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Current-Depth Refraction and Diffraction Model for Irregular Waves
水深 및 흐름의 影響에 의한 屈·回折을 考慮한 不規則波 模型

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Abstract□A new set of elliptic wave equations describing the deformations of irregular waves on a large-scale current field in water of irregular depth is given, and using finite difference scheme an efficient numerical method is also presented. The elliptic equations are solved in a similar way to initial value problem. This method is extensively used for the calculation of wave spectral transformation, and computation results agree very well with experimental data (Hiraishi, 1991). Finally numerical examples are presented concerning the interactions between waves and currents over a mildly sloping beach and also over a mound.

要 旨: 대규모 흐름이 존재하는 不規則한 海域에서 새로운 楕圓形 波動方程式을 유도하고, 有限差分法을 이용한 효율적인 數値模型을 개발하였다. 이때 楕圓形 方程式은 初期値 問題의 解法과 유사한 방법을 사용하여 해를 구하였다. 이 방법은 不規則波의 變形을 계산하는 데 특히 효과적이며 水理模型實驗結果(Hiraishi, 1991)와 잘 일치하였다. 마지막으로 水中淺堆가 존재하는 완경사 해역에서 波浪과 흐름의 상호작용에 의한 數値解를 예시하였다.

1. INTRODUCTION

Waves propagating near a tidal inlet will be transformed due to currents and irregular water depths. The wave-current interaction is one of the most interesting and important phenomena for the prediction of wave climate and resultant sediment transport in coastal area. Monochromatic waves may deviate by as much as 50 to over 100% from irregular waves with typical spectral shapes and directional spreads (Vincent and Briggs, 1989).

Recently, a number of studies have been made for the analysis of wave-current system. Booij (1981), Liu (1983) and Kirby (1984) proposed hyperbolic equations governing the propagation of waves in water of varying depth and currents in the mild-slope approximation. They used parabolic approximation in order to circumvent the difficulty in calculation of elliptic equations for regular waves.

Most of existing models employ parabolic or hyperbolic-type differential equations which are in general not so efficient to use in large area (order of hundreds wave length). In shallow water they need fine grid resolution to meet sufficient accuracy of numerical results. There is a definite need for an efficient method for the calculation of irregular wave transformation over a large coastal area.

2. GOVERNING EQUATIONS AND NUMERICAL SCHEMES

The mild-slope equation has been used successfully as a model equation for describing surface water waves propagating over a seabed of mild slope (Berkhoff, 1972). For a wave-current interaction Kirby (1984) derived a general equation. Recently Chae *et al.* (1990) and Jeong (1990) have rederived the mild-slope equation using variational prin-

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ciple and Green's theorem for linear water waves following Booij's method (1981). The equation can be written as

$$\frac{D^2\Phi}{Dt^2}+(\nabla\cdot\underline{U})\frac{D\Phi}{Dt}-\nabla\cdot(CCg\ \nabla\Phi)+(\sigma^2-k^2CCg)\Phi+W\frac{\partial\Phi}{\partial t}=0 \qquad (1)$$

where $D/Dt=\partial/\partial t+\underline{U}\cdot\nabla, \nabla=[(\partial/\partial x)i, (\partial/\partial y)j]$, and $\underline{U}=(u,v)$, Φ the complex velocity potential at the mean surface level, σ the intrinsic angular frequency, k wave number, C and Cg are the phase and group velocity respectively, which are defined according to $C=\sigma/k$, $Cg=\partial\sigma/\partial k$, $\sigma^2=gk\tanh kh$, and W dissipation coefficient,

$$\omega=\sigma+\underline{k}\cdot\underline{U} \qquad (2)$$

where ω is absolute angular frequency. The velocity potential at an elevation z is given by

$$\Phi(\underline{x},z,t)=f(z)\,\phi(\underline{x},t) \qquad (3)$$

where $f(z)=\cosh k(z+h)/\cosh kh$. Since the bottom is mildly sloping, the derivative of f with respect to $\underline{x}$ will be small. For a monochromatic wave with frequency ω the velocity potential is given by

$$\Phi(\underline{x},t)=Re[\hat{\phi}(\underline{x})e^{\ i\omega t}] \qquad (4)$$

