Numerical Analysis of Multi-Layer Multi-Coupled Microstrip Lines

수치해석에 의한 다층 다중 결합 마이크로스트립 선로 해석

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Abstract

Abstract-It is obtained the general expessions of the numerical method are applied for the TEM-mode analysis of multu-layer multi-coupled microstrip lines. In this paper, coupled microstrip are replaced by three-coupled microstrip lines in special applications. Three-layer versions of three-coupled microstrip lines are especially attactive because of the additional flexibilities offered by three-layer configuration. This structure can be used for obtaining large capacitance and preventing coupling among microstrip lines in filter and coupler. Sappihre is chosen for anisotropic substrates material. The permittivity parallel to the optical axis is higher than the permittivity in the plane perpendicular to this axis.

요 약

삼층 기판위의 삼중 결합 마이크로스트립 선로의 해석을 위한 수치해석의 일반식이 얻어졌다. 유한차분식을 얻기 위한 방법과 특이점 처리를 위한 최적의 방법이 구해졌다. 홀수 모드와 짝수 모드의 캐패시턴스와 특성 임피던스의 수치해석 결과를 보였으며 그 결과들은 이층의 두께와 삼층의 마이크로스트립 폭의 비에 따라 구해졌다. 수치해석으로 구한 짝수 모드 캐패시턴스는 다른 수치해석 결과와 비교되었다.

Ⅰ. Introduction

Many numerical analysis has been successfully applied to various microwave circuit probelem for more than ten years. The special advantages of the FDM techniques over other numerical methods are well illustrated by Duncan [1] and Sinnott [2] in their original paper on the method. Since then, several improvements have been made to this techniques by various authors order to enhance the accuracy of the solution and economize CPU time and memory space. Single and coupled microstrip lines on anisotropic substrates have been analyed by many authors [3]-[4]. The finite difference method was employed to obtain the

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characteristic impedance of microstripn on crystal sapphire substrates [1]. Unequal phase velocities can be found in the simple structure of coupled microstrip lines. Some versions of three layer three-coupled microstrip lines have been introduced to equalize the phase velocities.

In this paper, coupled microstrip lines are replaced by three layer three-coupled microstrip lines (Fig. 3) in special applications. Three-layer versions of three-coupled microstrip lines are especially attractive because of the additional flexibilities offered by three layer configuration. This structure can be used for obtaining large capacitance and preventing coupling among microstrip lines in filter and coupler. The finite difference method is used for analysis of three-layer three-coupled microstrip lines on anisotropic substrates.

The field is defined by a finite difference derivative of the calculated potential. Some methods to treat the singularities are extended in three-layer three-coupled microstrip line on anisotropic substrate. The numerical results are compured with those of the variational method.

Methods for deriving the appropriate finite difference equations are obtained and optimum methods for their solution considered in the treatment of singularities. Accurate numerical results are presented for the even and odd mode capacitances and the characteristic impedances for various ratio of the widths of microstrip line in the first layer to the thickness of first layer of three-layer three coupled microstrip lines on an isotropic substrates.

II. FORMULATION

Consider a configuration of three-layer three coupled microstrip lines on uniaxially anisotropic substrates (Fig.3) with permittivity dyadic given by the following,

$$\hat{\varepsilon}_i = \varepsilon_0 \begin{bmatrix} \varepsilon_{ix} & 0 & 0 \\ 0 & \varepsilon_{iy} & 0 \\ 0 & 0 & \varepsilon_{iz} \end{bmatrix} \quad i = 1,2,3,4 \tag{1}$$

A Solution to the potential problem can be obtained by solving the two dimensional Laplace's equation in the two anisotropic regions and the air region subject to the proper boundary conditions. The potential distributions, $\phi_i(x, y)$ $(i = 1, 2, 3, 4)$ in the two anisotropic regions satisfy Laplace's equation in two dimension,

$$\varepsilon_{ix} \frac{\partial^2 \phi}{\partial x^2} + \varepsilon_{iy} \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{2}$$

