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Optimal Design of the Nuclear Steam Generator Digital Water Level Control System

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증기발생기 디지탈 수위조절 시스템의 최적설계

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Abstract

A digital control system for the steam generator water level control is developed using the optimal control technique. To describe the more realistic situation, a feedwater valve actuator of the first order lag is included in the overall control system. The optimal gains are obtained by the LQ method which imposes the constraints on the feedwater valve motion as well as on the deviation between the input demand signal and the output feedwater. Developed also is a Kalman observer on account of the flow measurement uncertainty at low power. And a digital controller on the feedback loop is designed which makes the system maintain the same stability margins for all power ranges. The simulation results show that the optimal digital system has good control characteristics despite the adverse dynamics of the steam generator at low power.

요 약

증기발생기의 수위조절과 관련하여 최적제어이론을 이용한 디지탈 제어시스템을 설계하였다. 우선 급수변을 일차 지연함수로 취급하여 전체 시스템에 포함시킴으로써 보다 실제에 가까운 시스템이 되게 하였다. LQ 방법을 이용하여 급수변의 작동 및 요구신호량과 급수출력과의 차이를 최소화시킬 수 있는 최적이득상수를 결정하였으며, 아울러 저출력에서의 급수유량 측정이 불확실함을 고려하여 급수신호에 대한 칼만 관측기를 설계하였다. 그리고 전출력 구간에서 일정한 안정여유도를 유지시킬 수 있는 가변상수 디지탈 제어기를 설계하였다. 이러한 제어시스템은 보다 현실적인 상황을 반영하고 있으며 저출력에서 급수와 반대현상을 보이는 증기발생기의 동특성에도 불구하고 만족할만한 제어특성을 보이고 있다.

1. Introduction

In the various fields of industry, new techniques and theories employed in control applications have evolved significantly over the last decades, one reason being the wide spread automation of control operations by means of digital computers[1]. In the nuclear field also, a great effort has been made to improve the control system by use of digital technologies, and a long term schedule for the control system upgrade has been prepared by EPRI with an aim to implement in the next generation nuclear plants[2]. However, because of the conservatism of the nuclear field, it is often the case that conventional controllers have merely been replaced or updated by a digital based control, which ignores the advantages and improvements that can be obtained in overall plant performance.

In this paper, a digital steam generator water level control system has been designed by the application of optimal control technique. Of particular concern in a steam generator is the water level control in the low power ranges. Because of an adverse thermal-hydraulic characteristics of a steam generator, together with the uncertainty of feedwater flow measurement, the steam generator water level control at low power causes many problems in plant operation. Westinghouse and Combustion Engineering developed the digital control system such as ADFCS[3] and LPFCS[4] which can control the level automatically for all power ranges. But their control schemes are almost the same as those of classical analog systems, with the exception of variable gains which are programmed off-line.

This paper is based on several previous studies. In Refs. [5] and [6], the steam generator transfer functions between various inputs and the level were identified in terms of power level, and a continuous control system was studied. Then a digital control system was developed in Ref. [7]. The digital system of Ref. [7] is of two element control without the feedwater feedback loop. The feedwater feedback loop was dropped because of the uncertainty of flow rate measurement at low power. Further, all previous studies assumed the ideal feedwater valve operation which means there is no delay in the feedwater response to the input demand signal.

The outlines of this study are as follow. First, the delay of a valve actuator is taken into account. Since the rapid valve movements are not desirable, the demand signal to the feedwater station is considered as a constraint and is included in the linear quadratic cost function. Also taking the flow measurement uncertainty into account, a Kalman observer is applied to the feedwater station. Finally a power adaptive digital controller which makes the system have the constant stability margins for all power ranges is designed.

2. Digital Steam Generator Level Control System

The block diagram of Figure 1 shows the overall digital level control system of a steam generator. The level control is accomplished by the three elements of the steam flow rate change($\triangle W_s$), feedwater flow rate change($\triangle W_f$), and level change($\triangle L$). The regulated feedwater flow rate change is an input to the steam generator, and the unregulated inputs of the steam flow rate change, primary coolant temperature change($\triangle T_p$), and feedwater temperature change($\triangle T_f$) act on the steam generator as external disturbance injections.

