Hybrid Analysis of Dynamic Contact Problem by Impact

Jin－Wook KIM
(Kobe University)

NOMENCLATURE

b : Vector of body force
$C_{\beta c}$: Velocity of stress wave
E_β : Young's modulus
g_o : Vector of initial displacement between contact nodes
K : Stiffness matrix
$[K_\beta^c]$: Contact stiffness matrix of contact surface
M : Mass matrix
m_{ij}^β : Nodal mass of the contact surface
$R_{F\beta}$: Vector of the incremental contact force
S : Surface of contact body
S_β : Surface of boundary tractions
S_u : Constraint surface of displacement
t : Vector of boundary tractions
T : Vector of contact force
$\tilde{T}_i$: i－th nodal vector of external force
$\Delta\tilde{T}_\beta^*$: Incremental vector of the external force
U : Vector of displacement
$\dot{U}$: Vector of velocity
$\ddot{U}$: Vector of acceleration
V : Volume of the body
$\{f_{Ii}\}_N$: i－th nodal vector of the inertia force
$\{f_{Bi}\}_N$: i－th nodal vector of the body force
$\{P_{Ii}\}_N$: i－th nodal vector of the nodal force

$\{f_{Ti}\}_N$: i – th nodal vector of the boundary traction

α : Penalty coefficient(=10^7)

δ : Variational increment

δE^β : Virtual works of the internal elastic

δT^β : Virtual works of the inertia

δW^β : Virtual works of the traction boundary force

δW_c^β : Virtual works of the contact force

$\delta \epsilon_N$: Virtual strains of the N time step

μ : Friction coefficient

ν : Poisson's ratio

Ω_β : Domain of interest

Π : Function of system

ρ : Density

σ : Vector of stress

$V_{\beta c}$: Velocity or acceleration on contact surface

Subscripts

$(\cdot)_\beta$: Contact body A and B

$(\cdot)_c$: z and t direction on contact surface

$(\cdot)_{co}$: Open states

$(\cdot)_{csl}$: Slip states

$(\cdot)_{cst}$: Stick states

$(\cdot)_N$: Time step

$(\cdot)_t$: Tangential direction of global coordinates

$(\cdot)_z$: Normal direction of global coordinates

1. INTRODUCTION

Equations resulting from the finite – element method for the analysis of the stress wave propagation or fracture mechanism due to impact, may involve many thousand degrees of freedom. When the problem to be solved is dynamic and nonlinear, the computer time required can be prohibitive. Currently, there are two classes of algorithms for the time integration of the semidiscretizations: explicit and implicit. Explicit methods usually do not involve the simultaneous solution of systems of equations, so fewer computations are needed per time step compared to implicit methods, but numerical stability requires that the time step be small. Implicit methods, on the other hand, involve the simultaneous solution of a system of equations requiring matrix operations, but much larger time steps can be employed because they are unconditionally stable for linear and many nonlinear problems. These issues are elaborated in the description of Belytschko and Mullen(1978), and Hughes and Liu(1978, 1978).

In practice, the implicit method has pactical difficulties with respect to the calculation time for iteration and large computer memory, but it is preferable for numerical stability and accuracy of solution. The explicit method, on the other hand, has some practical merits of calculation time and computer memory, but it exhibits deterioration in solution accuracy compared to the implicit method. Considering these points, we seek to develop a method which has high solution accuracy and relatively short calculation time. This paper proposes a calculation algorithm for the dynamic contact problem which combines the merits of the explicit and implicit methods.

It is necessary to take into account the procedure for the analysis of contact force with friction on contact surface in impact, so mesh partitions in each body are divided into two groups : (1) the explicit groups, and (2) the implicit groups of the contact surface. In each time step the explicit nodes are integrated first. The calculated results of the incremental nodal force are used subsequently as the boundary conditions of the external nodal force for the integration of the implicit nodes. The incremental nodal displacements of the interface at each time step are re-evaluated using the results of the implicit calculation. In this method, a "strong coupling" procedure is employed in the interface which is composed of the same nodes for the explicit and implicit elements. A "weak coupling" procedure is used at the contact nodes of the implicit elements on the contact surface between two bodies. The hybrid algorithms employed are the Newmark $-\beta$ methods ; a central difference explicit integration and an implicit integration will be considered. After the separation between two bodies, the total system is solved using the explicit method exclusively.

