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Development of a Statistical Methodology for Nuclear Fuel Rod Internal Pressure Calculation

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통계적인 핵연료봉 내압 설계방법론 개발

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Abstract

A statistical methodology is developed for calculating the nuclear fuel rod internal pressure of Korean PWR fuel in order to reduce over-conservatism of the current KAERI deterministic methodology. The developed statistical methodology employs the response surface method and Monte Carlo calculation. The simple regression equation for the rod internal pressure is derived by taking into account the various fuel fabrication-related and fuel performance model-related parameters. The validity of the regression equation is examined by the F-test, R^2-method and Cp-test. The internal pressure predicted by the regression equation is in good agreement with that calculated by the computer code using the KAERI deterministic methodology. The distribution of the internal pressure from the Monte Carlo calculation is found to be normal. Comparison of the 95/95 rod internal pressure predicted by the developed statistical methodology with the maximum rod internal pressure by the deterministic methodology shows that the developed statistical methodology reduces significantly over-conservatism of the deterministic methodology.

요 약

가압경수로용 핵연료봉 내압을 계산하는 데 있어 현재의 결정론적 방법에 의한 과다한 보수성을 줄이기 위하여 통계적 계산 방법론을 개발하였다. 개발된 통계적 방법론은 반응표면 분석 방법과 Monte Carlo 계산 방법을 이용하였다. 반응표면 분석 방법을 이용하여 핵연료 제조관련 변수와 성능관련 변수를 고려하여 회귀식을 유도하였으며, 이 식의 검증은 F−test, R^2 및 C_p-test 방법을 사용하여 수행하였다. 회귀식으로 부터 예측된 봉내압은 결정론적 방법을 사용하여 계산된 값과 잘 일치하였다. Monte Carlo 계산으로 구한 핵연료봉 내압의 분포는 거의 정상분포로 나타났다. 본 연구에서 개발된 통계적 방법론으로 구한 95 / 95 봉내압과 현재 사용되고 있는 결정론적 방법론의 봉내압과 비교한 결과 결정론적 방법론의 과다한 보수성을 크게 줄일 수 있었다.

1. Introduction

A deterministic methodology has been employed for the fuel performance analysis of KOFA (KOrean Fuel Assembly) according to the current KAERI's (Korea Atomic Energy Research Institute) fuel rod design methodology. It is well known that the deterministic methodology would give too much conservatism in the fuel design calculations since the upper or the lower bound of the fuel fabrication and fuel performance model parameters are combined as a single input data set. In addition, it is very difficult to estimate the magnitude of over-conservatism of this methodology. However, fuel management and demand of fuel design for extended cycle length and high burnup make it necessary to reduce the over-conservatism contained in the deterministic methodology.

Various statistical methodologies can be proposed to examine the impact of each input parameter on the conservatism of the fuel design calculations. Kim et al. [1] proposed a statistical procedure for the rod internal pressure with a single parameter variation. The distributions of fuel performance parameters were assumed to be the same as input variable distributions in this procedure. However, it was also difficult to quantify the conservatism of this procedure.

In this work, a statistical methodology for calculating the rod internal pressure is developed using the response surface method and Monte Carlo calculation [2]. With this methodology, the conservatism included in the calculation of the internal pressure can be easily quantified. The impact of each fuel fabrication and each model related parameter on the internal pressure is also discussed. This work is intended for high burnup fuel design calculation with the increase in design margin.

2. Development of a Statistical Methodology for Rod Internal Pressure Calculation

The statistical procedure for calculating the rod internal pressure using the response surface method and Monte Carlo calculation is summarized in Fig.1. Each procedure given in Fig.1 is described in the following subsections.

2.1. Construction of Response Surface

2.1.1. Latin Hypercube Sampling

Latin hypercube sampling (LHS) has been developed for sampling input data sets by Mckey et al. [3]. The range of each input variable is divided into N non-overlapping intervals having equal probability.