The relations between each input and the level change were identified in Ref. [6], and were expressed in terms of percent power. In short, the non-linear thermal hydraulic model of a 857 Mw Westinghouse-F steam generator was linearized for several power regions and the transfer functions of the steam generator were found based on the linearized model. The same approach was used in Ref. [8]. As were in Ref. [7], the same notations are used hereinafter. $H_1(s)$ is the transfer function between the changes of feedwater flow rate and water level. Similarly $H_2(s)$, $H_3(s)$, and $H_4(s)$ are transfer functions for the changes of the steam flow rate, primary coolant temperature and feedwater temperature, respectively. The interrelations between those transfer functions were discussed in Ref. [6] and it was found that the coupling between them is negligible.

The dotted part in Figure 1 reperesents the feedwater station and is comprised of holders, samplers, a valve actuator of the first order lag and a feedback loop. With the input $a^*(s)$ and output $b^*(s)$, the transfer function of the feedwater station is

$$\frac{b^*(s)}{a^*(s)} = F^*(s) \tag{1}$$

where the starred function denotes the sampling process and can be directly transformed to z-domain.

Then the level and feedwater variations of the system are

$$\Delta L(z)\left(1 + G_c(z)F(z)\overline{G_hH_1}(z)\right)$$
$$= \Delta W_s(z)\left(F(z)\overline{G_hH_1}(z) + \overline{G_hH_2}(z)\right)$$
$$+ \Delta T_p(z)\left(\overline{G_hH_3}(z)\right) + \Delta T_f(z)\left(\overline{G_hH_3}(z)\right) \tag{2}$$

$$\Delta W_f(z)\left(1 + G_c(z)F(z)\overline{G_hH_1}(z)\right)$$
$$= \Delta W_s(z)\left(F(z) - F(z)G_c(z)\overline{G_hH_2}(z)\right)$$
$$- \Delta T_p(z)\left(F(z)G_c(z)\overline{G_hH_3}(z)\right)$$
$$- \Delta T_f(z)\left(F(z)G_c(z)\overline{G_hH_4}(z)\right) \tag{3}$$

where G_h is the holder, G_c is the controller and $\overline{G_hH_i}(z)$, i=1, 2, 3, 4 are the ZOH(Zero Order Holder) transformed discrete functions.

Therefore, the design of the digital water level control system is divided into two steps, that is, the determination of the F(z) related to the feedwater station and the design of the digital controller $G_c(z)$ which can maintain the level within the permissible range with sufficient stability.

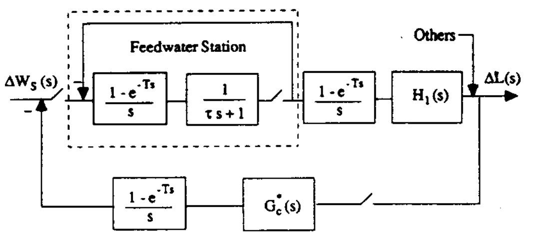

Fig. 1. Overall Digital Control Schemes

3. Feedwater Station Design

Figure 2 is the state variable block diagram of the feedwater station which is marked in Figure 1. The valve actuator plant is represented by A, B, and C. Since the system is SISO, with the first order actuator, they are scalars. The system input demand signal is r, and the plant input and output are u and y, respectively. The system described in the figure is a servo system of which the output should follow the input demand signal. For this reason an integrator is introduced to eliminate the dc error. K_1 is the feedforward gain and K_2 is the feedback gain.

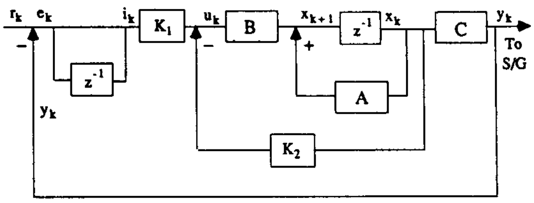

Fig. 2. Block Diagram of Feedwater Station

The state equations for the servo system as described in the figure are

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ u_k &= K_1 i_k - K_2 x_k \\ y_k &= Cx_k \\ i_k &= r_k - Cx_k + i_{k-1} \end{aligned} \tag{4}$$

where i_k is the integrator output signal and x_k is the state variable of the valve actuator plant.