To illustrate the method numerical results from the impact of steel and cemented carbides is considered. An impact velocity of $30m/sec$ is applied, and the numerical results, calculated with various frictional coefficients are then presented. The results are compared with the implicit method and conventional explicit method without considering the frictional force.

2. FORMULATION OF IIYBRID METIIOD AT DYNAMIC CONTACT

In our approach the mesh is partitioned into two explicit and implicit domains in each body. At the interface the nodes are not segregated into two sets, but rather the common nodes are considered. The primary consideration in establishing the governing equilibrium equations of the system using the hybrid technique of mesh partition is the way that boundary conditions in the interface can be generated and taken into account. The hybrid analysis is then based upon the following hypotheses :

a. The force boundary conditions at the interface must be satisfied simultaneously when invoking the stationarity of the functional of the total system.

b. The nodal displacements, velocities and accelerations of the interface are evaluated respectively by the same ones without the relation between mesh partitions.

c. The nodal masses of the interface are estimated by the common ones.

d. The stiffness matrices are individually partitioned into explicit and implicit categories.

The equations of the implicit method are formulated by the incremental displacement theory.

2. 1 Principle of virtual work

Two elastic bodies A and B in contact are considered as shown in Fig.1. The contact states are classified into : (1) opening state($(\cdot)_{co}$; where the two bodies are separatedt) ; (2) sticking state ($(\cdot)_{cst}$; where the two bodies are in contact and slipping doesn't occur) ; (3) slipping state($(\cdot)_{csl}$;

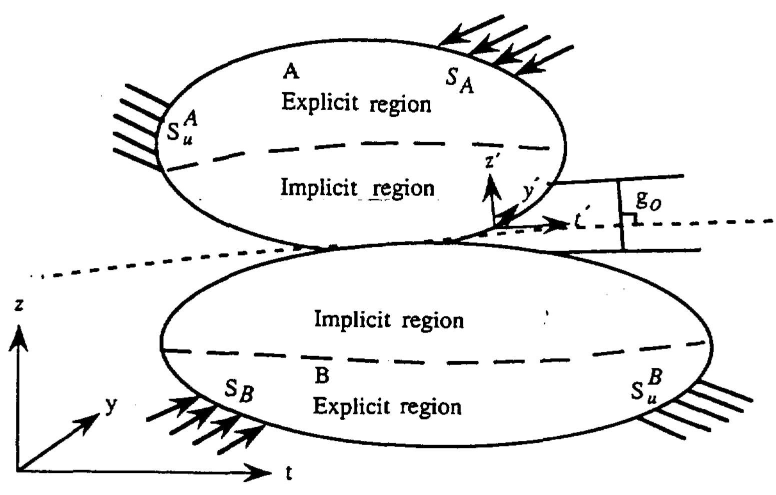


Fig. 1 Contact model of the hybrid method

where the two bodies are in contact and slipping occurs). It is necessary to satisfy the conditions which are the moment equilibrium and the continuity of displacement, respectively, and also to occasionally consider Coulomb's friction on contact states. The contact problem as shown in Fig. 1 is formulated by using the principle of virtual work. Using the functional Π, the variational formulation may be regarded as a generalization of the principle of virtual displacement. The formulation of the principle of virtual work to the explicit and implicit methods are shown in the following expressions respectively

$$\delta\Pi_{EXP}=\sum_{\beta=A,B}(\delta E^{\beta}-\delta T^{\beta}-\delta W^{\beta}-\delta W_c^{\beta})=0 \tag{1}$$

$$\delta\Pi_{IMP}=\sum_{\beta=A,B}(\delta E^{\beta}-\delta T^{\beta}-\delta W^{\beta}-\delta W_c^{\beta})=0 \tag{2}$$

