_\beta-v_t(D))dD \tag{5}$$

where N(D) is hydrometeor size distribution function at the cloud top, $v_t(D)$ is the terminal velocity of a hydrometeor of diameter D, D′ is the diameter such that v_β is greater than or equal to $v_t(D)$. We assume that $v_t(D)$ is a linear function of D. The propotional coefficient α is 3500 sec^{-1} for rain (Georgakakos and Bras, 1984a).

The output mass rate O_b is derived by the similar way to O_t,

$$O_b=\int_{D_{min}}^{D_{max}}\frac{\Pi}{6}D^3\rho_w N(D)(v_t(D)-v_\beta)dD \tag{6}$$

The mass of liquid water equivalent in storage X in the control volume is given by

$$X=\int_{D_{min}}^{D_{max}}\frac{\Pi}{6}D^3\rho_w N(D)dD \tag{7}$$

where D_{min} and D_{max} are the minimum and maximum diameters in the cloud. Due to evaporation in the subcloud layer, the rainfall rate at ground level is generally only a portion of O_b. The mass rainfall rate of liquid water equivalent per unit area at ground level is given by

$$R=\int_{D_L}^{\infty}\frac{\Pi}{6}D^3\rho_w\xi(D)(v_t(D)-v_\beta)N(D)dD \tag{8}$$

where the limit D_L is defined by $\max\{D_c, D'\}$. D_c is a critical minimum diameter to be evaporated completely in the subcloud layer before it reach the ground. $\xi(D)$ is a function that reduces particle mass which $\xi(D)$ may be given by the function of the ratio of critical diameter D_c to an initial diameter D_o at the cloud base elevation (Georgakakos and Bras, 1984a).

$$\xi(D)=1-\left(\frac{D_c}{D_o}\right)^3 \tag{9}$$

We can explicitly integrate the equation (5), (6), (7), and (8) if the function N(D) is given.

3. Skew-symetric hydrometeor size distribution

Hydrometeor size distribution function expresses the number of drops per unit size interval per unit volume of space. The typical HSD are inverse exponential HSD (IEHSD) which is proposed by Marshall and Palmer (1948), skew-symmetric HSD (SSHSD) by Fujiwara (1976), log-normal HSD (LNHSD) by Levin (1971). The formula of HSD is derived from the data observed under the different meteorological environments. For example, the rainfall intensity is varying from 1.0 mm/hr to 25 mm/hr in IEHSD, from 0.18 mm/hr to 200 mm/hr in SSHSD, and from 5.8 mm/hr to 39 mm/hr in LNHSD. We adopt SSHSD that will have high efficiency of accretion. SSHSD is given by

$$\left.\begin{array}{ll} N(D)=N_o\exp\{-\lambda(D-D_{cf})\} & D\ge D_{cf} \\ N(D)=N_o & D<D_{cf} \end{array}\right\} \tag{10}$$

where λ is constant, D_{cf} is the function of rainwater content, collection efficiency, and height from the cloud base. If we substitute N(D) in equation (8) with the formula (10), surface rainfall mass rate R is a nonlinear function of X. We want that this relationship is linear. Therefore, we will modify SSHSD size distribution so that D_{cf} is expressed by the function of moisture input. We can reconstruct the relationship (Lee *et al.*, 1992b) as follows:

$$D_{CC}=k_3 \cdot \ln(I/k_4) \tag{11}$$

where k_3 and k_4 are parameters. D_{cc} is critical hydrometeor diameter. Substituting D_{cf} with D_{cc}, λ with c, equation (10) is transformed as follows:

$$\left.\begin{array}{ll} N(D)=N_o \exp\{-c(D-D_{cc})\} & D\geq D_{cc} \\ N(D)=N_o & D<D_{cc} \end{array}\right\} \tag{12}$$

The parameter c in (12) is the inverse mean diameter size at a given level. The mean diameter of the hydrometeor should be larger near cloud bottom and smaller at the cloud top. It is very hard to establish the equation of this distribution because of difficulties in sampling. We assume reasonably that c is linearly distributed with height Z.

$$c(Z)=c_l+\frac{Z}{Z_c}(c_u-c_l) \tag{13}$$

where Z_c is the thickness of the cloud column, c_l and c_u are lower and upper values of c, and Z is height within the cloud measured from cloud bottom. According to Pruppacher and Klett(1978), several most important processes to determine the parameter c are as follows: the condensation, the collision coalescence, and the collisional breakup of the larger particles. They suggest that the stronger the updraft velocity, the larger the number of larger particles. It means that the average hydrometeor diameter increase as v increases. we assume that c is determined by v that is obtained by the relationship as follows (Pruppacher and Klett, 1978):

$$\frac{1}{c}=\varepsilon_4 \cdot v^k \tag{14}$$

where $\varepsilon_4(\sec^k m^{(1-k)})$ and k (dimensionless) are constant parameters.

