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# 유한 요소법에 의한 콘크리트 포장 구조의 평면 거동연구

## Finite Element Analysis of Planar Effect on the Concrete Pavements

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**Abstract**

Since horizontal movements due to shrinkage and thermal gradients in concrete pavements involve no actual load, the stresses induced will be those due to closing of the pavement joints and subbase friction. Consequently, complete derivations of stiffness matrix and equivalent nodal loads due to planar effects on the concrete pavements was throughly undertaken using the finite rectangular elements with two degrees of freedom at each node. The numerical example shows that the tensile stress induced in a pavement due to concrete shrinkage might be negligible except at very long slab and very high coefficient of frictions. However the stresses in conjunction with principal traffic loads might cause cracking problems.

**요 지**

콘크리트 포장구조에 있어서 온도팽창과 건조수축으로 인한 평면 변위 거동은 차량 하중에 관계없이, 포장슬래브와 노반과의 마찰에 의한 줄눈의 개폐에 주로 기인하고 있다. 따라서 콘크리트 포장구조의 평면 변위 거동을 해석하기 위하여 한 절점에서 dx, dy, 2 자유도를 갖는 직사각형 평면요소의 강성 행열과 마찰, 건조수축, 온도 등으로 인한 등가절점 하중을 유도하였다. 콘크리트의 건조수축으로 인한 포장슬래브의 구조해석결과, 슬래브 길이가 길거나 지반과의 마찰계수가 큰 경우를 제외하고는, 큰 응력은 발생하지 않았지만, 차량하중과 조합할때는 인장응력의 증가와 함께 포장 슬래브에 균열이 발생할 수 있음을 보여주고 있다.

## 1. Introduction

Traditionally, the design of concrete pavements has been based on the assumption of thin plate resting on an elastic foundation. The primary stresses considered in the design and analysis of pavements are those due to bending action, such as traffic loads, temperature differentials between top and bottom surfaces of the slab, etc. Subsequently, the in-plane planar effects on the concrete pavement mainly due to the concrete shrinkage, expansion and contraction of volume change have largely been ignored.

Westergaard presented a simple analysis, which was published in the proceedings of the Highway Research Board in 1926[1-3]: His analysis was ba-

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sed on a pavement subjected to a uniform shrinkage strain, and restrained at both ends. His analysis does not take into account the differences in frictional stress distribution across the width of the slab, differences due to the variation in vertical deflection due to its own weight and temperature curling.

Shrinkage, broadly defined, is volume change that is unrelated to load application. It is possible for concrete cured continuously under water to increase its volume. However, the usual concern is with a decrease in volume, mainly due to the moisture loss. the problem is that when the concrete pavement shrinks or expands owing to the moisture change, the horizontal displacement behavior of pavement slab is influenced by the friction resistance between the slab and subbase. Consequently, slab weight, overburden pressure, and coefficient of friction are considered here , based on the classical friction theory.

The shrinkage effects on concrete pavements might be generally not as high as the stress induced by traffic in conjunction with other environmental factors. Probably, this will have an effect on the magnitude of rotational and linear stiffnesses at the joints with opening and closing. A more accurate understanding of the shrinkage effect will help in determining all of the factors which affect the joint stiffness and the structural behavior of a concrete slab as well.

## 2. Rectangular In-plane Stress Element

A 4-nodes rectangular element[4] is selected to completely define the horizontal movement of concrete slab. This rectangular element has two degrees of freedom (DOF) at each node, giving a total of eight DOF per element as shown in Fig. 1. The eight DOF correspond to longitudinal and transverse displacements in x and y Cartesian coordinates.