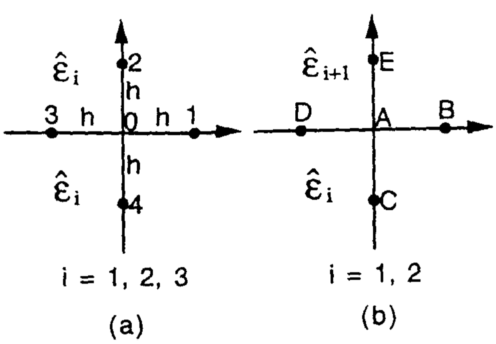

Fig 1. Ordinary and boundary point

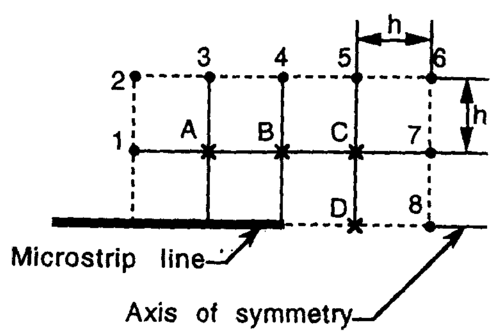

Fig 2. Near Singularity Point

It is assumed that the net point A is licated on the interface and points B, C, D, and, E are located distance h form point A in Fig.1. The difference equations are developed by expanding the potential at the boundary point A in Taylor's series as follows :

$$\phi_E = \phi_A + h\frac{\partial\phi}{\partial y} + \frac{1}{2!}h^2\frac{\partial^2\phi}{\partial y^2} + 0(h^3) \tag{3}$$

$$\phi_C = \phi_A - h\frac{\partial\phi}{\partial y} + \frac{1}{2!}h^2\frac{\partial^2\phi}{\partial y^2} + 0(h^3) \tag{4}$$

$$\phi_B = \phi_A + h\frac{\partial\phi}{\partial x} + \frac{1}{2!}h^2\frac{\partial^2\phi}{\partial x^2} + 0(h^3) \tag{5}$$

$$\phi_D = \phi_A - h\frac{\partial\phi}{\partial x} + \frac{1}{2!}h^2\frac{\partial^2\phi}{\partial x^2} + 0(h^3) \tag{6}$$

And considering boundary condition at point A on interface,

$$\varepsilon_{2y}\frac{\partial\phi}{\partial y})_{A^+} = \varepsilon_{1y}\frac{\partial\phi}{\partial y})_{A^-} \tag{7}$$

Summing equations (3), (4), (5) and (6), using (7), and applying Laplace's equation gives

$$\phi_A = \frac{\varepsilon_{i+1y}\phi_E + \varepsilon_i\phi_C}{\varepsilon_{ix} + \varepsilon_{i+1x} + \varepsilon_{iy} + \varepsilon_{i+1y}} + \frac{1}{2}\frac{(\varepsilon_{i+1x} + \varepsilon_{ix})(\phi_B + \phi_D)}{(\varepsilon_{ix} + \varepsilon_{i+1x} + \varepsilon_{iy} + \varepsilon_{i+1y})} \tag{8}$$

By the same process as used earlier the potential at ordinary point 0 can be expressed numerically in terms of the potentials of the neighboring nodes 1, 2, 3, and 4 as follows :

$$\phi_0 = \frac{1}{4}(\phi_1 + \phi_2 + \phi_3 + \phi_4) \tag{9}$$

A set of simultaneous equations is given above (8), (9). The original problem is reduced to solving simultaneous equations which are the approximate representations. Method of relaxation is used to solve a number of simultaneous equations. Successive over relaxation is suited to solve the problem of microstrip lines on anisotropic substrates. SOR may be defined as the relaxation cycle,

$$\phi_{0}^{j+1} = \phi_{0}^{j} - \frac{\alpha}{n}\sum_{k=1}^{n}\phi_{k(j)} \tag{10}$$

where

j measures the number of iteration cycles,
n is the number of neighboring points,
α is the accelerating factor.

The method is convergent if $0<\alpha<2$, and most rapidly convergent for α between 1 and 2. It is very important to determine the optimum accelerating factor α.