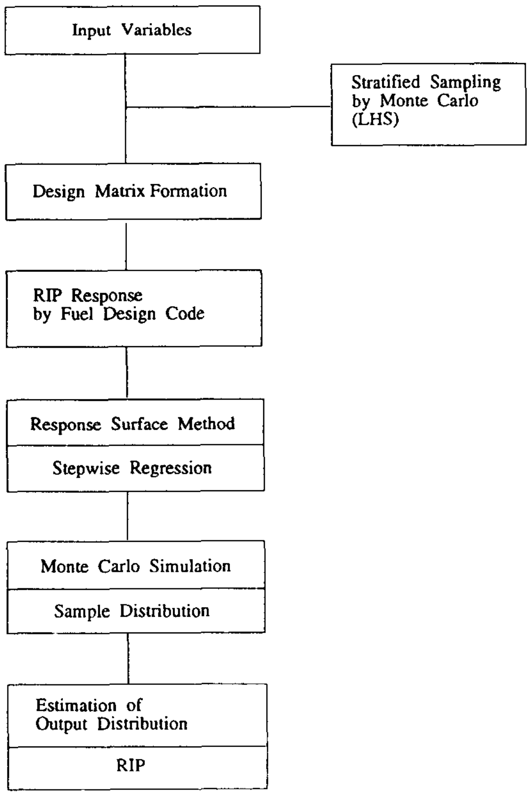

Fig. 1. Flowchart for the Procedure of Statistical Calculation of Rod internal Pressure

One value from each input variable is selected at random. The N values thus obtained for x1 are paired in a random manner with the N values of x2 to give the N pairs. Then these N pairs are combined in a random manner with the N values of x3 to form N triples, and so on, until N K-tuplets are selected. Here K is the number of variables. It should be noted that the Latin hypercube sampling method is better than other sampling methods in a view of estimators of the mean, the variance, and the population distribution function of the output of a computer code if the input variables have monotonic relation with dependent variables [3]. In addition, it is reported that sample size is sufficient if N > 2K.

2.1.2. Derivation of Regression Equation

In order to derive a regression equation for the rod internal pressure within the range of all the input variables, the stepwise regression technique [4] is employed. This technique is a modified forward and backward regression technique.

For the purpose of estimating goodness of the derived regression equation, F-test, R^2-method and Cp-test [5] can be utilized. F-test is first done by selecting the most significant independent variable, which has the highest partial correlation coefficient. After that, the selected variable is checked whether it is significant or not with an aid of partial F-test under the given significant level (α). Typically, significance level of $\alpha = 0.05$ is chosen conservatively. If the significant independent variable is found to be not significant, $Y = \bar{Y}$ is considered to be the best model. Otherwise, the partial correlation coefficients of all variables not included in the regression equation are calculated and the next variable with the highest partial correlation is selected. At this step, partial F-tests for the selected two variables are also performed. This procedure will be repeated until partial F-tests for all possible regression variables do not satisfy the given significance level.

As a criterion of the goodness of fit for the regression equation, R^2 (determination coefficient)-method may also be used, and the definition of R^2 is as follows:

$$R^2 = \frac{\sum(\hat{Y} - \bar{Y})^2}{\sum(Y_i - \bar{Y})^2} = \frac{\text{explained variation}}{\text{total variation}} \tag{1}$$

where Y_i = experimental value

$\bar{Y}$ = mean value of the experimental values

$\hat{Y}$ = predicted value from the regression equation

The regresion equation can also be checked by Cp-test. According to this test result, a proposed regression equation is considered to be adequate when the Cp value is close to the number of selected variables.

2.2. Monte Carlo Calculation

Monte Carlo calculation with the derived regression equation has been performed to generate a distribution of rod internal pressure under the given range of all input variables. From the distribution of the internal pressure, the value of the 95/95 rod internal pressure can be obtained. The 95/95 internal pressure means the rod internal pressure at 95% probability with 95% confidence level.

3. Preparation of Input Data and Calculation of Rod Internal Pressure

As an example, key input data for calculating the internal pressure are listed in Table 1. Seven variables from the fabrication-related parameters and four variables from the model-related parameters were selected. The distribution of clad inner and outer diameters are shown in Figs. 2 and 3, as examples. As shown in these figures, the fabrication-related parameters are safely assumed to have normal distribution.