Substitutions of Eq. (4) into Eq. (3), and further them into Eq. (1) produce an elliptic equation as follows:

$$-i\omega[2\underline{U}\cdot\nabla\hat{\phi}+\hat{\phi}(\nabla\cdot\underline{U})]+(\underline{U}\cdot\nabla)\ (\underline{U}\cdot\hat{\phi})+(\nabla\cdot\underline{U})(\underline{U}\cdot\nabla\hat{\phi})-\nabla\cdot(CCg\nabla\hat{\phi})+(\sigma^2-\omega^2-k^2CCg)-i\omega W\hat{\phi}=0 \qquad (5)$$

If $\underline{U}=(0,0)$, Eq. (5) reduces to Berkhoff's (1972) mild-slope equation. Here the complex velocity potential $\hat{\phi}$ can be written in terms of the amplitude a and the phase S as

$$\hat{\phi}=-ig\frac{a}{\sigma}e^{is} \qquad (6)$$

where g is the acceleration due to gravity, and $S(\underline{x})$ phase function given by

$$S(\underline{x})=\underline{k}\cdot\underline{x} \qquad (7)$$

Then Eq. (5) with the substitution of Eq. (6) reduces to a set of elliptic equations by separating the resulting equation into real and imaginary parts as follows

$$\nabla\cdot\left[\underline{U}\frac{a^2}{\sigma^2}(\omega-\underline{U}\cdot\nabla S)+CCg\frac{a^2}{\sigma^2}\nabla S\right]+W\frac{a^2}{\sigma}=0 \qquad (8)$$

$$CCg\frac{a}{\sigma}(\nabla S)^2-(\underline{U}\cdot\nabla S-\omega)^2\frac{a}{\sigma}+(\sigma^2-k^2CCg)\frac{a}{\sigma}-\nabla\cdot\left(CCg\frac{a}{\sigma}\right)+(\nabla\cdot\underline{U})\left(\underline{U}\cdot\nabla\frac{a}{\sigma}\right)+\underline{U}\cdot\nabla\left(\underline{U}\cdot\nabla\frac{a}{\sigma}\right)=0 \qquad (9)$$

These are the final forms of the wave equation for this numerical model study. In the present study we are concerned with the problems where W is assumed zero for simplicity and the mean current is in the following condition

$$|\underline{U}|^2 < CCg \qquad (10)$$

Eq. (9) then can be simplified as follows:

$$CCg\frac{a}{\sigma}(\nabla S)^2-(\underline{U}\cdot\nabla S-\omega)^2\frac{a}{\sigma}+(\sigma^2-k^2CCg)\frac{a}{\sigma}-\nabla\cdot\left(CCg\frac{a}{\sigma}\right)=0 \qquad (11)$$

If we set $\underline{U}=(0,0)$, equations (8) and (11) reduce to the ones of Ebersole (1985) model for depth refraction-diffraction. Furthermore the equation of wave action conservation for steady waves can be simply obtained from Eq. (1) by defining $\hat{A}=\rho ga^2/(2\sigma)$, $\underline{k}=\nabla S$, and $\sigma=\omega-\underline{U}\cdot\nabla S$. The equation is

$$\nabla\cdot\{\hat{A}(\underline{U}+\underline{C}_g)\}=0 \qquad (12)$$

The main wave direction θ can be given from Eq. (13) with the combination of Eqs. (8) and (11). The irrotationality condition of wave number vector is

$$\frac{\partial(|\nabla S|\sin\theta)}{\partial x}-\frac{\partial(|\nabla S|\cos\theta)}{\partial y}=0 \qquad (13)$$

If we neglect wave reflections from boundaries, and also if approximate intermediate values of wave properties can be provided at all grid points using a refraction model, the problem can be converted into initial value problem for wave diffraction (eg. Ebersole, 1985).