By eliminating i_k the above equation can be put in the matrix equation of

$$\begin{pmatrix} x \\ u \end{pmatrix}_{k+1} = \begin{pmatrix} A & B \\ K_2 - (K_2 + K_1C)A & 1 - (K_2 + K_1C)B \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix}_k + \begin{pmatrix} 0 \\ K_1 \end{pmatrix} r_{k+1}$$

$$y_k = Cx_k \tag{5}$$

In the steady state, $x(\infty)=\bar{x}$, $u(\infty)=\bar{u}$, $r(\infty)=\bar{r}$, and the servo system can be written in the error dynamic system of $\tilde{x}=x-\bar{x}$, $\tilde{u}-\bar{u}$, and $\tilde{y}=y-\bar{y}$.

$$\begin{pmatrix}\tilde{x}\\ \tilde{u}\end{pmatrix}_{k+1}=\begin{pmatrix}A & B\\ 0 & 0\end{pmatrix}\begin{pmatrix}\tilde{x}\\ \tilde{u}\end{pmatrix}_k+\begin{pmatrix}0\\ 1\end{pmatrix}p_k,$$

$$\tilde{y}_k=(C \quad 0)\begin{pmatrix}\tilde{x}\\ \tilde{u}\end{pmatrix}_k \tag{6}$$

or,

$$\xi_{k+1}=\tilde{A}\xi_k+\tilde{B}p_k,$$
$$p_k=-\tilde{K}\xi_k \tag{7}$$

where, $\xi_k=\begin{pmatrix}\tilde{x}\\ \tilde{u}\end{pmatrix}_k$, $\tilde{A}=\begin{pmatrix}A & B\\ 0 & 0\end{pmatrix}$, $\tilde{B}=\begin{pmatrix}0\\ 1\end{pmatrix}$, $\tilde{C}=(C \quad 0)$,

and

$$\tilde{K}=-(K_2-(K_2+K_1C)A \quad 1-(K_2+K_1C)B)$$

Hence the servo system of Eq.(5) is converted into the ordinary regulator system of Eq.(7). To determine the gain matrix $\tilde{K}$ the linear quadratic regulator(LQR) method is used. In LQR, Eq.(7) is to be solved under the constraint of which the linear quadratic cost function of Eq.(8) be minimum.

$$J=\frac{1}{2}\sum_{k=0}^{\infty}\left[\xi_k^T Q\xi_k+p_k^T Rp_k\right] \tag{8}$$

In the above equation, the first term stands for the system error and the second term for the amount of the input energy to the plant. The original system error, e_k, is $e_k=r_k-y_k$, and since $\tilde{e}_k=e_k-\bar{e}=-\tilde{C}\xi_k$, the error of the regulator deviation system has the relation of $\tilde{e}_k^T\tilde{e}_k=\xi_k\tilde{C}^T\tilde{C}$, from which the matrix $\mathbf{Q}$ is a positive semidefinite Hermitian of $\tilde{C}^T\tilde{C}$. Then Eq.(8) becomes

$$J=\frac{1}{2}\sum_{k=0}^{\infty}\left[\xi_k^T(Q+\tilde{K}^TR\tilde{K})\xi_k\right] \tag{9}$$

Since Q and R of the described system satisfy the Lyapunov condition, Eq.(9) yields the Riccati equation of

$$\tilde{A}_c^T\mathbf{P}\tilde{A}_c-\mathbf{P}+\mathbf{Q}+\tilde{K}^TR\tilde{K}=\mathbf{0},$$
$$\text{where } \tilde{A}_c=\tilde{A}-\tilde{B}\tilde{K}. \tag{10}$$

When considering random initial conditions, the cost function can be written as

$$J=\frac{1}{2}\xi_0^T\mathbf{P}\xi_0=\frac{1}{2}\mathrm{tr}\left(\mathbf{P}\xi_0\xi_0^T\right) \tag{11}$$

There are many techniques for solving Eq.(10) and a Hamiltonian constraint equation with Lagrange multiplier[9] is used. Once obtaining the gain vector from the Riccati equation, the elements of the gain vector, K_1 and K_2, can be determined from Eq.(7).

The relation between the valve stem position and the flow rate is not exactly linear. But an equal percentage valve shows a good linearity when actually installed on the system by the fluid pressure drop [10], and the linearity is assumed in this study. It is also assumed that the time constant of the lagger is 1 sec. Further, since the natural frequency of the steam generator thermal-hydraulics is very low[6, 7], the sampling period is determined as 1 sec. Then A, B, and C of Figure 2 are found to be 0.3679, 0.6321 and 1, respectively.