Table 1. Input Data for Calculating the Rod Internal Pressure by Fuel Performance Analysis Code

Variables	Value		
1. Porosity(%)	3.74	5.10	6.50
2. Pellet O.D(mm)	8.04	8.05	8.06
3. Clad O.D(mm)	9.45	9.50	9.55
4. Clad I.D(mm)	8.18	8.22	8.26
5. Dishing Volume(mm^3)	8.00	11.00	14.00
6. Plenum Volume(mm^3)	5.70	6.40	7.00
7. Pressure(bar)	21.50	22.50	23.50
8. Fission Gas Release Model Constant	8.00		38.00
9. Radial Relocation Model Constant	0.48		0.831
10. Swelling Model Constant	0.34		0.46
Densification Model Const.	3.48		4.10
11. Creep Model Constant	0.77E−20		1.1E−20

The comparison of measured and calculated fission gas release [6] is shown in Fig.4. Since the distribution of the fission gas release model constant is unlikely known, we assume that this model constant has the uniform distribution. It is found that the uniform distribution will give more conservative

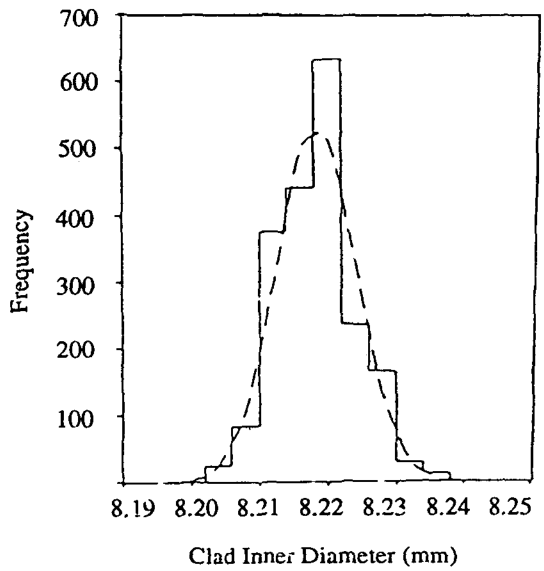

Fig. 2. Plot of Distribution of the As-Fabricated Clad Inner Diameter

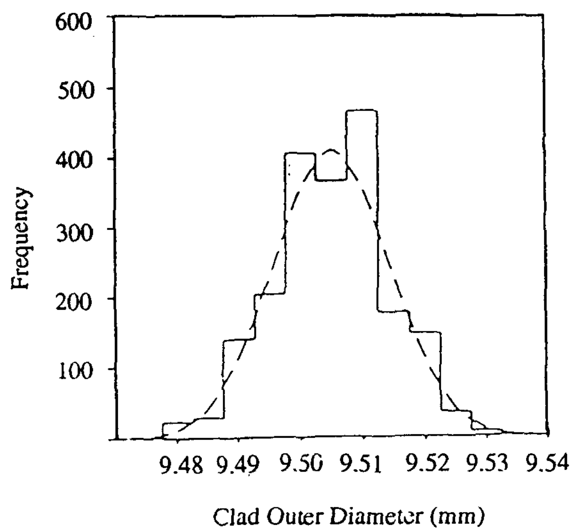

Fig. 3. Plot of Distribution of the As-Fabricated Clad Outer Diameter

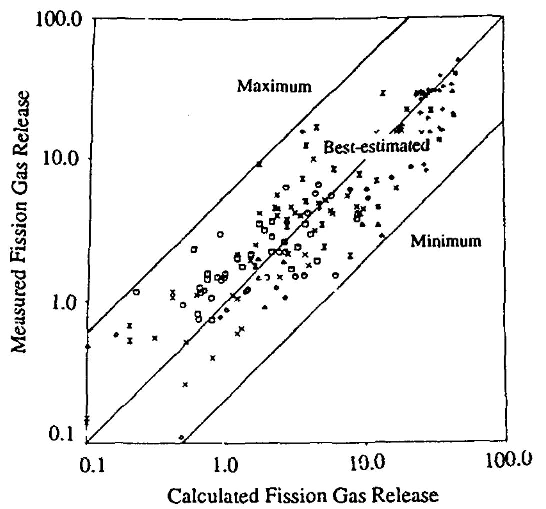

Fig. 4. Comparison of the Measured and Calculated Fission Gas Release