Finite difference method is adopted to solve the governing equations (8), (11) and (13). The coordinate and grid systems as shown in Figure 1 are employed. Forward difference scheme is used in

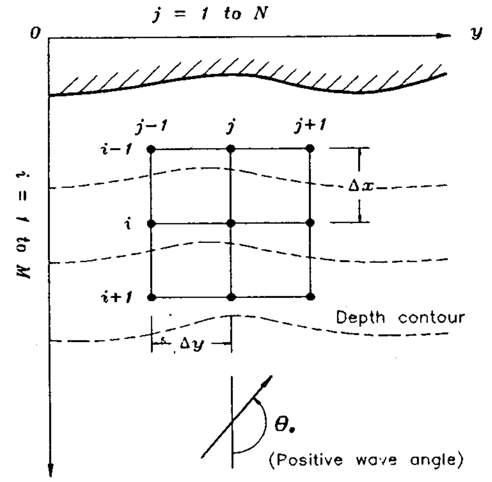

Fig. 1. Definition of coordinate system, grid cell, and wave angle conventions.

x-direction and centered scheme in y-direction to approximate the Eq. (8). Detailed schemes are given in Jeong (1990).

Input wave conditions are to be given along the offshore boundary. At side boundaries waves will be transmitted without reflection. Near the land boundary waves will break and be fully absorbed. Initial wave field is defined at all grid points using Snell's law. Intermediate values of wave heights and directions over the modelled area can be provided from current-depth refaction model (eg. Chae and Song, 1986) in order to calculate wave diffraction.

As we use steady-wave iteration approach, the simple iterative method for the solution of the equations may have no stability restrictions (Roache, 1982). The grid size of the present model does not much depend on wave length, while the restriction is strictly applied to parabolic and hyperbolic models. This is one of the major advantages of the present model. The computation is made row by row and proceeds toward the shoreward direction such as the method for initial value problem.

3. CALCULATION OF WAVE SPECTRAL CHANGES

The input directional spectrum is defined as

$$S_o(f, \theta) = S(f)\ G(f, \theta) \tag{14}$$

where $S_o(f)$ is the Bretschneider-Mitsuyasu (B-M hereafter) frequency spectrum, and $G(f, \theta)$ is the Mitsuyasu type directional spreading function (Goda, 1985).

The frequency spectrum and directional spreading function are divided into equal segments. The lower and upper frequency limits of the spectrum are 0.07 Hz and 0.37 Hz. $\Delta f = 0.02$ Hz (15 frequency bins) and $\Delta\theta = 10°$ (17 directional bins) are used.

The input wave amplitude for a particular frequency-directional component is $a_o = \sqrt{2S_o(f, \theta)\Delta f \Delta\theta}$. The resulting wave amplitude at any location can be computed using the model, and then the transformed spectrum $S(f, \theta)$ can be obtained as

$$S(f, \theta) = (a/a_o)^2\ S_o(f, \theta) \tag{15}$$

Applying the govering equations to each component of directional spectrum transformed solutions can easily be obtained.

4. COMPUTATION RESULTS AND DISCUSSIONS

Some results of the computations are compared with analytical solutions (Jeong, 1990), and to demonstrate the applicability of the equations and methods, numerical computations are made for two cases. The first case is for the refraction-diffraction due to rip-current in a mild sloping beach as shown in Figure 2 (studied by Arthur, 1950).

The computational domain is divided into square grids ($\Delta x = \Delta y = 10m$) and numerical calculations are performed. Normal incident waves of $H_o = 1m$, $T = 8s$ are used as an incident wave condition at the offshore boundary. The dimensionless wave heights, H/H_o, for two transections are plotted in Figure 3. For the purpose of comparison, parabolic model results (Kirby, 1984) are also shown in the same figure. A comparison of the figures shows that they are in good agreement.