In Eq.(8), the system response speed and the amount of input energy become different depending on the ratio of R and Q where Q is the Q_{11} element of matrix $\mathbf{Q}$. Figure 3 shows the system response speed and the input energy for various values of R. As shown in the figure, the larger value of R imposes a larger constraint on the input energy but makes the system speed slower. This is also explained in Figure 4 which describes the pole location versus the values of R/Q. As R increases, the pole location moves father from the center of unit circle, and the system gets more sluggish. Hence it is necessary to trade off these two factors, and the value of $R=1$ is used. With the assumptions made above, the system gains are such that $K_1=1.1781$ and $K_2=0.582$, with the cost function value of 0.536. And the transfer function of the feedwater station, Eq.(1), is

$$F(z)=\frac{1-z_r}{z(z-z_r)}=\frac{0.7447}{z(z-0.2553)} \tag{12}$$

As is well known, the flow measurement is unreliable at low power. Therefore, an estimated value rather than the measured one is preferrable at low

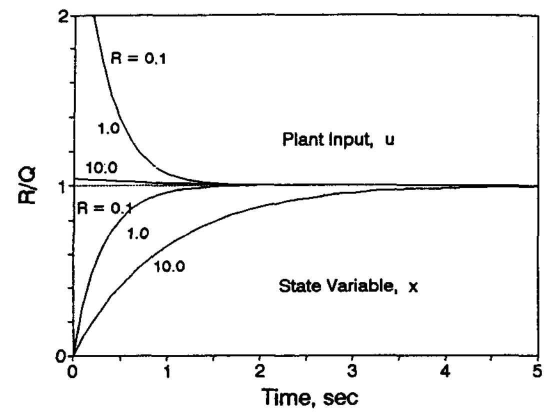

Fig. 3. Responses of x and u vs. R/Q

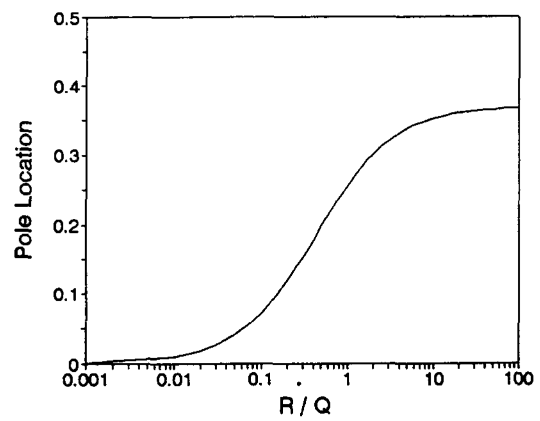

Fig. 4. Regulator Pole Locations

power by the application of a Kalman observer. The current observer equation for the first order system is

$$\hat{x}_{k+1} = (A\hat{x}_k + Bu_k) + G(y_{k+1} - C(A\hat{x}_k + Bu_k))$$

$$= (A - GCA)\hat{x}_k + GCAx_k + Bu_k \quad (13)$$

where G is the observer gain and $\hat{x}_k$ is the estimated variable.

From Eqs.(4) and (13), the system is described by theZ following matrix equation.

$\mathbf{x}_{k+1} = \mathbf{L}\mathbf{x}_k + \mathbf{M}r_{k+1}$, where

$$\mathbf{x} = (x \quad \hat{x} \quad u)^T, \quad \mathbf{M} = (0 \quad 0 \quad K_1)^T \quad (14)$$

$$\mathbf{L} = \begin{pmatrix} A & 0 & B \\ GCA & A - GCA & B \\ -K_1CA - K_2GCA & K_2(1 - A + GCA) & 1 - K_1CB - K_2B \end{pmatrix}$$

With the presence of the system and measurement noises, the state equation of the system described in Figure 2 is

$$x_{k+1} = Ax_k + Bu_k + Hw_k$$

$$y_k = Cx_k + v_k \quad (15)$$

where w_k and v_k are uncorrelated white noises of the system and measurement, respectively.