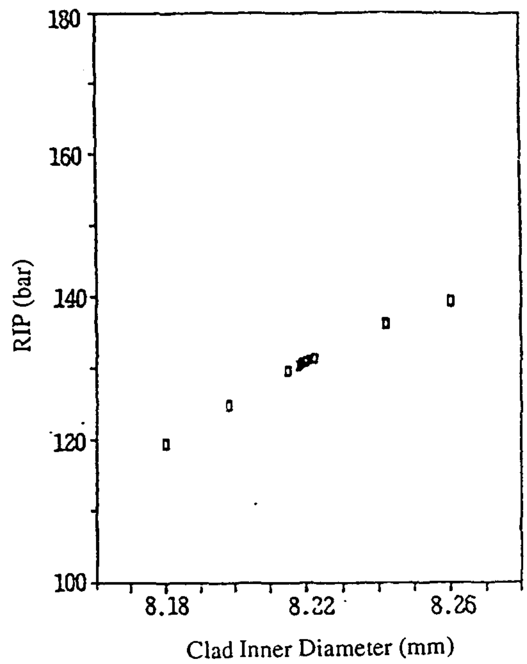

Fig. 5. Variation of the Rod Internal Pressure With the Clad Inner Diameter

results than normal distribution because the variation of the rod internal pressure with the minimum or maximum model constant of the uniform distribution becomes large compared with the internal pressure of the normal distribution. Similarly, other performance model-related parameters can be also safely assumed to have uniform distribution.

With an aid of Latin hypercube sampling, one hundred samples from each input parameter were selected. This sample size is thought to be sufficient since N is much greater than 2K, as described in the previous subsection 2.1.1. The applicability of the Latin hypercube method to the calculation of the rod internal pressure may be checked by plotting the variation of rod internal pressure as a function of each input parameter, as shown in Figs. 5, 6 and 7. Since the rod internal pressure varies monotonically with each parameter, such as clad inner diameter, pellet dishing volume for the fabrication-related parameters and fission gas release model constant, the Latin hypercube sampling method can be safely used.