The second case is for irregular wave propagation over a shoal as shown in Figure 4, which was recently simulated on a hydraulic laboratory equipped with multi-directional random wave generators (Hiraishi, 1991). The shoal is similar to that used in the experiments of Ito and Tanimoto (1972) with

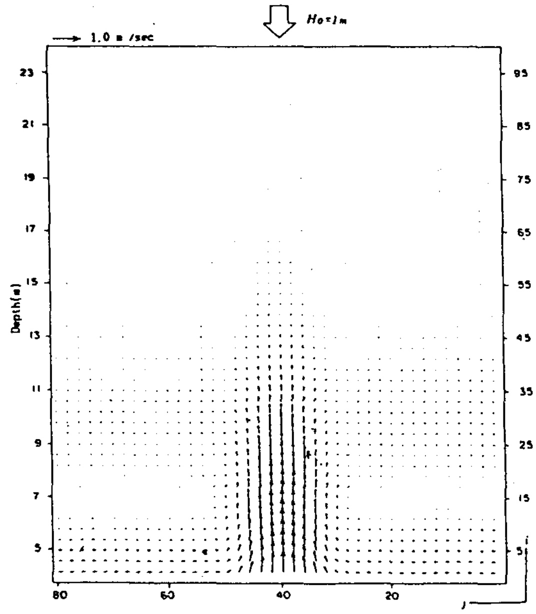

Fig. 2. Rip-current field.

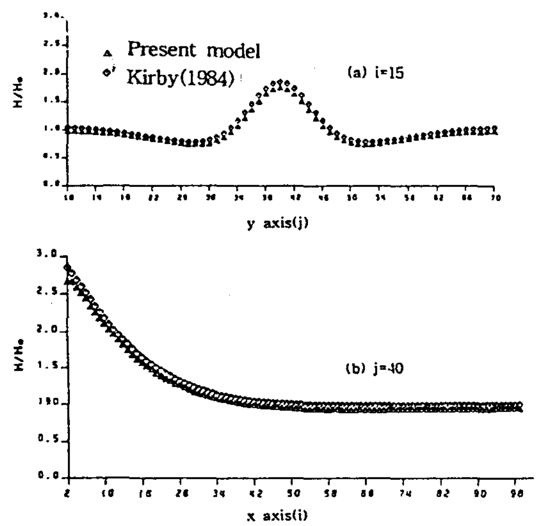

Fig. 3. Wave height relative to incident was for waves interacting with rip-current.

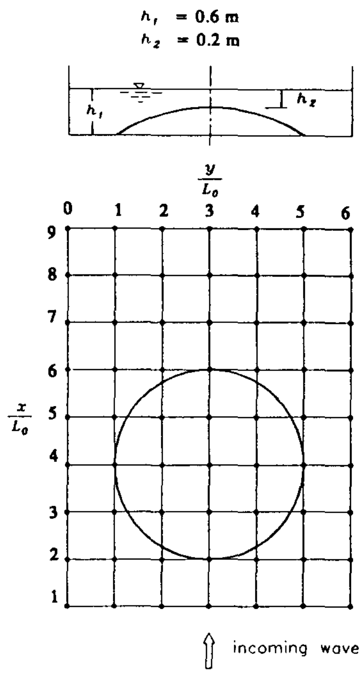

Fig. 4. Experimental configuration (Hiraishi, 1991).