2. 2 Formulation of explicit method

To establish the governing equations of the explicit mesh partitions, we use the finite element method. Form Eq.(1), if the virtual contact work is equal to zero, then the virtual work equation can be obtained by invoking the stationarity of Π with respect to the displacements of time step (Owen and Hinton 1980) as

$$\begin{aligned}&\int_{\Omega A}[\delta\varepsilon_N]^T\sigma_N d\Omega_A-\int_{\Omega A}[\delta\mathbf{U}_N]^T i\mathbf{b}_N-\rho_N\ddot{\mathbf{U}}_N j d\Omega A-\int_{\Omega A}[\delta\mathbf{U}_N]^T\mathbf{t}_N dS_A\\&+\int_{\Omega B}[\delta\varepsilon_N]^T\sigma_N d\Omega_B-\int_{\Omega B}[\delta\mathbf{U}_N]^T i\mathbf{b}_N-\rho_N\ddot{\mathbf{U}}_N j d\Omega_B-\int_{SB}[\delta\mathbf{U}_N]^T\mathbf{t}_N dS_B=\mathbf{0}\end{aligned} \tag{3}$$

Using integration by parts we obtain the following equation from Eq.(3)

$$\{\mathbf{f}_{Ii}\}_N-\{\mathbf{f}_{Bi}\}_N+\{\mathbf{P}_{Ii}\}_N-\{\mathbf{f}_{Ti}\}_N=\mathbf{0} \tag{4}$$

where the nodal force is

$$\{\mathbf{P}_{Ii}\}_N=\int_{\Omega}\{\mathbf{B}_i\}^T\sigma_N d\Omega \tag{5}$$

The incremental nodal force is obtained simply by using the relationship between the $N-1$ time steps,

$$\{\Delta P_{Ii}\}=\{\mathbf{P}_{Ii}\}_N-\{\mathbf{P}_{Ii}\}_{N-1} \tag{6}$$

2. 3 Formulation of implicit method

Using the penalty function method, the continuous condition of the displacement on the contact surface between two contact bodies, and invoking the stationarity condition of $\delta\Pi=0$ as Eq.(2), we can obtain the governing equations for the functional. This relation is in fact the principle of incremental virtual work corresponding to the nonlinearity of the contact surface(Yagawa et al. 1983). We obtain

$$\begin{aligned}\int_{t(N)}^{t(N+1)} \{ & \int \Omega_\beta (\sigma_{ij}^{(N)}\delta\Delta\varepsilon_{ij} + 1/2\Delta\sigma_{ij}\delta\Delta\varepsilon_{ij} - 1/2\rho\Delta\dot{U}_i^{(N)}\delta\Delta\dot{U}_i - 1/2\rho\Delta\dot{U}_i\delta\Delta\dot{U}_i)dV_\beta \\ & - \int_{S\beta}\tilde{\mathbf{T}}_i^{(N)}\delta\Delta\mathbf{U}_i dS_\beta - \int_{S\beta}\Delta\tilde{\mathbf{T}}\delta\Delta\mathbf{U}_i dS_\beta \\ & + \int_{co+csl+cst}\boldsymbol{T}_c^{(N)}(\delta\Delta\mathbf{U}_c^A - \delta\Delta\mathbf{U}_c^B)dS \\ & + \int_{cst}\Delta\mathbf{T}_c(\delta\Delta\mathbf{U}_c^A - \delta\Delta\mathbf{U}_c^B)dS + \int_{csl}\Delta\mathbf{T}_t(\delta\Delta\mathbf{U}_t^A - \delta\Delta\mathbf{U}_t^B)dS \\ & + \int_{csl}\Delta\mathbf{T}_z(\delta\Delta\mathbf{U}_z^A - \delta\Delta\mathbf{U}_z^B)dS + \int_{co}\mathbf{T}_c(\delta\Delta\mathbf{U}_c^A - \delta\Delta\mathbf{U}_c^B)dS\}dt = 0 \end{aligned} \quad (7)$$