Let the generic displacement of element $u=\{u, v\}$

and the nodal displacements

$q=\{u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4\}$

Then, the shape functions ($f_i$), which relates nodal displacements (q) to generic element displacement

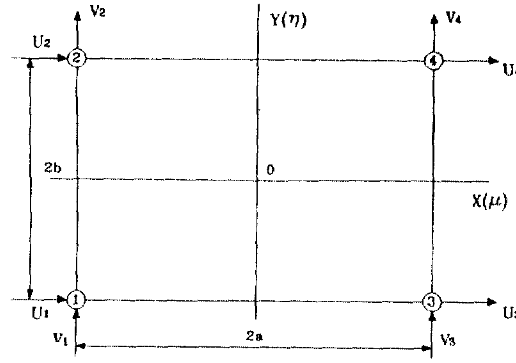



Fig. 1. Rectangular in-plane element

(u), are derived as follows.

$$u=f\cdot q$$
$$f_1=0.25(1-\mu)\ (1-\eta)$$
$$f_2=0.25(1-\mu)\ (1+\eta)$$
$$f_3=0.25(1+\mu)\ (1-\eta)$$
$$f_4=0.25(1+\mu)\ (1+\eta)$$

where, μ, η are the value of −1 to 1 in the normalized coordinate system. We can simply check these shape functions using the Lagrange method with a positive unit displacement at the correct node while the other three nodes are fixed at zero displacements.

then, strain-displacement relationship shows that

$$\varepsilon=(\varepsilon_x,\ \varepsilon_y,\ \varepsilon_{xy})^T$$
$$=[\partial u/\partial x,\ \partial v/\partial y,\ \partial u/\partial y+\partial v/\partial x]^T$$
$$=d\cdot u$$

where, d is the differential operator,

$$d=\begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}$$

By Substituting $u=f\cdot q$ to above equation,

$$\varepsilon=d\cdot f\cdot q$$
$$=B\cdot q$$

where the matrix $B_i=\begin{bmatrix} f_{i,x} & 0 \\ 0 & f_{i,y} \\ f_{i,y} & f_{i,x} \end{bmatrix}$

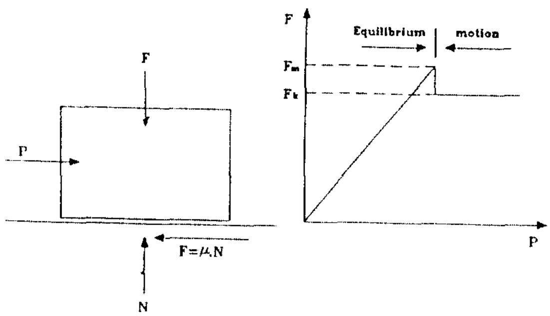



Fig. 2. Classical Friction model

(i=node number)

relates nodal displacements to strains.
With the stress-strain relationships,

$\sigma = \{E\} \cdot \{\varepsilon_T - \varepsilon_0\}$
where, $\varepsilon_T$=Total Strain
$\varepsilon_0$=initial Strain

The element stiffness matrix, and equivalent nodal loads due to initial strains are derived as follows.

$$K = \int_v B^T \, EB \, dv$$

$$= t \, ab \int_\mu \int_\eta B^T \, EB \, d\mu d\eta$$

$$\{F\varepsilon_0\} = -\int [B]^T \, [E]\{\varepsilon_0\} \, dv$$

### 3. Friction Resistance

Since moisture and thermal effects involve no actual applied loads on a highway pavement system, the stresses induced in the pavement will be those due to pavement movement, expansion or contraction, and subbase friction.

Friction resistance between the concrete slab and the subbase is derived based on the simple classical model as shown in Fig. 2.

Since, friction resistance is a function of vertical reaction and coefficient of friction ($\phi$),

In-plane friction stress can be obtained as follows:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \end{bmatrix} = \phi \cdot \{\sigma_N\} = \phi \cdot K_0 \cdot \begin{bmatrix} w \\ w \end{bmatrix}$$

where, $\{\sigma_N\}$ is the vertical surface pressure on the differential element.

$$\{\sigma_N\} = K_0 \cdot \begin{bmatrix} w \\ w \end{bmatrix}$$

$K_0$=Subgrade stiffness
w=Vertical deflection.
By equating internal work and external work,

$$W_T = \phi \cdot K_0 \cdot \{q\}^T \int_a [f_b]^T [f_b] \cdot [q_b] \, dA$$

$$W_e = \{q\}^T \cdot [F_{fr}]$$

where, b subscripts means plate bending element, which has 3 DOF (one vertical deflection, and two rotations per each node.) $[F_{fr}]$ is the equivalent nodal load due to friction.