The estimator error between the real and estimated values is $\hat{e}_k = x_k - \hat{x}_k$, and the constraint of the optimal estimate of is such that the error covariance makes $E[\hat{e}_k \hat{e}_k^T]$ minimum. For the case of infinite horizon, the Kalman gain is obtained by the error covariance P which satisfies the ARE(Arithmetic Riccati Equation) of

$$G = PC^T(V + CPC^T)^{-1},$$

$$P = APA^T - APC^T(V + CPC^T)^{-1}CPA^T + HWH^T \quad (16)$$

where W and V are covariances of the system and measurement noises.

Since the noises of Eq.(15) are assumed to be white, the design of the feedwater station control is LQG(Linear Quadratic Gaussian).

The Kalman filter has two inputs of the system noise and measurement noise. For a given system, the relative values of the covariances are more important rather than the absolute value of each one. As the system noise becomes more significant, the ratio of W to V gets high and vice versa.

Figure 5 shows the effect of the covariance ratio. The Kalman gains and the observer pole locations are calculated for various values of V, with W being fixed. The figure explains that with the increasing measurement noise, the Kalman gain decreases and the observer pole becomes large, which indicates the slower convergence of the estimated signal to the real one.

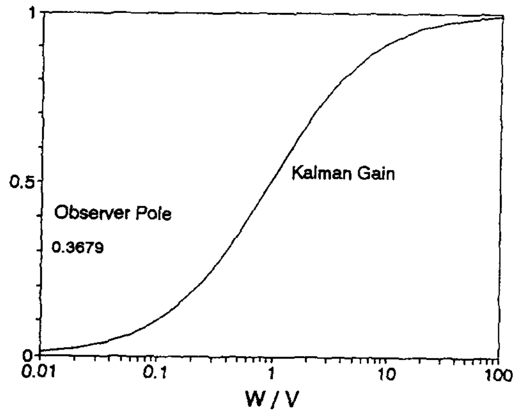

Fig. 5. Pole Locations and Gains of Observer

If the state equation, Eq.(14), is converted to the transfer function with the Kalman gain obtained from Eq.(16), by assuming all initial values of zero, the transfer function has the form of

$$F(z) = \frac{(1 - z_r)(z - z_p)}{z(z - z_r)(z - z_p)} \tag{17}$$

where z_r is the pole obtained by the regulator design, and z_p is the pole determined from the observer design. For the case of $W = V = 1$, the Kalman gain G is 0.5169, z_r=0.2553 and z_p=0.1777. Hence the observer pole is more dominant than the system pole as desired.

The above equation has the same zero and pole, therefore this system is non-minimal[9] as can be confirmed by the rank conditions. The non-minimal zero and pole play their roles in the initial stage of the transient when the real and estimated values are different from each other. Since the system is non-minimal, it is possible to hardwire the observer with the less number of integrators. However the transfer function of Eq.(17) has an assumption of initial values of zero. For the most cases the steam generator transient starts from the equilibrium state and the feedwater station can be described by the transfer function of Eq.(12).

4. Digital Controller Design

On determining the feedwater station transfer function, it is necessary to design the digital controller $G_c(z)$. From Eqs.(2) and (3), the characteristic equation of the system is

$$H_c(z) = 1 + G_c(z)F(z)\overline{G_hH_1}(z) \tag{18}$$

The frequency response diagram of the digital system F(w), mapping from z-domain to w-domain, is shown in Figure 6. The figure shows that the magnitude is almost constant. The phase is also constant over the significant frequency band of the steam generator. This means that the Bode diagram of Eq.(18) is very similar to that of $1 + G_c(s)H_1(s)$, which was

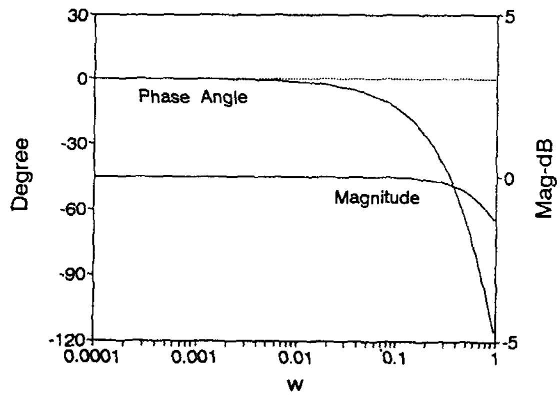

Fig. 6. Bode Diagram of F(w)