Near singularities, such as the corners of microstrip line, the usual finite difference formulas become inaccurate because of the unbounded nature of the field derivatives in these regions. There is a concentration of energy near the singularity and any serious inaccuracy on the overall energy computation. The error in the vicinity of the singularity can be reduced as the mesh width decreases, but the convergence of the iteration is considerably slowed. A known eigenfunction can be used to obtain finite difference equation for nodes near the singularity in terms of the potentials of the more remote nodal points. However, application of this method is limited to those for which an eigenfunction expansion is known. An efficient method is expanded to treat singularities in near of edge of microstrip line.

Consider for example Fig. 2. The region enclosed by nodes 1 to 8 is taken as the vicinity of the singularities and finite difference equations are derived for nodes A to D in terms of nodes 1 to 8. A large number of cells are in the region. Unity potential is set to one of 1 to 8 with the remainder to zero. Interpolation is obtain potentials at intermediate points on the boundary. If point 4 is at unity potential, boundary points between it ad its zero potential neighboring points 3 and 5 is set to nonzero values found by interpolating through a triangle function while all other intermediate boundary nodes is set to zero.

Ⅲ. DETERMINATION OF MICROSTRIP LINE PARAMETERS

The way is open to compute the constants of microstrip line if a nodal potential distribution is known. To obtain the capacitance it is prerequisite to determine the charges on the microstrip line. These may be found by Gauss' theorem requiring the integration of the normal component of electric displacement over a surface enclosing the microstrip line. Forming this surface by lines joining nodal points drawn parallel to the coordinate directions :

$$D_n = \varepsilon_n E_n = -\varepsilon_n \cdot \frac{\partial\phi}{\partial n} \quad (11)$$

where

D_n is the normal component of displacement,
E_n is the normal compoent of electric intensity,
n is normal coordinate.

By the same Taylor expansion as used earlier the derivatiove of the potential at arbitrary node i may be expressed numerically in terms of the known potentials of the neighboring two nodes on each side of it, with an error in the order of h^2, as

$$\frac{\partial\phi_i}{\partial n} = \frac{\phi_{i+1} - \phi_{i-1}}{2h} \quad (12)$$

It is now easy to apply Gauss' theorem. Thus, if the surface containing the microstrip line consists of straight line segments each containing p ondes,

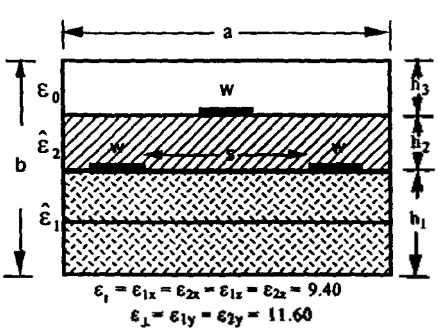

Fig 3. Three-layer three-coupled microstrip lines on anisotropic substrate

the capacitance per unit length normal to the cross-section is given by

$$C = \sum_{s} \sum_{p=1}^{P} \left(\varepsilon_n \frac{\partial\phi}{\partial n} \right) \quad (13)$$

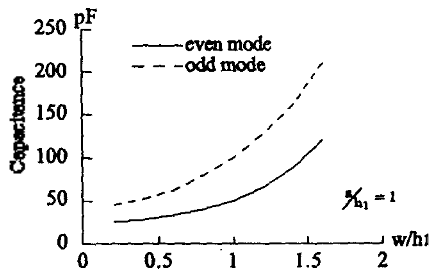

Fig 4. Capacitances vs. w/h_1 for even and odd mode on microstrip lines.

where the second summation is used to indicate that the first and last terms in the summation are halved. This is seen to be equivalent to integration by the trapezoidal rule, known to involve a dominant error in the order h^2, and is therefore consistent with the whole finite difference process. S is surface of microstrip and ε_n is ε_{2y} aloug the top surface and ε_{1y} along botton surface of the microstrip. We have taken an average value of $(\varepsilon_{ix} + \varepsilon_{i+1x})$ normal to the two edges.