where

$$\Delta\mathbf{T}_c = \alpha(\Delta\mathbf{U}_c^A - \Delta\mathbf{U}_c^B) \quad (8)$$

$$\Delta\mathbf{T}_t = \pm\mu(\mathbf{T}_{zo} + \alpha(\Delta\mathbf{U}_z^A - \Delta\mathbf{U}_z^B + \mathbf{g}_o)) - \boldsymbol{T}_{to} \quad (9)$$

$$\Delta\mathbf{T}_z = \alpha(\Delta\mathbf{U}_z^A - \Delta\mathbf{U}_z^B + \mathbf{g}_o) \quad (10)$$

$$\boldsymbol{g}_o = \mathbf{U}_{zo}^A - \mathbf{U}_{zo}^B \quad (11)$$

The equilibrium equations of the dynamic contact system are obtained by idealizing Eq.(7) as

$$+\begin{bmatrix} [\mathbf{M}_A] & 0 \\ 0 & [\mathbf{M}_B] \end{bmatrix}\begin{Bmatrix} \Delta\ddot{\mathbf{U}}_A \\ \Delta\ddot{\mathbf{U}}_B \end{Bmatrix} + \begin{bmatrix} [\mathbf{K}_A] & 0 \\ 0 & [\mathbf{K}_B] \end{bmatrix}\begin{Bmatrix} \Delta\mathbf{U}_A \\ \Delta\mathbf{U}_B \end{Bmatrix}$$

$$+\begin{bmatrix} [K^c_{AA}] & [K^c_{AB}] \\ [K^c_{BA}] & [K^c_{BB}] \end{bmatrix}\begin{Bmatrix} \Delta\mathbf{U}_A \\ \Delta\mathbf{U}_B \end{Bmatrix} = \begin{bmatrix} \Delta\tilde{\mathbf{T}}_A^* \\ \Delta\tilde{\mathbf{T}}_B^* \end{bmatrix}\begin{Bmatrix} \mathbf{R}_{FA} \\ \mathbf{R}_{FB} \end{Bmatrix} \quad (12)$$

The vector of the contact force with friction at any time step is shown by using Coulomb's frictional condition to slipping states and the incremental contact force

$$\begin{Bmatrix} \Delta\mathbf{T}_A \\ \Delta\mathbf{T}_B \end{Bmatrix} = \begin{bmatrix} [K^c_{AA}] & [K^c_{AB}] \\ [K^c_{BA}] & [K^c_{BB}] \end{bmatrix}\begin{Bmatrix} \Delta\mathbf{U}_A \\ \Delta\mathbf{U}_B \end{Bmatrix} + \begin{bmatrix} \mathbf{R}_{FA} \\ \mathbf{R}_{FB} \end{bmatrix} \quad (13)$$

$$\mathbf{T}_{to} = +\Delta\mathbf{T}_t = \pm\mu(\mathbf{T}_{zo} + \Delta\mathbf{T}_z)\,|\,\phi(\bar{v})\,| \quad (14)$$

$$\phi(\bar{v}) = tanh(\xi(\dot{\mathbf{U}}_t^A - \dot{\mathbf{U}}_t^B)),\ 0<\xi\leq 1. \quad (15)$$

We now introduce Eq.(14) where Eq.(15). The $\pm$sign is Eq.(14) is opposite to the direction of the slip direction. $\phi(v)$ is the hyperbolic function used to relax the numerically unstable problem due to a rapid change of slipping direction. In a practical calculation, the hyperbolic function is implemented to evaluate the frictional force. This form may be used to obtain the stable solution of the nonlinear frictional system.

