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# 난류전단 흐름에서의 기포응집에 관한 수치모의: 1. 모형의 개발

# Numerical Simulation of the Coalescence of Air Bubbles in Turbulent Shear Flow: 1. Model Development

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**Abstract**

A Monte-Carlo simulation model is developed to predict size distribution produced by the coalescence of air bubbles in turbulent shear flow. The simulation consists of generating a population of air bubbles into the initial positions at each time step and tracking them by simulating motions and checking collisions. The radial displacement of air bubbles in the simulation model is produced by numerically solving an advective diffusion equation. Longitudinal displacements are generated from the logarithmic flow velovity distribution and the bubble rise velocity. Collision of air bubbles for each time step is detected by a geometric test using their relative positions at the beginning of the time step and relative displacements during the time step. At the end of the time step, the total number of bubbles, their positions, and sizes are updated. The computer program is coded such that minimum storages for sizes and positions of bubbles are required.

**요 지**

난류전단 흐름에서의 기포응집에 따른 기포의 크기분포를 예측하기 위한 Monte-Carlo 모의모형을 개발하였다. 임의로 선택된 각 초기위치에 일련의 기포들을 매시각 발생시키고, 각 기포들의 움직임과 충돌을 모의함으로써 각각의 위치와 크기를 추적하도록 하였다. 기포의 횡방향 변위는 이송확산 방정식의 수치해를 이용하여 부여하였으며, 종방향 변위는 흐름의 대수유속분포 및 기포 상승속도로부터 주어지도록 하였다. 각 기포들간의 초기 상대위치와 상대변위를 이용한 기하학적 해석에 의하여 매 시간단계에서의 기포응집을 탐지하여, 시간단계 말기에서의 기포 총수, 각 기포의 위치 및 크기를 결정하였다. 기포들의 크기 및 위치를 나타내기 위하여 소요되는 기억용량을 최소화하도록 전산모형을 구성하였다.

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## 1. Introduction

Bubble size is an important factor for the performance of an aeration system[1-4] that utilizes air bubbles. Many small bubbles are preferable to a few large bubbles as they provide larger air-water interface area. For a given air volume, bubbles with twice the radius give half the total bubble surface area and thus half the rate of oxygen mass transfer. Together with the total amount of air volume, bubble size distribution is essential to determine the total surface area which is required for predicting oxygen uptake.

In an experimental investigation of oxygen transfer in turbulent shear flow[5] in which air was injected into downward water flows in a vertical circular conduit through a number of 0.33 mm diameter orifices around the periphery of the conduit, it was observed that an air pocket formed near the pipe wall, and air bubbles developed at the lower end of the air pocket by the shearing action of turbulence. Different air diffusers were tested to observe the behavior of air injected through the diffuser. The formation of air pockets did not depend on the size or the number of orifices. Bubbles generated in the limited portion of the pipe cross section near the wall were mixed over the cross section due to turbulent diffusion as they traveled downward. The measurements of bubble sizes indicated that the bubbles were smaller in size but larger in number near the air diffuser, and they were larger in size but fewer in number far downstream. It was also observed that the change in bubble size became less significant as the bubbles traveled further downstream.

The simulation of size distribution of air bubbles as observed in the experimental investigation requires modelling of the following processes: (1) formation of the air pocket around the orifices; (2) generation of air bubbles from the air pocket; (3) motion of air bubbles; and (4) disintegration and coalescence of air bubbles. The bubble formation from an orifice has been studied extensively.[6] At low air flow rate through the orifice, bubbles form at, and detach from the orifice one after the other. The duration between the release of one bubble and the formation of the next bubble decreases with increasing air flow rates. At a critical flow rate this duration goes to zero and the bubble by bubble regime transfer to the continuous bubble regime forming an air pocket around the orifice. The formation of the air pocket in the above-mentioned experiments was due to air flow rates larger than the critical flow rate. The air pocket is broken into air bubbles as a result of the dynamic influence of the liquid. However, the process of bubble formation from the air pocket is not well understood to allow simulation of size distribution of air bubbles released from the air pocket. Therefore, the generation of the bubbles is not modeled. Instead, an initial bubble size distribution as an input to the simulation model is prescribed, and the model sensitivity to the initial size distribution is investigated. The motion of air bubbles is simulated from the known distribution of the longitudinal velocity of the flow and the radial concentration distribution of air given by the advective diffusion equation. Bubble coalescence occurs when they collide. The collision of air bubbles is detected by a geometric test using their initial positions and displacements. Air bubbles which grow to a size larger than a critical value[7,8] break up into smaller bubbles due to deformation and internal dynamic pressure; the latter in turn is due to the gas motion induced by the drag exerted by the liquid. The maximum bubble size measured in each test in the experimental investigation was smaller than the maximum diameters computed from the relation proposed by Hinze.[7] It is, therefore, judged that the air bubbles did not grow to the critical size, and bubble breakup did not occur.