In this modified SSHSD(MSSHSD) function, if hydrometeor diameter is less than D_{cc}, N(D) has constant value. But if hydrometeor diameter is greater than D_{cc}, N(D) is exponentially decreased. MSSHSD has parameters N_o, c and D_{cc}.

The possible objection to (12) is that it implies hydrometeor at diameters approaching zero. The attractive alternative would be to use a distribution starting at zero and peaking somewhere in the small diameter region. Nevertheless, given the acknowledged uncertainties of measuring the number of small hydrometeors, (12) is adequate (Lee *et al.*, 1992b).

4. Parameter estimation

Our conceptual model consists of state (1) and output equation (8). Input variables of the model are temperature T_o, dew-point temperature T_d, and pressure p_o on the ground. The required variables for modeling are parameterized in the previous section. Substituting N(D) with MSSHSD and integrating equation (5), (6) and (8), yields linear function of state X with parameters. Thus the rainfall model equations can be expressed as follows (Georgakakos and Bras, 1984a):

$$\frac{dX}{dt}=f(u;\ \varepsilon_1,\ \varepsilon_2,\ \varepsilon_3)-(u;\ \varepsilon_4,\ k,\ \gamma,\ \beta,\ k_3,\ k_4)X \tag{15}$$

$$R=\phi(u;\ \varepsilon_1,\ k,\ \gamma,\ \beta,\ k_3,\ k_4)X \tag{16}$$

where u is meteorological input vector. γ is the ratio of the avarage diameter at cloud base to the average diameter at cloud top. The function $f(u;\ \varepsilon_1,\ \varepsilon_2,\ \varepsilon_3)$ represents moisture input and $h(u;\ \varepsilon_4,\ k,\ \gamma,\ \beta,\ k_3,\ k_4)$ X is moisture output. $\phi(u;\ \varepsilon_1,\ k,\ \gamma,\ \beta,\ k_3,\ k_4)X$ is a nonlinear function of u and parameters acting linearly on the state X. Eventually, we can obtain the rainfall rate from state equation (15) and output equation (16) using meteorological input vectors (T_o, p_o, T_d) if the embeded parameters and initial condition of moisture content X are given. Conversely, we can estimate the parameters and initial condition in the model equation to minimize root mean square error (RMSE) between the predicted by the equation (16) and the observed rainfall. For this purpose the objective function can be defined as follows (Lee *et al.*, 1992 a):

Table 1. The list of storm events selected

No.	Event Year	Month	Day	Time	Duration (hr)	Total Rainfall (mm)	Cause of Storm
1	1983	July	21	21	17	103.1	Cyclone
2	1984	July	21	21	11	119.0	Front
3	1985	July	7	2	15	105.8	Front
4	1986	June	24	7	26	144.8	Typhoon
5	1986	August	27	23	18	80.0	Typhoon
6	1987	July	22	16	12	110.6	Typhoon
7	1987	August	4	11	10	98.9	Cyclone
8	1990	June	19	11	13	83.3	Cyclone