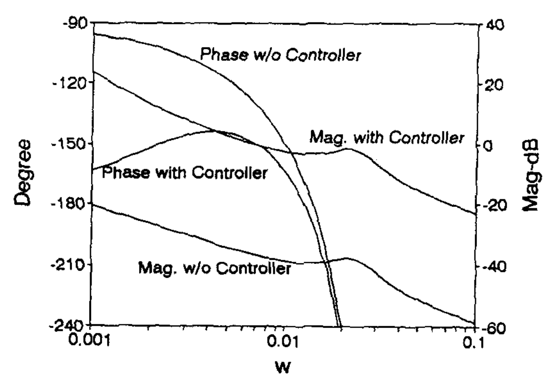

Fig. 7. Bode Diagram of Characteristic Function

discussed in Ref.[7], and the stability boundary of the digital controller does not change from that of Ref. [7].

Figure 7 shows the frequency response diagram of $F(w)\overline{G_hH_1}(w)$ together with that of the compensated system of $G_c(w)F(w)\overline{G_hH_1}(w)$ for the case of 5% power. The controller G_c is assumed to be a PI controller of $G_c(w)=61(1+410w)/410w$, and the compensated system has a gain margin of 3.3dB with a phase margin of 30.3 degrees. An algorithm has been developed to determine the PI constants which give the same stability margins for all power ranges including zero power. The algorithm calculates the gain and integration constants easily with the input of desired margins.

The controller constants are found to be as

$$G_c(w)=K\left(\frac{1+T_I w}{T_I w}\right)$$
$$K=34.26+3.85\,P+0.20\,P^2$$
$$T_I=641.3-60.0\,P+2.1\,P^2 \qquad (19)$$

where P is the initial power in percent.

Converting the above controller into the z-domain, the controller is

$$G_c(z)=K_P+\frac{K_I T}{2}\left(\frac{z+1}{z-1}\right) \qquad (20)$$

where $K_P=K$, $K_I=\frac{K}{T_I}$ and T is a sampling period.

5. Level Transient Calculation

The level transient simulation have been conducted for the various cases, among which the severe case of power increase from the initial power of 5% to 10% within one minute is discussed below. The variations of the steam flow rate and primary coolant temperature are the same as those of Ref.[7], say, the steam flow rate starts to increase from t = 10 sec to t = 70 sec by the ramp change of 0.273 kg/sec, and the primary temperature starts its change from t = 25 sec to t = 70 sec by the ramp change of 0.03°C/sec, and again increases from t = 70 sec to t = 80 sec by 026 °C/sec. There is no feedwater temperature change in this power range.

The water level variation is calculated by Eq.(2) and the details of $\overline{G_hH_1}(z)$, $\overline{G_hH_2}(z)$, and $\overline{G_hH_3}(z)$ along with the $G_c(z)$ are summarized in Table 1. The first term of Eq.(2) is the level variation by the steam and feedwater flow rate changes, and the second term accounts for the level variation by the primary coolant temperature. The first term is of the sixth order and the second term of the ninth order. Since the high order transfer functions are susceptible to the numerical instability, the high order polynomials are separated into the first and the second order functions[11], then a series of low order IIR filters are constructed and simulated[12].

Figure 8 shows the level transient up to t = 1000 sec. Curve A is the level variation by the change of the steam and feedwater flow rates, B is by the change of the primary coolant temperature and the curve C, sum of A and B, is the total water level variation. The total water level transient shows the similar trend to that of Ref.[7], but there are larger peak values with a long term oscillation. However the peak values(max. of 24cm at 100 sec, min. of −15cm at 390 sec) stay in the permissible range with a sufficient margin.

The transient of the feedwater flow rate, calculated by Eq.(3) is shown in Figure 9. Much the same as the level transient, the feedwater variation of this

Table 1. Discrete Transfer Functions at 5% Power

$H_1(z)$	$10^{-4}\left(\frac{1.0853z^2-2.2058z+1.1211}{z^3-2.9873z^2+2.9752z-0.9879}\right)$
$H_2(z)$	$10^{-4}\left(\frac{5.0575z^2-5.0842}{z^2-1.9763z^2+0.9763z-0.9802}z^{-82}\right)$
$H_3(z)$	$10^{-3}\left(\frac{8.9207z^2-1.7802}{z^3-2.8103z^2+2.62103z-0.8142}z^{-2}\right)$
$G_c(z)$	$58.6\left(\frac{z-0.9975}{z-1}\right)$