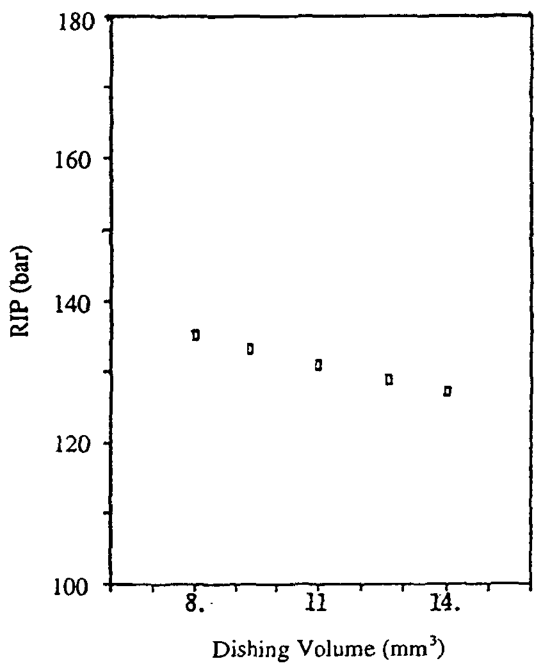

Fig. 6. Variation of the Rod Internal Pressure With the Pellet Dishing Volume

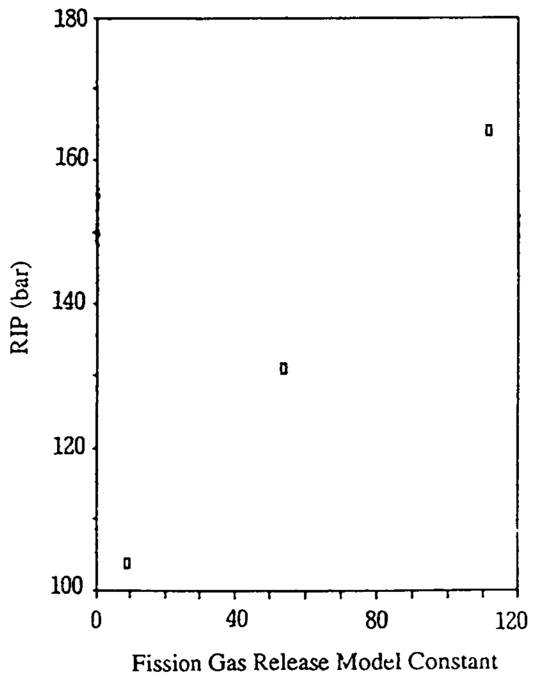

Fig. 7. Variation of the Rod Internal Pressure With the Fission Gas Release

4. Results and Discussion

The selected input variables used for the calculation of rod internal pressure are to be standardized to exclude the effect of the variable range on the derivation of regression equation. The regression equation approximating the response surface was derived as follows:

$$Y = 140.16871 - 2.80626\ Z1 + 3.76117\ Z4 - 1.43826\ Z5 - 1.67950\ Z6 + 11.02865\ Z8 - 1.49943\ Z9 - 3.26779\ Z10 - 1.3510\ Z11 \quad (2)$$

where Zi = (Xi − Xi,o)/ σi = standardized value
Xi = real value
Xi,o = nominal value
σi = standard deviation of the variable Zi
Z1 = pellet porosity
Z4 = clad inner diameter
Z5 = pellet dishing volume
Z6 = fuel rod plenum volume
Z8 = fission gas release model constant
Z9 = radial relocation model constant
Z10 = swelling / densification model constant
Z11 = cladding creep model constant

As listed in Table 2, the significance levels of all the variables in the above regression equation are less than 0.01%, which is sufficiently lower than the significance limit of $\alpha = 5\%$. It is found that the value of R^2 is 95.4% and the value of Cp is 9 for the derived regression equation. These indicate that the regression equation has been adequately derived. Fig.8 also shows that the internal pressure predicted by the

Table 2. Summary of Statistical Test Results For the Regression Equation

Variable Entered	Number In	Partial R**2	Model R**2	C(P)	Prob ⟩ F
X8 (FGR)	1	0.7020	0.7020	489.3481	0.0001
X4 (CID)	2	0.0811	0.7831	332.0886	0.0001
X10 (SWL)	3	0.0657	0.8488	205.0643	0.0001
X1 (POR)	4	0.0520	0.9008	104.9195	0.0001
X6 (PLE)	5	0.0156	0.9164	76.2793	0.0001
X9 (REL)	6	0.0150	0.9313	48.8910	0.0001
X5 (DSV)	7	0.0119	0.9433	27.4461	0.0001
X11 (CRP)	8	0.0104	0.9537	9.0000	0.0001

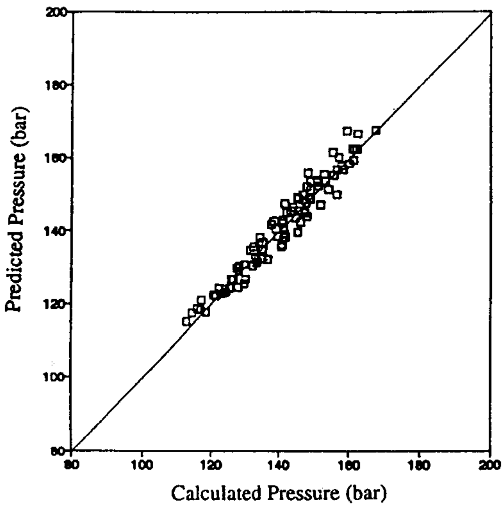

Fig. 8. Comparision of the Internal Pressure Predicted by the Regression Equation With That Calculated by the Computer Code