$$\therefore [F_{fr}] = \phi \cdot K_0 \int_{area} [f]^T \cdot [f_b] \cdot [q_b] \, dA$$

$$= \phi \cdot a \cdot bK_0 \cdot \int_\mu \int_\eta [B]^T [E] \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} d\mu \cdot d\eta$$

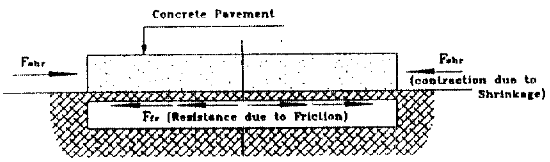
### 4. Shrinkage Load

When the pavement's temperature or internal moisture increases, the pavement will expand, but subbase friction resists the movement, causing compressive stresses to develop in the concrete. Since concrete is strong in compression, this is usually not a problem. However, in situations where the internal moisture decreases (say, shrinkage), the pavement will contract. Subbase friction, however, counters this movement, resulting in tensile stresses in the pavement.

The equivalent nodal loads due to uniform concrete shrinkage $\varepsilon_{shr}$ are derived as follows; By setting $\{\varepsilon_0\}$ equal to $\varepsilon_{shr}$

$$[F_{shr}] = -tab \; \sin(\alpha)\varepsilon_{shr} \int_\mu \int_\eta [B]^T [E] \begin{bmatrix} 1 \\ 1 \end{bmatrix} d\mu \cdot d\eta$$

A careful check should be made to see, if the magnitude of $F_{shr}$ is greater than that of frictional force. If so, the sum of $F_{shr}$ and $F_{fr}$ will be applied to the pavement. If not, the contraction force due to concrete shrinkage will be ignored,




If $|F_{shr}|$ > $|F_{fr}|$ .

Then Total Active Load(PLOAD)

PLOAD = $F_{shr}$ + $F_{fr}$

Else PLOAD = 0.0

Fig. 3. Friction-shrinkage force relationships

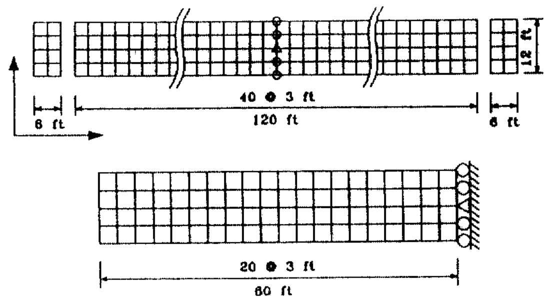



Fig. 4. Pavement system model for analysis of variation in longitudinal stress along longitudinal centerline

## 5. Temperature Load

The thermal gradient in the slab is assumed to be a second degree polynomial, defined by three measured temperatures across the slab thickness, one at the surface ($T_t$), one at the center ($T_m$) and the third at the bottom of the slab ($T_b$). These three temperature readings and a reference temperature ($T_0$) are then fitted to the polynomial equation $F(z)=A+Bz+Cz^2$. $T_0$ is the assumed temperature at which there are no thermal strains in the slab.

By equating an initial strain ($\varepsilon_0$) equal to $\varepsilon_{temp}$, the previous equation yields to the equivalent nodal loading due to thermal effects as follows:

$$[F_t]=\alpha \int [B]^T[E]\begin{bmatrix} F(Z) \\ F(Z) \\ 0 \end{bmatrix} dX\ dY\ dZ$$

The thermal gradient function F(Z) may be integrated over the slab thickness and the temperature readings substituted for the function constants to obtain:

$$T=\int_{-t/2}^{t/2} F(z)\ dZ$$
$$=t[Tm-T_0+(T_t+T_b-2Tm)/6],$$

and for the normalized coordinate system,

$$[F_t]=\alpha ab\ \sin(\alpha) \iint [B]^T[E]\begin{bmatrix} T \\ T \\ 0 \end{bmatrix} d\mu\ d\eta.$$

After computing equivalent nodal forces due to thermal gradients, the sum of shrinkage and thermal induced stresses is compared with frictional forces.