Table 2. Estimated values of Parameters

Cause of storm	Event No.	ε_1	ε_4	ε_3
Cyclone	1	0.2110E−01	0.2425E−04	0.8075E−04
	7	0.3970E−01	0.3100E−04	0.4000E−05
	8	0.1720E−01	0.3700E−04	0.3000E−05
Front	2	0.1580E−01	0.3505E−05	0.1220E−03
	3	0.3180E−01	0.3300E−04	0.1300E−04
Typhoon	4	0.9200E−02	0.4600E−04	0.1500E−04
	5	0.8300E−02	0.3706E−04	0.1050E−03
	6	0.1880E−01	0.3300E−04	0.2400E−04

$$G = \mathrm{Min}\left\{ \frac{1}{m} \sum_{i=1}^{m} [R_p(y,\ \Delta t \cdot i) - R_o(\Delta t \cdot i)]^2 \right\}^{1/2} \quad (17)$$

where R_p is the predicted rainfall intensity, R_o is the observed rainfall intensity and y are parameters (ε_1, ε_2, ε_3, ε_4, k, γ, β, k_3, k_4). $\Delta t \cdot i$ is time increment that Δt is time interval and i is integer. $i=0$ and $i=m$ indicates the start and the end of rainfall, respectively.

5. Case study

Heavy storms usually bring disastrous damages to the southern part of Korean peninsula. The occurrence of heavy rainfall is mostly associated with the frontal activities, extratropical cyclones and typhoons from June to September. Our case study focuses on the heavy rainfall events. Hence, eight events at CWSS are selected (Table 1). Details on Table 1 is described by Lee *et al.* (1992b).

For our model, it is available to use meteorological input and rainfall data recorded at ground level. But all parameters included in state equation (15) and observation equation (16) are not known. We classify the parameters into two groups to prevent them from their interferencing among others during optimization. The major parameters have dominant effect on output of the model while the minors are trivial. There are 3 major parameters; ε_1 embeded in updrafts, ε_4 in hydrometeor diameter at cloud base, k_3 in HSD. We take the values of the minor parameters given previously by Lee *et al.* (1992b). To estimate the parameters, direct search algorithm proposed by Hook and Jeeves (1961) is applied to minimize RMSE between the predicted and observed rainfall intensity.