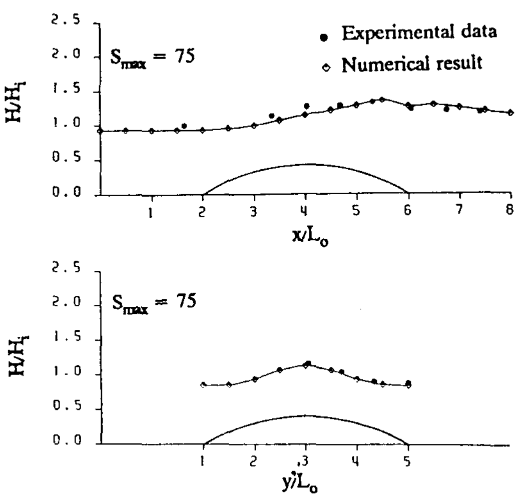

Fig. 5. Comparisons between present model results and experimental data.

a minimum water depth of 0.05 m at the center of the shoal and constant depth (0.15 m) in the region outside the shoal. B-M spectrum is used for the input spectrum for which $H_{1/3}=0.1m$, $T_{1/3}=1.5s$, and $S_{max}=75$ (narrow directional spectrum) are used. The grid sizes used are $\Delta x=\Delta y=0.1m$. The results are presented in Figure 5, in the form of normalized wave height against the input wave height. The computations agree very well with experimental data which are for the case of non-breaking waves. As the frequency and directional spectra are not available, the comparison between computation and experiment are not be made for those spectra. However, the spectrum can be simulated by linear superposition of monochromatic wave components

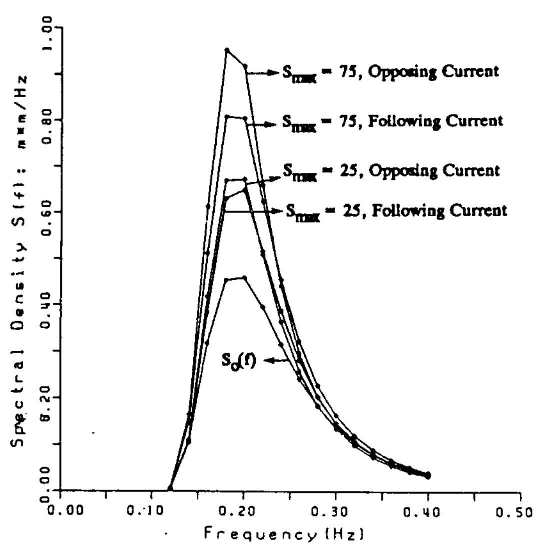

Fig. 6. Input frequency spectra ($S_o(f)$) and output frequency spectra ($S(f)$) at $x/L_o=7$, $y/L_o=3$.

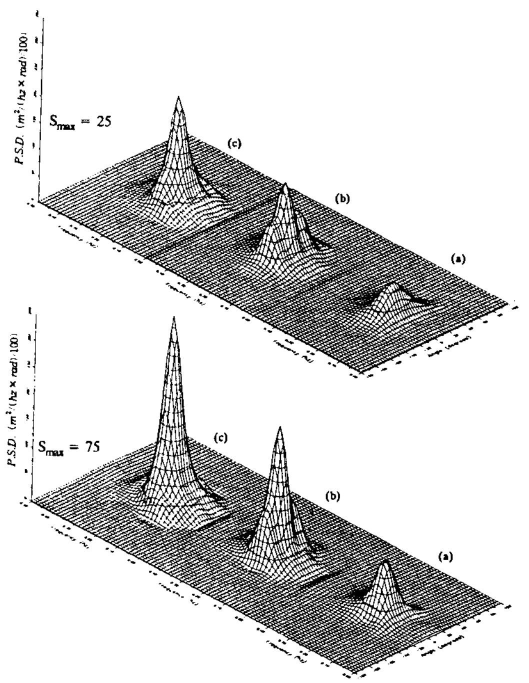

Fig. 7. Input directional spectra ($S_o(f,\theta)$) and output directional spectra ($S(f,\theta)$) at $x/L_o=7$, $y/L_o=3$ for different S_{max} and current conditions. (a) $S_o(f,\theta)$, (b) $S(f,\theta)$ with following current, (c) $S(f,\theta)$ with opposing current.