If an absolute magnitude of $F_{shr}$, $|F_{shr}|$, is less than $|F_{fr}|$, then check to see if sum of $|F_{temp}|$ and $|F_{shr}|$ is greater than $|F_f|$. If so, total applied load will be the sum of $[F_{shr}]$, $[F_{temp}]$ and $[F_{fr}]$. Otherwise the total applied load will be zero.

## 6. Computer Model

The pavement is modeled by using a three slab system with two intermediate joints.(6) To provide stability to the system, it is necessary to restrain some degrees of freedom. Since the nodes along the slabs centerlines will have the smallest displacements perpendicular to those centerlines, (equal to zero for a symmetric loading), rollers were placed along the lateral (Y-direction) centerlines of the slabs, and pins at the centers. These boundary conditions, shown in Figure 4, will prevent the nodes along the lateral centerlines from displacing perpendicular to those centerlines and provide structural stability. In addition to that, in order to account for the friction resistance at the subbase boundary, the general structural analysis program with finite plate bending element must be run prior to using this program so as to obtain the vertical nodal displacements needed to determine the frictional forces at the nodes.

## 7. Numerical Examples

The slab system was first analyzed to calculate the tensile stresses that may be induced in a con-

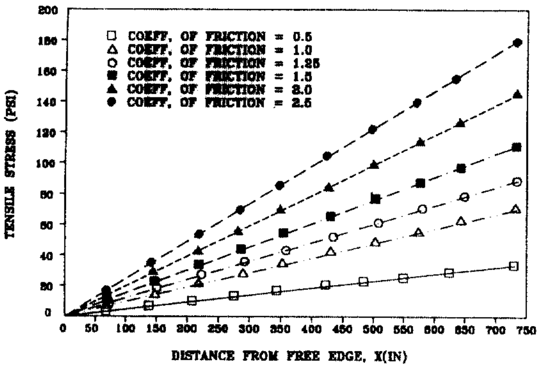



Fig. 5. Variation in stress along longitudinal centerline of unrestrained slab due to concrete shrinkage ($\varepsilon_{sh}=0.0003$, 1 psi=0.07 kg/$cm^2$)

crete slab due to shrinkage strains using the model shown in Figure 4.

Varing levels of shrinkage strains, $\varepsilon_{Shr}=0.0001$, 0.0002, 0.0003, 0.0004, 0.0005, where applied to the slab system for various coefficients of friction ($\phi=$ 0.5, 1.0, 1.5, 2.0, and 2.5). This amounted to a total of 25 analysis cases.

The stresses shown in Fig. 5 are those due to concrete shrinkage only along the longitudinal centerline, the variation in stress is essentially linear. The longitudinal stresses are constant across the pavement width. The variation in lateral stresses across the width of the slab is similar to the longitudinal stress distribution in that it is linear and peaks at the longitudinal centerline.

## 8. Conclusions

The stiffness matrix and the equivalent nodal loads due to planar effects on the concrete pavements, such as concrete shrinkage, thermal expansion or contraction was developed using finite rectangular in-plane element. The derivations are quite good with test problems. To obtain greater accuracy in the results, higher order functions and larger meshes are also of interest. Furthermore, the relationships between the planar forces due to concrete expansion or contraction and the friction resistance at the subbase boundary was well-examined. Since the in-plane displacements in a pavement due to shrinkage and thermal effects involve no applied loads, the stresses induced in the pavement will be those due to closing of the pavement joints and subgrade friction.

From the analysis of the pavement slab system under concrete shrinkage effects, it was determined that

The tensile stress induced in a concrete pavement system due to concrete shrinkage will generally be less than 50% of the tensile strength of the concrete. However, the stress in conjunction with traffic loads might cause cracking problems.

This result show that the stress induced by planar effects would not become significant except at very long slab lengths or extraordinarily high coefficients of friction, large temperature variation, high thermal coefficient, and lateral cracking of the pavement is more likely due to curling of the slab. Consequently, in addition to the traffic load several effects of the environmental variations on the concrete pavements should be studied in the near future to investigate the complicated cause of crack failures in the concrete pavement system.

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