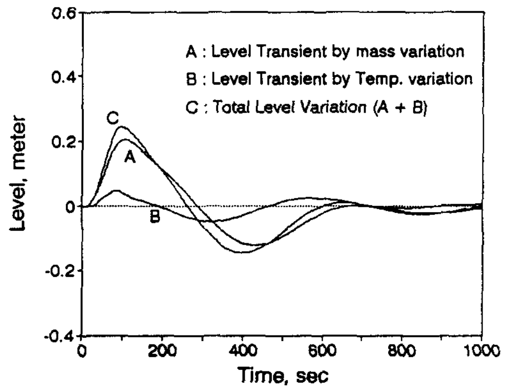

Fig. 8. Level Transients, 5 to 10% up

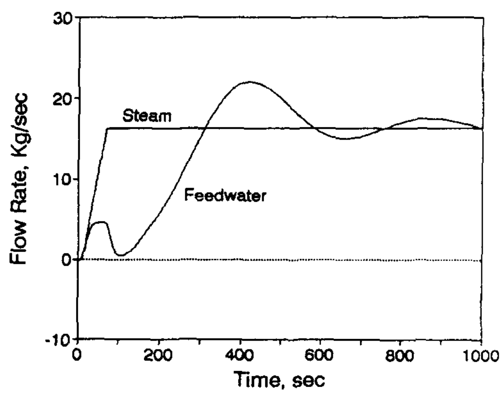

Fig. 9. Feedwater Transients, 5 to 10% up

study shows a long term oscillation with a peak value of exceeding the steam flow rate at about 400 sec, while the result of Ref. [7] converges to the steam flow rate monotonically. But during the initial stage of the transient the feedwater variation is milder than that of Ref. [7]. This is due to the constraint on the input energy which is included in the linear quadratic cost function.

The slow dynamics of both the level and feedwater transient has its reason in including the valve actuator lag in the system. In the previous studies, the feedwater valve was assumed to respond to the demand signal without delay, which is far from the real situation.

6. Conclusion

In designing the digital steam generator water level control system, the valve dynamics is treated as the first order lag and is included in the feedwater station. In consequence of the valve lag, the system responses both of the level and feedwater transients are slow. With the feedwater feedback loop, the feedwater station is a servo system and the linear quadratic constraints are imposed both on the input and output signals to determine the optimal system gains. On account of the flow measurement uncertainty, a non-minimal Kalman filter is proposed also.

Along with the feedwater station design, a digital PI controller has been designed whose control constants are scheduled to the reactor power. This controller ensures the constant stability margins for all power ranges.

The numerical simulation shows somewhat slower dynamics than that of the case of which no delay in valve actuation is assumed. But the results describe the more realistic situation and the level transient stays within the permissible range. The feedwater shows a milder variation in the initial stage of the transient.

References

1. Kevin Warwick(Ed.), *Failsafe Control Systems*, Chapman and Hall (1991)
2. *Integrated Instrumentation and Upgrade Plan*, *Rev.3*, EPRI NP−7343 (1992)
3. "Feedwater System Operability Improvement," Westinghouse material for WOG 1988 Trip Reduction Conference (1988)
4. L.W. Pearce et al., "How Digital Low Power Feedwater Control Can Minimize Plant Trips," *Nuc. Eng. Intl.*(June, 1988)
5. Y.J. Lee, U.C. Lee, "Steam Generator Water Level Control by the Control Scheme of Predictor," *Procd. of the 6th KAIF/KNS Annual Conf.*

701−704, KNS (1991)

6. Y.J. Lee, "Water Level Control of Nuclear Steam Generator," *Trans. on KSME J.*, 16, 753−764 (1992)
7. Y.J. Lee, U.C. Lee, "Digitalization of the Nuclear Steam Generator Water Level System," *J. of KNS*, 25, 126-135 (1993)
8. S.K. Menon, A.G. Parlos, "Gain-Scheduled Non-Linear Control of U-Tube Steam Generator Water Level," *Nucl. Sci. and Eng.*, 111, 294−308 (1992)
9. F.L. Lewis, *Applied Optimal and Estimation*, Prentice Hall (1992)
10. *Control Valve Handbook*, Korea Motoyama (1990)
11. S. Wolfran, *Mathematica*, Addison Wesley (1991)
12. MATLAB/Simulink, Ver.4.0, Math Work Inc. (1992)