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A Practical Method for Computing Wave Resistance

by

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조파저항 계산을 위한 실용적인 방법

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Abstract

This is a continuing work of Van & Lee[1]. Some unresolved results of theirs are first discussed more, and then Tulis's[2] exact theory is briefly reviewed. A second order theory derived from Tulin's is used as a basis to judge the accuracy of the Poisson and the Dawson[3] free surface boundary condition(FSBC) in the low speed region for a two-dimensional submerged body. In search of a new FSBC, a purely numerical approach is adopted, and we show one candidate and its performance, which is satisfactory to a certain degree.

요 약

본 논문은 Van & Lee[1]의 후속 연구이다. [1]에서 얻어진 결과 중 제대로 설명되지 못했던 부분에 대한 논의를 하고, Tulin[2]의 엄밀해에 대해 간략하게 살펴보았다. 다음 Tulin의 엄밀해를 2차항까지 근사한 2차이론을 사용하여 2차원 몰수체가 저속으로 운동하는 경우에 대해 Poisson 형태와 Dawson[3] 형태의 자유표면조건이 얼마나 정확한 해를 주는지 알아보고자 하였다. 결과적으로 보다 나은 결과를 줄 수 있는 자유표면조건이 필요함을 보이고, 순전히 수치적인 관점에서 새로운 자유표면조건을 유도하였으며, 이를 사용하여 만족할 만한 결과를 얻을 수 있음을 보였다.

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1. Introduction

Van & Lee[1], which will be denoted as VL afterwards, reported on the performance of various FSBC in computing the wave resistance for a submerged body and of a ship. They compared the Poisson, the Ogilvie[4] and the Dawson[3] FSBC using the same panel code, and concluded that 'we are still in need of a theory which gives a BC on the FS more accurate than those tested, and more practically applicable than the exact nonlinear BC'.

This paper is a continuing work of VL. We first have more discussions of the result of VL, then a theory put forward by Tulin[2] some years ago is briefly reviewed since his theory gives an exact method for predicting the surface wave produced by a moving submerged body. Examining the results of the Tulin's theory with those of the other approximate FSBC's used in VL, we shall reconfirm the need of a new FSBC. Then follows the newly proposed FSBC, its result and the discussion, consecutively.

2. More Discussions on Van & Lee [1]

We begin with summarizing the result of VL, which may have sounded controversial when presented. They gave computational results for a submerged circular cylinder and for the Salvesen[5]'s hydrofoil using 3 different FSBC's, namely the Poisson's, the Dawson's and the Ogilvie's.

First of all, they were then not aware of the continued works of Ogilvie's school on the low speed theory and his FSBC,e.g., Ogilvie & Chen [6] an Chen & Ogilvie[7]. In these works they claimed that the nonhomogeneous term in the Ogilvie FSBC is not proper, which had been pointed out by Dagan[8] and also by Keller[9] and thus that the works following the Ogilvie's fashion, for instance, Baba & Takekuma[10], may not be a valid approximation for the low speed problem. Furthermore, they showed that for the two-dimensional surface-piercing body the wave resistance is proportional to U^{48} as $U \to 0$, where U is the forward speed. Although their estimate may be correct from the viewpoint of the perturbation theory, it is not in accordance with our experience, and as they noted 'not likely to be useful to a naval architect.' Therefore in the sequel we will not consider the Ogilvie FSBC any more.

VL obtained 0.037(by Poisson) and 0.46(by Dawson), respectively, as the amplitude of waves far downstream(A_D) produced by a circular cylinder moving with $F\left(=\frac{U}{\sqrt{gL}}\right)$ being 0.4, and whose submerged depth(h) is 1.0 where g is the gravitational acceleration, and L the body length used for nondimensionalizing all lengths. One clear point which they did not catch then is as pointed out by Banner & Phillips[11], that there exists an upper bound for the surface elevation(η) of the steady flow given by

$$\eta \le \frac{F^2}{2}, \tag{1}$$

which can be easily shown from the dynamic FSBC, namely the Bernoulli's equation. Thus for $F = 0.4$, the surface elevation cannot be greater than 0.08, and we see that the Dawson FSBC not only overpredicts the surface elevation but also violates the basic principle demanded by the Bernoulli's equation. With this upper bound in mind, when we looked at the linear results given in other past publications, we found out many of them violating Eq.(1). The reason for this violation is the neglect of the second order terms in perturbation velocity from the dynamic FSBC, and hence in the region where the perturbation velocity is not so small those linear theories cannot observe the upper bound given by Eq.(1).For example, for a submerged circular cylinder A_D is given by(see e.g. Lamb[12])