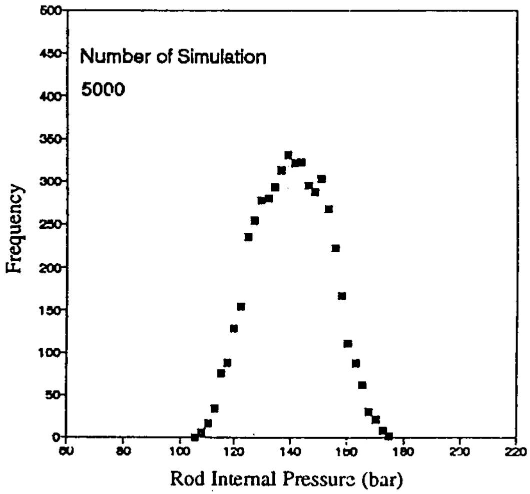

Fig. 9. The Statistical Distribution of the Rod Internal Pressure Predicted by Monte Carlo Simulation

eq. (2) is relatively well fitted into the value calculated by the KAERI performance computer code [7].

From the derived regression equation, the fission gas release model constant is found to be the most significant variable in calculating the rod internal pressure and the clad inner diameter to be the second most significant variable. Therefore, one can say that pellet microstructure optimization against fission gas release is the most important factor in reducing the rod internal pressure.

The maximum, minimum and average rod internal pressure calculated by the computer code using the deterministic methodology are 211, 100 and 140 bar, respectively, whereas the corresponding values predicted by the regression equation are 199, 81 and 140 bar. The derived regression equation predicts less conservatively the maximum internal pressure to be 12 bar smaller than the computer code, while it predicts conservatively the minimum pressure to be 19 bar smaller. These differences are caused by the error in deriving the regression equation. As discussed above, the error is estimated to be 4.6% from the value of R^2=95.4%. However, this error would affect negligibly the statistical distribution of the rod internal pressure predicted by Monte Carlo simulation. The reason for that can be explained from Fig.9. This figure shows the distribution of rod internal pressure predicted by the Monte Carlo simulation with the number of 5000. From this figure, it can be seen that the maximum, minimum and average internal pressures are 177, 103 and 140 bar, respectively. It is found that the probability of the internal pressure higher than 177 bar and lower than 103 bar is estimated to be 0.02% from the distribution shape. Therefore, one can see that the maximum and minimum internal pressures are safely assumed to be 177 and 103 bar. Considering that the maximum and minimum pressures predicted by the regression equation are greater than 177 bar and smaller than 103 bar, respectively, the impact of the error contained in the regression equation on the statistical distribution of the internal pressure seems to be negligible.

From Fig.9, the 95/95 rod internal pressure is 170 bar. The maximum rod internal pressure calculated by the computer code using the deterministic methodology is 211 bar. Comparison of the statistical methodology with the deterministic one indicates that the amount of decrease in the internal pressure is 41 bar due to the introduction of the statistical methodology.

5. Conclusions

A statistical methodology for calculating the rod internal pressure is developed in order to reduce over-conservatism of the deterministic methodology currently employed in the design of Korean PWR fuel. The conclusions are summarized as follows:

(1) The developed statistical methodology uses the response surface method and Monte Carlo calculation. The validity of the derived regression equation for the internal pressure is examined by the F-test, R^2-method and Cp-test, and the results show the regression equation is derived properly. The regression equation for the internal pressure indicates that the fission gas release model constant is the most significant and the clad inner diameter is the second most significant parameters.

(2) The predicted internal pressure by the regression equation is in good agreement with the calculated one by the computer code using the deterministic methodology. It is found that the distribution of internal pressure from the Monte Carlo calculation appears to be normal. Comparison of the deterministically obtained maximum internal pressure with the 95/95 internal pressure indicates that the amount of decrease in the internal pressure is 41 bar.

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