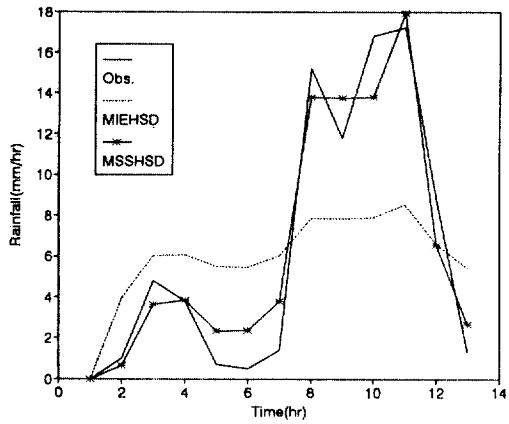

Fig. 2. Comparison of the predicted with the observed hyetograph, June 1990

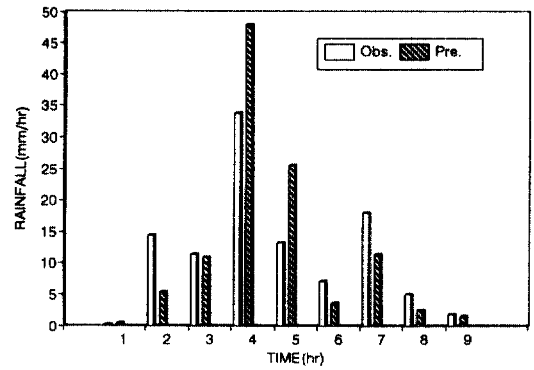

Fig. 3. Comparison of the predicted with the observed hyetograph. June 1993

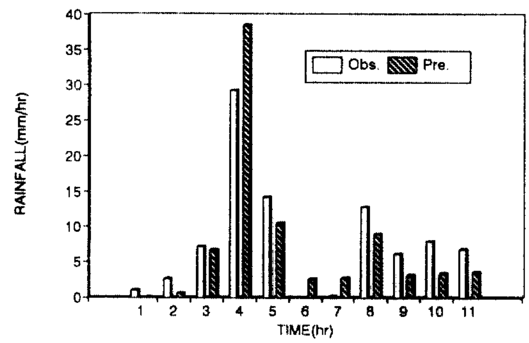

Fig. 4. Comparison of the predicted with the observed hyetograph. July 1993

To solve state equation numerically we take one hour as a time interval. Also we need nominal values of parameters for optimization. For the minors we take nominal values of the parameters suggested by Georgakakos and Bras (1984b) while for the new minors which we introduces $k_3=3\cdot 10$ E−05 and $k_4=1.0\times 10^{-3}$. We conduct numerical experiments with meteorological inputs and rainfall data which are hourly averaged, the given and nominal values of parameters. The results of optimization are listed in Table 2. It shows that k_3 depends highly on the environment of raincloud. RMSE for each event is arranged in Table 3. They are comparable to mean rainfall intensities (MRI). The ratio of RMSE to MRI ranged from 0.26 to 0.45. The mean ratio in cyclone is 0.29, in front 0.44 and in typhoon 0.4. We predicted hyetographs with optimal parameters. In Fig. 2 the stared line is predicted by our model equipped with MSSHSD, the dashed line with MIEHSD. In Fig. 2, dashed line is predicted by the same procedure as the model with MSSHSD. From the Fig. 2, We can see that the predicted by the model with MSSHSD shows good fit to solid line of the observed hyetograph.

To test the performance of our proposed model, hyetographs are predicted by the model with medium values of the optimized parameters in Table 2 and the observed input data. There are two events in hand which are not included in parameter optimization. The storm events for testing are given in Table 4. RMSE between observed and predicted hyetograph was to investigate the performance of the model. The results are given in Table 5. As shown in Fig. 3 and 4 the observed hyetographs are compared to the predicted hyetographs. The ratio of RMSE to MRI for No. 1 event is comparable to the values of the previous cases with the optimal constants in Table 3. However, the ratio for No. 2 event is higher than previous value.

6. Conclusions

Point heavy rainfall model based on cloud physics is developed and tested for heavy storm events observed at CWSS in Korea. In point heavy rainfall model, every subprocess of rainfall was

Table 3. Root Mean Square Error of Rainfall Intensity

Cause of Storm	Event No.	Observed Total Rainfall (mm)	Predicted Total Rainfall (mm)	Mean Rainfall Intensity (mm/hr)	Root Mean Square Error (mm/hr)	RMSE / MRI
	1	103.1	109.1	6.1	2.7	0.44
Cyclone	7	98.9	100.3	9.9	4.5	0.45
	8	83.3	87.0	6.4	2.0	0.31
		Algebraic mean		7.4	3.0	0.40
Front	2	119.0	115.0	10.8	4.7	0.43
	3	105.8	111.5	7.1	3.2	0.45
		Algebraic mean		8.9	3.9	0.44
	4	144.8	145.3	5.6	1.5	0.26
Typhoon	5	80.0	78.0	4.4	1.6	0.36
	6	110.6	109.4	9.2	2.5	0.27
		Algebraic mean		6.4	1.8	0.29

Table 4. Storm events

		Event			Duration	Total Rainfall	Cause of
No.	Year	Month	Day	Time	(hr)	(mm)	Storm
1	1993	June	29	01	9	104.8	Front
2	1993	July	12	02	11	88.0	Front

Table 5. Root Mean Square Error of Rainfall Intensity

Cause of Storm	Event No.	Observed Total Rainfall (mm)	Predicted Total Rainfall (mm)	Mean Rainfall Intensity (mm/hr)	Root Mean Square Error (mm/hr)	RMSE / MRI
Front	1	104.8	108.9	11.6	3.9	0.34
Front	2	88.0	80.7	8.0	6.2	0.77
		Algebraic mean			5.0	0.55

parameterized and the coefficient were optimized. We solved the state equation with hourly averaged input variables and obtained hyetographs at the ground level. The results of this study are summerized as follows:

1. Dominant parameters ε_1, ε_4 and k_3 ranged from 8.3·10E−0.3 to 3.9·10E−0.2, from 3.5·10E−0.6 to 4.6·10E−0.5, from 1.2·10E−0.4 to 3.0·10E−0.6, respectively.

2. Optimal parameter k_3 depends strongly on the meteological environment of raincloud.

3. The hyetograph predicted by the model with MSSHSD showed a good fit to the observed. It is proved indirectly that accretion efficiency of

MSSHSD is higher than MIEHSD.

4. It is suggested that the proposed model is useful in order to approximately predict the hyetograph by using the medium value of parameters and the meteorogical data observed at ground level.

Acknowledgements

We especially benefited from discussion with Slobodan P. Simonovic. This research was partially supported by the Korea Electric Power Company.

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(接受 : 1994. 2. 28)