$$A_D = \pi F^{-2} e^{-\frac{h}{F^2}} \tag{2}$$

Accordingly, in order to satisfy Eq.(1), h must be larger than the critical depth(h_c) given by

$$h_C = F^2(-4\ln F + \ln 2\pi). \quad (3)$$

We note that the maximum of h_c is 1.846 at F = 0.96 and that for F greater than $(2\pi)^{\frac{1}{4}}$= 1.583 there is no such lower bound for h(see Fig. 1). Eq.(3) says that h must be larger than 0.880 for F=0.4,and, in the case treated in VL though this condition $h > h_c$ is observed, the Dawson model still does violate the condition given by Eq.(1). A related question to this is how an approximate model would behave when h is close to or smaller than h_c. We will get back to this point later again.

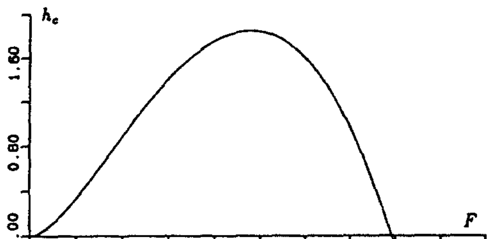

Fig. 1 Critical depth h_c for a submerged circular cylinder as a function of F

VL also got 0.008(by Poisson), and 0.024(by Dawson), respectively, as A_D for waves generated by the Salvesen's hydrofoil whose F=0.422 and h = 0.9174. In connection with this, it is worth noting that Salvesen[5] demonstrated that his second order theory gives much better comparison with his experiments than the linear one, which belongs to the same category as the Poisson model. Furthermore, he also showed that for very low speed ($\frac{F}{\sqrt{h}} < 0.5$), the wave elevation given by the second order theory is several times that by the linear one in its value. Though A_D by the Dawson is three times that by the Poisson, it is only about half the experimental value of Salvesen's. For the Salvesen's hydrofoil, the Dawson FSBC is better than the Poisson, but it is still smaller than the second order result of Salvesen's as well as his experimental one, as pointed out by Dawson[3] himself.

Judging from all the observation made above, it is highly desirable to have an exact(or almost so) theory as a reference to be able to tell which BC is better in the low speed region as well as whether we need a new BC on the FS or not. Fortunately, for the problem we are interested, Tulin[2] developed an exact theory, and so next we briefly review his major findings.

3. Tulin's Theory[2]

The difficulty in getting an exact theory for the surface wave problem is due to the fact that the location of the FS is not known *a priori*, even when the flow is assumed as irrotational. However, this difficulty disappears if one works on the complex velocity potential plane, that is the Φ-plane, where $\Phi = \phi + i\psi$, instead of the physical plane, which we call the z-plane, where $z=x+iy$ .On the Φ-plane, ψ =0 may be taken as the free surface, which is known in advance and a straight line indeed. Consequently, we need only one BC on the FS, namely the dynamic one given by

$$\frac{1}{2}q^2 + F^{-2}y = \frac{1}{2}, \text{ on } \psi = 0, \quad (4)$$

where q and θ are the modulus and the argument of the complex velocity, respectively.
Differentiating Eq.(4) with respect to θ once and setting $K=F^{-2}$, we get

$$\frac{1}{2}\frac{\partial q^2}{\partial\phi} + K\frac{\partial y}{\partial\phi} = 0, \text{ on } \psi = 0. \quad (5)$$