3. COMBINING THE EQUATIONS OF MOTION

The vectors of the incremental nodal forces, which are fist calculated by the explicit time integration from Eq.(5), are imposed on the total equation to solve the unknown variable in implicit time integration. Hence, the relation between Eq.(6) and Eq.(12) can be rearranged as

$$[\mathbf{M}_\beta]\{\Delta\ddot{\mathbf{U}}_\beta\}_N+[\mathbf{K}_\beta]\{\Delta\mathbf{U}_\beta\}_N+[\mathbf{K}_C]\{\Delta\mathbf{U}_{\beta c}\}_N$$
$$=\Delta\tilde{\mathbf{T}}_\beta{}^*\}_N-\{\mathbf{R}_{F\beta}\}_N-\{\Delta\mathbf{P}_{Ii}\}_N. \tag{16}$$

3. 1Criteria of contact states

Consider two contact bodies A and B as shown in Fig.1. The contact conditions are classified into three categories as described previously. If the assumed contact state is open, the criteria of the open and contact (sticking or slipping) states are respectively given as follows by relative displacements.

$$\Delta\mathbf{U}_Z{}^A-\Delta\mathbf{U}_Z{}^B+\mathbf{g}_o>\mathbf{0} \tag{17}$$
$$\Delta\mathbf{U}_Z{}^A-\Delta\mathbf{U}_Z{}^B+\mathbf{g}_o\leq\mathbf{0} \tag{18}$$

If the assumed contact state is sticking or slipping, the criteria of the open and contact(sticking or slipping) states are respectively given as follows by contact forces.

$$\mathbf{T}_{zo}+\Delta\mathbf{T}_z<\mathbf{0} \tag{19}$$
$$\mathbf{T}_{zo}+\Delta\mathbf{T}_z\geq\mathbf{0} \tag{20}$$

The criteria of contact states are as follows. If the inequalities (21) and (22) are satisfied, the contact state will be the sticking state.

$$|\mathbf{T}_{to}+\Delta\mathbf{T}_t|\leq|\mu(\mathbf{T}_{zo}+\Delta\mathbf{T}_z)| \tag{21}$$
$$(\mathbf{T}_{to}+\Delta\mathbf{T}_t)(\Delta\mathbf{U}_t{}^A-\Delta\mathbf{U}_t{}^B)\geq 0 \tag{22}$$

the states are the sticking states.

$$|\mathbf{T}_{to}+\Delta\mathbf{T}_t|\geq|\mu(\mathbf{T}_{zo}+\Delta\mathbf{T}_z)| \tag{23}$$
$$(\mathbf{T}_{to}+\Delta\mathbf{T}_t)(\Delta\mathbf{U}_t{}^A-\Delta\mathbf{U}_t{}^B)\leq 0 \tag{24}$$

If the inequalities (23) and (24) are satisfied, the contact state will be the slipping state.

3. 2 Velocity and acceleration on contact surface

The continuity of velocities and accelerations between two contact points should be satisfied at the instant of the impact. The updated velocities and accelerations at the contact points are formulated in a similiar fashion as in the references(Yagawa et al. 1983, Chen and Tsai 1986). However, in the slipping states the tangential values of velocities and accelerations are free.

$$\mathbf{V}_{\beta c}=\frac{\rho_A \mathbf{C}_{Ac}\mathbf{V}_{Ac}+\rho_B \mathbf{C}_{Bc}\mathbf{V}_{Bc}}{\rho_A \mathbf{C}_{Ac}+\rho_B \mathbf{C}_{Bc}} \tag{25}$$

where $\mathbf{C}_{\beta c}(\beta=A, B\ ;\ c=z, t)$ are the velocities of the stress waves which are the longitudinal and transverse waves respectively. Then