Given the capacitance, the characteristic impedance follows without difficulty, although two cases need to be distinguished. If the medium is inhomogenous, two steps are necessary : the capacitance is determined twice, once with all dielectrics removed, and then with them present. Since inductance per unit length is not altered by the introduction of the dielectric (assuming, of course, that it is nonmagnetic) if follows that

$$Z_0 = \frac{1}{v_0 \sqrt{CC_0}} \quad (14)$$

where

C_0 is capacitance without dielectrics,
C is the capacitance with dielectrics present,
v_0 is the velocity of electromagnetic waves.

Ⅳ. NUMERICAL RESULTS

The geometrical configuration is a = 2 b = 4cm, $h_1 = h_2 = 1.5$mm, and $s/h_1 = 1$ in Fig.3. The recent accurate determination of sapphire relative pe rmittivities was reported by the infrared results The results indicate that there is less than 0.1 percent bulk material dispersion in sapphire below 300 GHz, so the choice of $\varepsilon_{\perp} = 9.40$ and $\varepsilon_{\parallel} = 11.60$ for the present work is well justified. Implicit in the finite difference method solution is the assumption that the propagation wave on microstrip line is TEM mode. This assumptions is only satisfactory up to GHz and dispersion effect should be taken into account over 10 GHz frequency range. The boundaries are not considered sufficiently far from the microstrip line that the calculated solutions are virtually independent of boundary position. The relative effects of side and top wall on the calculated capacitances depend on the width of microstrip line and the thickness of anisotropic substrate. For wide microstrip line the relative position of the side wall is important, while for narrow microstrip line the location of the top wall has greater effect. The total chosen nodes and error bound are 250 × 150 and 10^{-5} respectively, and 1.7 is chosen for accelerating factor to obtain numerical efficiency and rapid convergence.

Fig.4 shows that the even and odd mode capacitances of two-layer three-coupled microstrip lines are proportional to w/h_1. There was 40 iterations until it is found to change by less than 0.01 percent. The error in estimating the capacitance increased due to slow convergence for $w/h_1 < 0.2$. At the other end of the scale, although convergence was rapid for the line $w/h_1 > 5$, the results were very sensitive to box size. The capacitances are calculated over the range $0.1 \leq w/h_1 \leq 2.2$ The characteristic impedance associated with a given microstrip line is derived from the capacitance using quais-static relationship. Fig.5 shows that the characteristic impedances are shown inversely proportional to w/h_1. The even mode capacitances of the finite difference method are compared with those of the variational method in Table 1. Thable 1 shows that the results of the finite difference method are between the upper and lower bounds of the variational method.

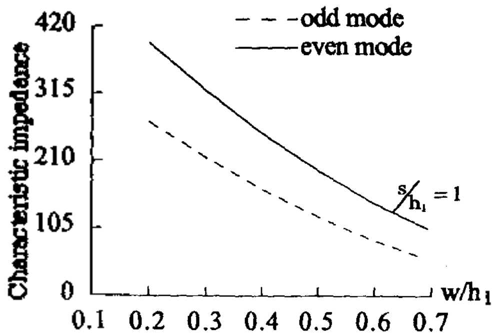

Fig 5. Characteristic impedances vs. w/h_1.

Table 1. Comparisons of the even mode capacitances of FDM with those of the variational method (pF).

W/h_1	upper bound	FDM	lower bound
0.2	23	30	40
0.4	27	38	49
0.6	31	45	59
0.8	39	55	70
1.0	49	65	90
1.2	60	79	107
1.4	84	99	120
1.6	116	150	170

Ⅴ. CONCLUSIONS

In this paper, the analysis of three-layer two-layer three-coupled microstrip lines on uniaxially anisotropic sapphire substrates is presented in this paper. The finite difference method is applied to obtain the capacitance and the characteristic impedance. The finite difference equations in isotropic and anisotroipic regions are derived to obtain the capacitances in this structure for the first time. The capacitances and the characteristic impedances are shown with respect to w/h_1. These microstrip line parameters are obtained in the range $0.1 \le w/h_1 \le 2.2$. Numerical results compared with those of variational method. There is 7 percents improvement over variational method in the calculation of even mode capacitances.

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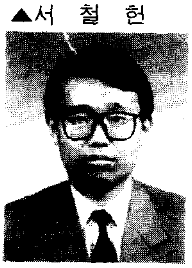

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