(eg. Panchang *et al.*, 1990). From those comparisons, the present model appears to be used effectively for the calculation of irregular wave propagation with respect to computation accuracy and times (26 min. with IBM 386 PC).

The present model is used for the analysis of irregular wave transformation due to combined refraction-diffraction while the waves propagate over a circular shoal (Ito and Tanimoto, 1972). The input spectrum is descretized into segments of Δf and $\Delta\theta$. $H_{1/3}=1.0m$ and $T_{1/3}=5.0s$ are used for the frequency spectrum (Figure 6) and angular spreading parameter $S_{max}=25$ and 75 for the broad and narrow directional spectra, respectively. Current velocity fields are generated using a standard depth-averaged flow model, and assumed frozen during the wave propagation over the field. A uniform current field is assumed at the incoming boundary where the maximum velocity is 0.5 m/s.

The results are shown in Figures 6 and 7, the frequency and frequency-directional spectra are for opposing and following current conditions, and also for broad and narrow directional spreading conditions at a specified points ($x/L_o=7$, $y/L_o=3$) behind the circular shoal.

As shown in Figure 7, we can clearly see the differences on spectral shapes of input $S_o(f,\theta)$ depending on the value of S_{max}. The smaller value of S_{max} yields less peaky spectral shape and broader band of energy distribution than those with larger S_{max}. When the waves propagate on a current field, the wave height and direction are strongly dependent on the magnitude and direction of the current.

In the following current field the velocities over the shoal are generally larger than those in the other region. This will increase the celerity and decrease focusing effect of wave rays propagating over that region, but in the opposing current the effect will be adverse. Such a wave-current interaction causes a large peak around centered direction in the opposing current field and a small peak with side humps in the following current. The waves with directionally narrow banded spectrum will produce very sharp peak, which is contributed mainly from the peak region.

The computed frequency spectra are shown in Figure 6. The spectral peaks are almost at the same frequency, and the amplification is prominent in

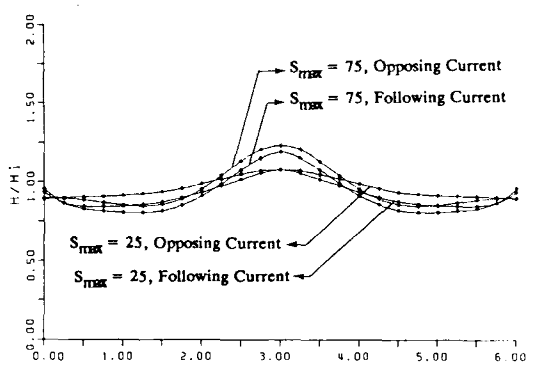

Fig. 8. Wave height comparisons, for narrow and broad directional spectra.

the peak frequency region, where the current effects are also dominant.

The propagation of wave spectra with narrow or broad directional spread shows a little difference between the wave heights in the following and opposing current conditions. The wave heights in the opposing current field are generally larger than those in the following current field (Figure 8).

5. CONCLUSIONS

A set of elliptic type mild-slope equations has been derived for wave-current interactions over a slowly varying topography. Numerical computation method to solve the equations has been presented. The model solves the elliptic equations in a way similar to an initial value problems. Accuracy of numerical computation does not greatly depend on grid size. It can be said that the present model is efficient for wave propagation problems in a large coastal area. Numerical results are shown for the transformation of waves propagating on a rip-current in a mildly sloping beach. They are in good agreement with published ones (Kirby, 1984). It is also shown that the spectral transformation of irregular wave can be satisfactorily simulated by summing up the results from a monochromatic refraction-diffraction model for component waves of a spectrum. From the analysis of frequency-directional spectrum for waves propagating on currents flowing over a mound we can see large differences in spectra depending on current directions, but there is a little difference in wave heights. When the waves propagate on strong currents in shallow water, nonlinearity of the waves and wave breaking will be significant, and therefore this model should not be applied.

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