★v : NU

$$C_{\beta z}=\sqrt{\frac{E_\beta}{\rho_\beta(1+\nu_\beta)}\cdot\frac{1-\nu_\beta}{1-2\nu_\beta}} \tag{26}$$

and

$$C_{\beta t}=\sqrt{\frac{E_\beta}{2\rho_\beta(1+\nu_\beta)}} \tag{27}$$

3. 3 Nodal mass estimation in interface

Considering the same accelerations in each interface without the relation between mesh partitions, and for the sake of the numerical stability after separation of two bodies, the lumped masses of each interface are estimated by the same ones. The lumped masses at the interface between the explicit and implicit regions are estimated as

$$\mathbf{m}_i^{\beta}=\sum_{j=1}^{n}\mathbf{m}_{ij}^{Exp}+\sum_{j=1}^{n}\mathbf{m}_{ij}^{Imp} \tag{28}$$

where i is the interface node of mesh partition and j is the element of the interface node. The superscripts, $(\cdot)^{Exp}$ and $(\cdot)^{Imp}$, respectively, denote the explicit and implicit mesh partitions in interface, and n is the number of elements which include the node i.

4. NUMERICAL ANALYSIS AND DISCUSSIONS

The Newmark – β method is employed to integrate Eq.(16), and the parameters β and γ of the

Newmark – β method are $\beta=0$, $\gamma=1/2$ in the case of the explicit time integration and $\beta=1/4$, $\gamma=1/2$ in the case of the implicit time integration. Incremental time, Δt, is taken to be 0.054μsec. The analytical model is shown in Fig. 2. It is a two – dimensional axisymmetric model, and the mechanical properties of steel and cemented carbide are shown in Table 1. The length and diameter of the striker are 8mm and 10*mm* respectively, and its impact velocity is 30*m/sec*.

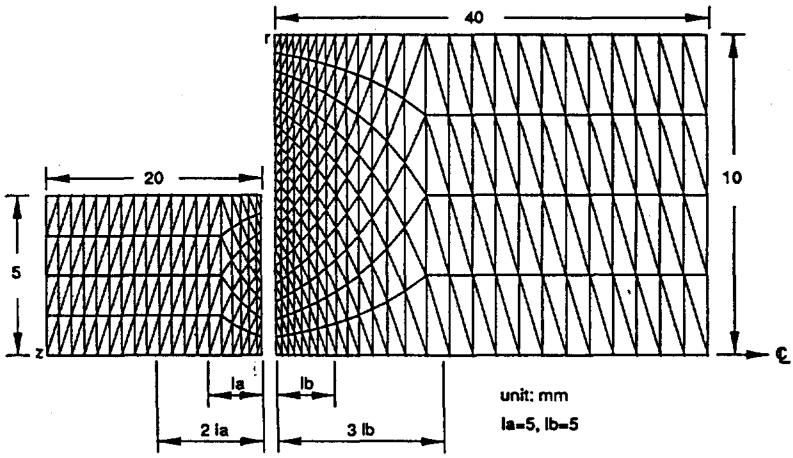
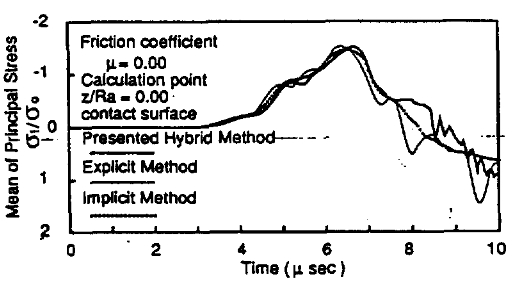
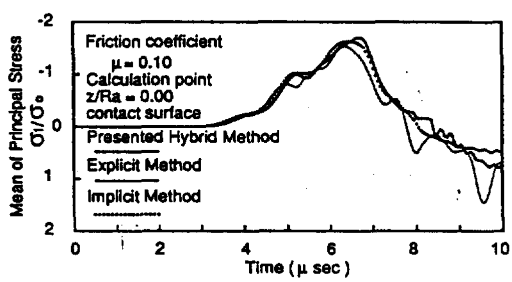


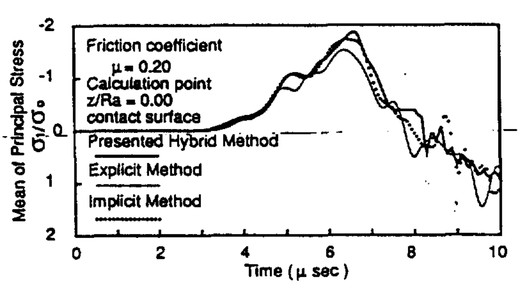
Fig. 2 Calculation model of the axisymmetric solid