These observations suggest that in the experimental investigation bubbles initially crowded in a limited cross-sectional area collided and coalesced with each other, grew in size, and reached an equilibrium size distribution as they were mixed over the cross section by turbulent diffusion. A schematic representation of this behavior is given in Fig. 1. This paper describes the Monte-Carlo simulation model which simulates the cross-sectional turbulent mixing and collision processes of air bubbles to obtain the bubble size distribution. Sensitivity analysis and model application are

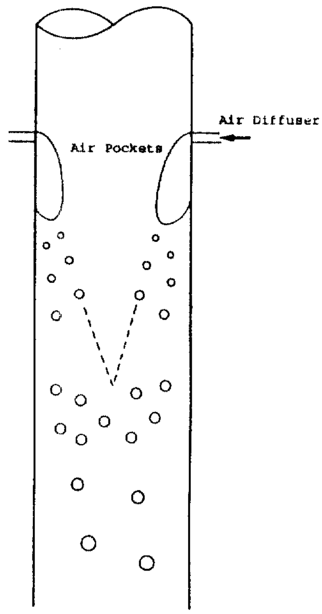


Fig. 1. Schematic of the Behavior near the Air Diffuser

presented in the companion paper.[9]

## 2. Description of the Numerical Model

### 2.1 Outline

A schematic representation of the computational domain is given in Fig. 2. The flow domain is discretized as D(1), D(2), ⋯, D(i), ⋯, D(m) according to the longitudinal location; the longitudinal axis is along the downward vertical direction. Each D (i) is subdivided as D(i, 1), D(i, 2), ⋯, D(i, j), ⋯, D(i, n) according to the radial position. D(0) is an artificial volume for the initial introduction of the air bubbles.

The simulation starts by generating, at each time step, a certain number of spherical bubbles of uniform size randomly placed in some subdomains of D(0). If any two bubbles are placed close enough to each other to collide, they are assumed to coalesce to form a larger one, thereby reducing the total number of bubbles by one. Therefore, the population of bubbles consists of bubbles of a unit volume and integer multiples of the unit volume. This is so throughout the entire simulation as well as initially.

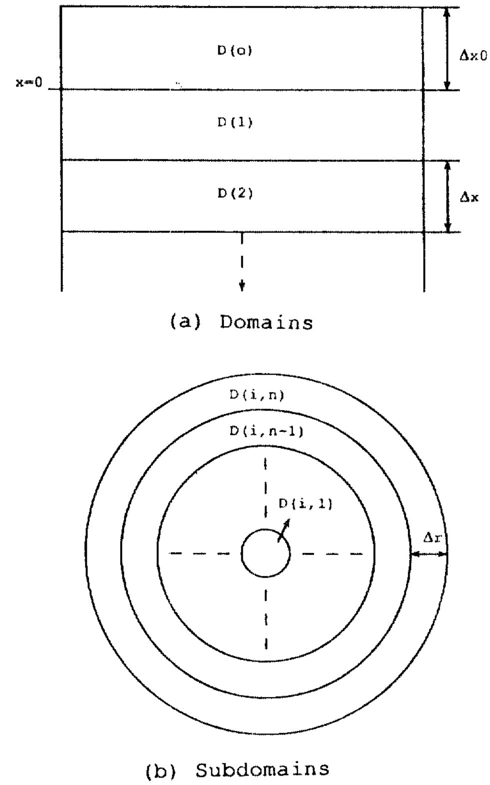


Fig. 2. Schematic Representation of the Computational Domain

After introducing a new air volume in an initial configuration, one generates the displacements of bubbles during each time step. Longitudinal displacements are generated from the known velocity distribution of the flow and the bubble rise velocity. The radial displacements at each longitudinal location are generated based on the radial concentration distribution of air volumes which is prescribed by the solution of the advective diffusion equation presented in the ensuing section. The method for generating displacements is described in detail later.

It is assumed that each bubble travels on a straight-line path at constant speed during each small time step. Based on this assumption the collisions between bubbles are detected by a collision algorithm developed by Pearson *et al*[10]. At the end of the time step, the total number of bubbles, their positions, and sizes are updated. Required computer storages are minimized by replacing those for bubbles moving out of the aeration region with those for bubbles newly introduced into the system.

## 2.2 Turbulent Diffusion of Air Bubbles

The advective diffusion equation for air bubbles in a steady turbulent shear flow in a downward vertical pipe can be written as

$$u_a\frac{\partial c}{\partial x}-\frac{\varepsilon}{r}\frac{\partial c}{\partial r}-\frac{\partial}{\partial r}(\varepsilon\frac{\partial c}{\partial r})=0 \quad (1)$$

in which $c(x, r)$ = volume concentration of air bubbles; $u_a(r)$ = longitudinal velocity of bubbles; $\varepsilon(r)$ = turbulent diffusion coefficient; $r$ = radial coordinate; and $x$ = longitudinal coordinate. Diffusive transport in the x direction, the relative motion of bubble to the water in the r direction, and change in bubble volume by compression are considered negligible in Eq. (1).

Longitudinal bubble velocity($u_a$) of individual bubbles can be expressed as

$$u_a=u-u_0 \quad (2)$$

where $u(r)$ = longitudinal velocity of water; and $u_0$ = bubble rise velocity. The bubble rise velocity for air bubbles of the order of millimeters in diameter is fairly constant as 22.5 cm/s.[11]

If a logarithmic velocity distribution is assumed except for the region very near the wall, u can be written as

$$u(r)=U+\frac{u_*}{\kappa}[\frac{3}{2}+\ln(\frac{a-r}{r})] \quad (3)$$

where $U$ = cross-sectional mean flow velocity; $u_*$ = shear velocity; $a$ = pipe radius; and $\kappa$ = von Karman constant. From the linear shear stress distribution and Reynolds analogy, the following parabolic diffusivity distribution can be derived[12]:

$$\varepsilon(r)=\kappa u_* r(1-\frac{r}{a}) \quad (4)$$

The boundary conditions at the pipe axis ($r=0$) and on the wall ($r=a$) are respectively given by symmetry and no-flux conditions as

$$\frac{\partial c}{\partial r}(x, 0)=0 \quad (5)$$

$$\frac{\partial c}{\partial r}(x, a)=0 \quad (6)$$

The initial condition for air bubbles of volume flow rate $Q_a$ introduced at $x=0$ through an annulus area $A_s$ between $r=a_1$ and $r=a_2$ can be expressed as

$$c(0, r)=\frac{Q_a}{U_s A_s}(a_1<r<a_2) \quad (7a)$$

$$=0 \text{ (otherwise)} \quad (7b)$$

in which

$$A_s=\pi(a_2^2-a_1^2) \quad (8)$$

and $U_s$ = mean velocity of air bubbles in $A_s$ and $Q_a$ = air flow rate. The initial condition represents the air concentration at the location at which air bubbles are released from the air pocket. The Crank-Nicholson scheme was used to solve the advective diffusion equation; the details can be found elsewhere.[13]

## 2.3 Simulation of Initial Conditions

The total volume of air ($V_a$) introduced to the aeration system during each time step ($\Delta t$) is

$$V_a=Q_a\Delta t \quad (9)$$

in which $Q_a$ is the rate of air injection. Let $V_0$ be a unit volume of the initially generated bubbles, i.e., the volume of the minimum size bubbles. If the bubbles are spherical, $V_0$ can be written in terms of radius ($R_0$) as

$$V_0=\frac{4}{3}\pi R_0^3 \quad (10)$$

For bubbles which are not spherical, $R_0$ is the equivalent radius of a spherical bubble with volume $V_0$. Then, the number of bubbles ($N_0$) of volume $V_0$ which should be added to the system in each time step is