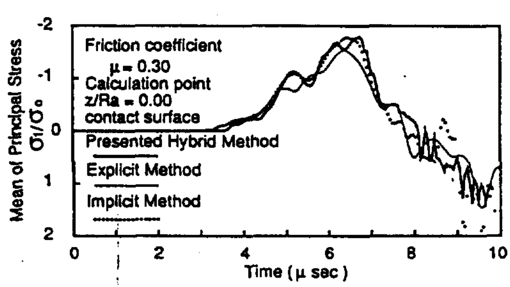
(a) μ=0.0



(b) μ=0.1



(c) μ=0.2

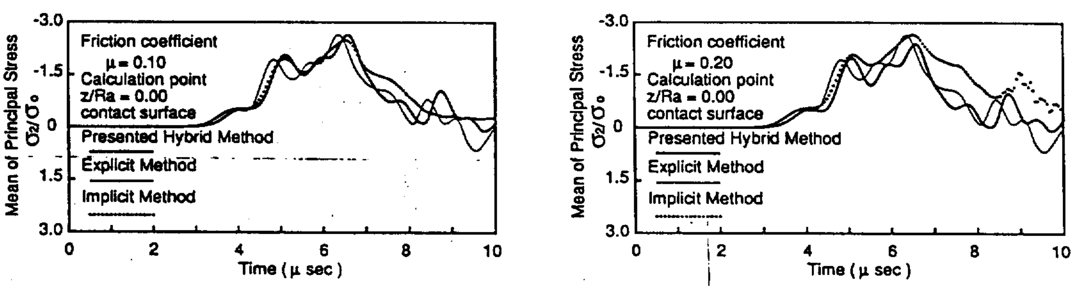


(d) μ=0.3

Fig.3 Time history of the mean value of principal stress on contact surface(la=10mm, lb=15mm)

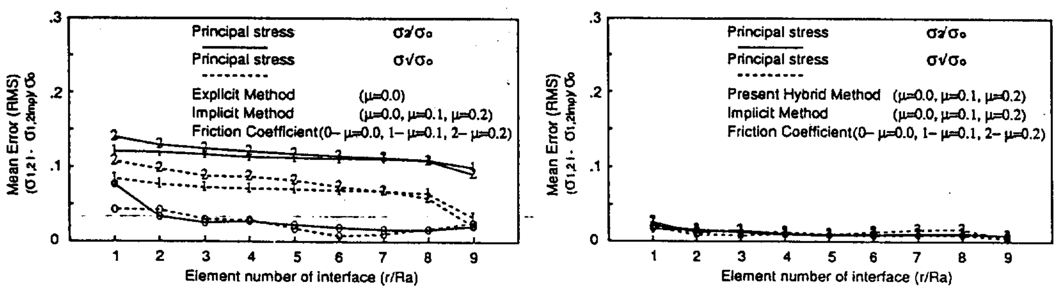
Numerical calculations are performed for the cases that Coulomb's frictional coefficients between A and B, μ, are 0.0, 0.1, 0.2 and 0.3.

In the description in Fig. 3, the normalized stress, σ_1/σ_0, ($\sigma_0=608MPa$), is shown as the case of mesh partition, $l_a=10mm$ and $l_b=15mm$ in Fig. 2. For various frictional coefficients, $\mu=0.0$, 0.1, 0.2 and 0.3, the time histories of the mean values of the normalized principal stress on the contact surface are given in Fig. 3. During the contact periods, the numerical results obtained by using the hybrid method show generally good agreement with those of the implicit method(Yagawa et al. 1983). After separation of the two contact bodies, however, because the calculation is performed with the explicit method to the total system, the results of the hybrid method disagree with those of the implicit method. In case $l_a=5mm$ and $l_b=5mm$ in Fig. 2, and if the number of elements of implicit partition are considered to be few, the accuracy of the hybrid method deteriorates, and this is shown in Fig. 4. Next the accuracy of the presented hybrid method is checked. The relative errors of results of the explicit method regardless of frictional force and the presented hybrid method are compared to those of the implicit method. The root mean square errors (RMS) of the principal stress on the contact surface which were obtained during the contact periods are shown