$$N_0=\frac{Q_a\Delta t}{V_0} \quad (11)$$

This number of bubbles is placed in a subdomain or subdomains of D(0) near the wall corresponding to the area $A_s$ in the pipe cross section through which bubbles are initially introduced. The size of D(0) is taken as

$$\Delta x_0 = U_s \Delta t \tag{12}$$

which is the distance a bubble in $A_s$ travels during $\Delta t$ with an average velocity $U_s$. Positions of individual bubbles are assigned such that they are randomly distributed inside the annulus of volume $A_s \Delta x_0$, and an initial unit volume ($V_0$) is assigned to each bubble. Before simulation of any motion, collisions of bubbles are examined by simply checking whether the distance between the centers of any two bubbles is shorter than the sum of their radii. Then the numbers and sizes of the initially-generated bubbles are updated by discarding one member of the colliding pair and adjusting the size of the other to conserve the total air volume.

### 2.4 Displacement of Air Bubbles

The longitudinal temporal mean velocity of an air bubble of an equivalent radius $r_b$ located at a distance r from the center of the pipe cross section is

$$u_a(r,\ r_b) = u(r) - u_0(r_b) \tag{13}$$

The bubble rise velocity, $u_0(r_b)$, is prescribed using the experimental results of Haberman and Morton.[11] The longitudinal displacement of each bubble during $\Delta t$ is obtained by multiplying $u_a(r,\ r_b)$ by $\Delta t$, and from this, bubbles in the system are repositioned longitudinally. Then, the total volume of bubbles in each domain, D(i) is obtained by counting all the bubbles in it and adding up their volumes. Also obtained is the volume of bubbles in each D(i, j). From the cross-sectional volume distribution of air bubbles for each D(i) prescribed by the solution of the advective diffusion equation and total air volume in D(i) obtained above, volumes which should belong to each subdomain, D(i, j) can be computed. The air volume in each D(i, j), which has been obtained after the longitudinal repositioning, is adjusted to satisfy the prescribed cross-sectional distribution for each D(i) by generating the radial displacement in the following way. Determined first are the air volumes to be moved between subdomains in each D(i): from D(i, n) to D(i, n−1), D(i, n−2), ⋯ from D(i, n−1) to D(i, n−2), D(i, n−3), ⋯ etc. The criteria used to determine these is to let as much air volume as possible move to subdomains nearest to the one to which it belongs in the previous time step. What this implies physically is that $\Delta r$ is the most significant scale of turbulent eddies which contribute to the radial mixing of air bubbles. It also represents the minimum scale such that eddies smaller than it is not significant to the diffusion of air bubbles. Bubbles to be moved to other subdomains are chosen randomly. The radial position of a bubble in a subdomain to which it moves is also given at random. The radial displacement of a bubble is given such that it moves on a straight-line path towards the center of the pipe. Although large scale eddies are important for the geometric-scale mixing, there also exist motions driven by eddies which are smaller than $\Delta r$. The small scale motion of bubbles which stay in the same subdomain as in the previous time step was simulated by moving them to an arbitrary position belonging to the same subdomain.

### 2.5 Collision Algorithm

The simulation of longitudinal and lateral displacements gives displacement vectors at each time step. Whether two bubbles collide during their displacements can be detected by a geometrical test using their relative initial position and displacement. An algorithm developed by Pearson *et al.* [10] was adopted. A description of the algorithm is given below.

Fig. 3 gives a schematic illustration for the collision algorithm. Let $\mathbf{P}_1$ and $\mathbf{P}_2$ be initial position vectors of bubbles 1 and 2 of radius $r_1$ and $r_2$, respectively. Let $\mathbf{D}_1$ and $\mathbf{D}_2$ be displacement vectors such that position vectors after $\Delta t$ become $\mathbf{P}_1 + \mathbf{D}_1$ and $\mathbf{P}_2 + \mathbf{D}_2$, respectively. Define $d\mathbf{P}$ and $d\mathbf{D}$ as

$$d\mathbf{P} = \mathbf{P}_1 - \mathbf{P}_2 \tag{14}$$

$$d\mathbf{D} = \mathbf{D}_2 - \mathbf{D}_1 \tag{15}$$

Then, the initial distance between bubbles 1 and 2 is $|d\mathbf{P}|$. The distance after $\Delta t$ is $|d\mathbf{P} - d\mathbf{D}|$. Let $\theta$ be an angle formed by $d\mathbf{P}$ and $d\mathbf{D}$. For $|d\mathbf{P} - d\mathbf{D}|$ to be less than $|d\mathbf{P}|$, cos $\theta$ or the inner product of $d\mathbf{P}$ and $d\mathbf{D}$ should be positive. This

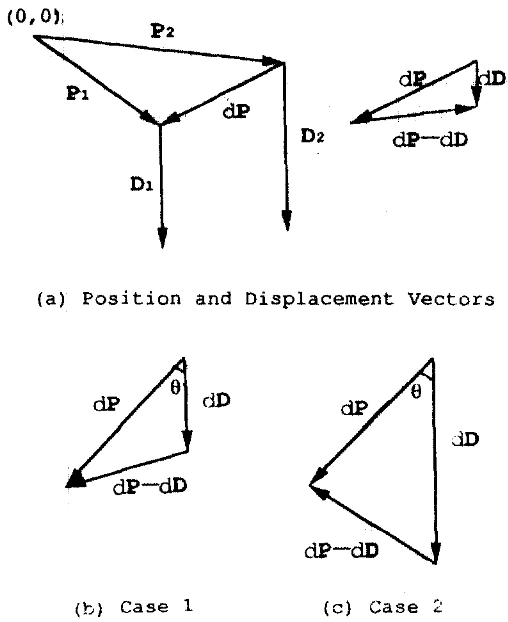


Fig. 3. Geometry for Collision Algorithm

is a necessary condition for the collision of two bubbles. In addition, the minimum distance between bubbles 1 and 2 during $\Delta t$ should be less than the sum of radii, $r_1$ and $r_2$. There are two possible cases of collisions. If the minimum distance occurs at $\Delta t$, i.e., at the end of time step, then

$$|\mathbf{dD}| \leq |\mathbf{dP}| \cos\theta \tag{16}$$

In this case (Case 1 in Fig. 3), the following relation should be satisfied for a collision:

$$|\mathbf{dP} - \mathbf{dD}| \leq r_1 + r_2 \tag{17}$$

If the minimum distance occurs sometime before $\Delta t$, then

$$|\mathbf{dD}| > |\mathbf{dP}| \cos\theta \tag{18}$$

In this case (Case 2 in Fig. 3), the minimum distance is $|\mathbf{dP}| \sin\theta$. Thus, for a collision the following condition should be satisfied:

$$|\mathbf{dP}| \sin\theta \leq r_1 + r_2 \tag{19}$$

Either Eqs. (16) and (17) or Eqs. (18) and (19) constitute a condition for collisions together with $\cos\theta > 0$.

It is very costly to detect which bubbles have collided at each time step. It generally requires $N^2$ operations, where N is the total number of bubbles in the system. To save computing time, it is first checked whether each pair of bubbles are within a longitudinal distance of $U_a \Delta t$ at the beginning of the time step, where $U_a$ is the cross-sectional mean velocity of air bubbles. Only if this is so, they are considered as candidates for a collision and the computation is performed to check for collision. If two bubbles collide, one of those randomly chosen is assigned a larger volume equal to the sum of the two volumes before the collision. A zero volume is assigned to the other. In this way, the radial volume distribution after the collision remains almost the same as the prescribed distribution. Also assigned a zero volume are bubbles which have travelled far enough to reach a certain downstream position corresponding to the outlet of the aeration region. Bubbles with zero volume are removed from the system. The computer storage for them is reused for those introduced at the initial position in the next time step. This prevents the increase of required computer storage with increasing number of time steps after bubbles occupy the entire downstream region of interest.

## 3. Conclusions

The cross-sectional turbulent mixing and collision processes of air bubbles in turbulent shear flow are numerically simulated by a Monte-Carlo simulation technique to obtain the bubble size distribution. The simulation proceeds by tracking the positions and sizes of a variable number of spherical bubbles. Longitudinal displacements are generated from the given velocity distribution of the flow and the bubble rise velocity. An advective diffusion equation is solved to provide the radial displacement. Bubble collisions are simulated on the basis of straight-line trajectories during each time step.

In the following paper,[10] the model is applied to the case of laboratory experiment and some of the possibilities for its use and further improvement are explored.

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