(a) $\mu=0.1$ (b) $\mu=0.2$

Fig.4 Time history of the mean value of principal stress on contact surface(la=5mm, lb=15mm)



(a) Conventional Explicit Method. (b) Presented Hybrid Method.

Fig.5 Mean error(RMS) of the principal stress on contact surface(la=10mm, lb=15mm)

Table 1 Mechanical and physical properties of materials

Body	Metal	Young's modulus(GPa)	Poisson's ratio	Density(g/cm³)
A	Steel	207	0.3	7.86
B	Cemented Carbide	626.8	0.207	14.9

Table 2 Comparison of properties of the conventional method and presented hybrid method

Methods	Frictional Coefficient	Calculation Time
Implicit	$\mu=0.0$ $\mu=0.1$ $\mu=0.2$	798sec 801sec 815sec
Presented Hybrid	$\mu=0.0$ $\mu=0.1$ $\mu=0.2$	 492sec 600sec
Conventional Explicit	$\mu=-0.0$	26sec

in Fig. 5. The RMS error is defined as

$$RMS=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\sigma_i}{\sigma_o}-\frac{\sigma_{impi}}{\sigma_o}\right)^2} \tag{29}$$

where σ_i is the principal stress by the explicit method or the presented hybrid method. The relative errors for the execution of the presented hybrid method are smaller than those of the explicit method without friction. In addition, the calculation time(KUBO. COMPUTER TITAN3000) of the presented hybrid method is checked. In case of $\mu=0.1$ and the same incremental time step, about 492*sec* are required for the presented hybrid method, but for the implicit method about 801*sec* are needed. The results are summarized in Table 2. Thus, the presented hybrid algorithm in which the frictional force and separation is considered has the advantage of faster calculation time as well as the numerical stability

5. CONCLUSIONS

The hybrid method algorithm has been formulated on the basis of a penalty procedure obtained from the implicit formulation by using the results calculated explicitly in the explicit mesh partitions. The presented method accounts well for the dynamic contact problem by using the incremental type theory. The solution of the contact surface depends on the number of meshes in implicit partition, it is necessary to place sufficient number of elements in the interface for the sake of numerical stability. As a result, the accuracy on the contact surface agrees well with the results of the implicit method. This hybrid method algorithm is easy to incorporate in a computer program, and there are the pragmatic merits of calculating the dynamic contact problem. Since this algorithm is calculated as the explicit method to the total system after separation between the two contact bodies, the solution depends on the explicit method. At that point it may become numerically unstable due to a sudden change.

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Hybrid Analysis of Dynamic Contact Problem by Impact

충격에 의한 동적접촉문제의 하이브리드해석

김 진 욱

(고베대학교)

본 논문에서는 접촉하는 두 물체의 동적접촉문제를 해석하기위한 음해법과 양해법을 합성한 새로운 하이브리드 해석방법을 제안하였다. 그리고 양해법의 결과를 기준으로 하여 그 계산정도를 비교하고 제안한 방법의 정도와 효율의 유효성을 조사하였다.

얻어진 결과를 요약하면 아래와 같다.

1) 마찰을 고려한 음해법과 양해법의 정식화를 통하여 접촉상태에서 분리가 일어날때 까지를 표현할 수 있는 새로운 알고리즘을 제시하였다.

2) 본 방법은 종래의 음해법에 비해 계산시간이 짧고 계산정도는 거의 비슷한 결과를 보였다.

3) 하이브리드법은 알고리즘의 변경이 간단하고, 동적접촉문제의 해석을 위한 실용적인 면에 큰 장점을 가지고 있다.