Using $\frac{\partial y}{\partial\phi} = q^{-1}\sin\theta$, $\sin\theta = \frac{1}{3}(\sin 3\theta + 4\sin^3\theta)$,
Eq.(5) can be rewritten as

$$\frac{\partial \ln q^3}{\partial\phi} + K\frac{\sin 3\theta}{q^3} + 4K\frac{\sin 3\theta}{q^3} = 0, \text{ on } \psi = 0, \quad (6)$$

or equivalently

$$Re\left[\frac{1}{G}\left\{\frac{dG}{d\Phi}-iK+KT\right\}\right]=0, \text{ on } \psi=0, \tag{7}$$

where Re stands for 'the real part of', and
$G=\left(\frac{d\Phi}{dz}\right)^3=q^3e^{-3i\theta}$, $Re\left(G^{-1}T\right)=4\dot{q}^3\sin^{-3}\theta$, on $\psi=0$.
When there is a submerged body, { } in Eq.(7) is not regular everywhere in the lower half plane, and we nust have

$$\frac{1}{G}\left\{\frac{dG}{d\Phi}-iK+KT\right\}=-iK+Q(\Phi), \text{for } \psi\le 0, \tag{8}$$

where $Q(\Phi)=iQ_i(\phi)$, on $\psi=0$. We note that Q represents the effect of the submerged body. So far, what we have done is deriving a differential equation satisfied in the lower half plane $\psi\le 0$, from the dynamic BC on $\psi=0$. In doing so the wave generation resulting from a submerged body is also taken into account.

Setting $G=1+H$, where H is due to the perturbation velocity, we get a first order ordinary differential equation for H as

$$\frac{dH}{d\Phi}+(iK-Q)H=Q-KT, \text{ for } \psi\le 0, \tag{9}$$

from which we obtain a solution on $\psi=0$ as

$$H(\phi)=\int_{-\infty}^{\phi}\left[iQ_i\left(\hat{\phi}\right)-KT\left(\hat{\phi}\right)\right]e^{-i\int_{\hat{\phi}}^{\phi}\left\{K-Q_i\left(\bar{\phi}\right)\right\}d\bar{\phi}}d\hat{\phi}, \tag{10}$$

Nonlinearity of the problem is now shown up by the terms T and Q. Since Q is also present on the left-hand side of Eq.(9) as a part of the coefficient of H, it seems to affect also the phase of the resulting wave system strongly. Furthermore, as Tulin pointed out, we may regard that $Q=O(\varepsilon)$ and $T=O\left(\varepsilon^3\right)$, where ε is a small parameter, thus neglecting T in Eq.(10) leads to a second order theory. Now, in Eq.(10), question is how to relegate Q_i with a real submerged body. Tulin provides a way to do this by introducing the concept of surrogate body, and he gave

$$Q_i(\phi)=-6\frac{d\theta_s}{d\phi}+2K\left[1-\frac{\cos 3\theta_S}{q_S}\right]+2KIm\left(\frac{T}{G}\right), \text{ on } \psi=0, \tag{11}$$

where Im means 'the imaginary part of', $q_se^{-i\theta_S}=\frac{d\Phi_s}{dz}$ and Φ_S is the complex velocity potential for the flow around the surrogate body. This body corresponds to the slit in the Φ-plane located at (ϕ, $-h$) where $\phi\in[0,1]$. We know from the theory of analytic function (see e.g. Tulin [13]) that

$$\tau_s(\Phi)\equiv ln\left(q_se^{-i\theta_S}\right)=\frac{1}{\pi}\int_0^1\frac{\theta_U\left(\hat{\phi}\right)}{\Phi-\left(\hat{\phi}-ih\right)}d\hat{\phi} \tag{12}$$

where θ_u is the angle between the x-axis and the tangent of the upper surface of the body in the z-plane. Here, we assume that a submerged body is symmetric with respect to its own longitudinal axis. In principle, given a geometry of the body, now we are able to construct q_s and θ_s by Eq.(12), and in turn obtain Q_i and H(ϕ) from Eq.(11) and Eq.(10), respectively. Once H (ϕ)is known, we get back to the physical plane by the transformation

$$z=\int\frac{d\phi}{(1+H)^{\frac{1}{3}}}=\int\frac{\varepsilon^{i\theta}}{q}d\phi, \text{ on } \psi=0, \tag{13}$$

which gives the equation for the surface elevation.

4. Computational Results by Tulin [2]

Since it is rather cumbersome to compute $\theta_u(\phi)$ for the Salvesen's hydrofoil, here we take a simple shaped body, i.e., a symmetric body whose $\theta_u$is given by

$$\theta_u = -4\delta\left(\phi - \frac{1}{2}\right). \quad (14)$$

where δ is the maximum thickness of the body at $\phi = \frac{1}{2}$. For small δ, a rough estimate of the body shape in the z-plane may be given by

$$y = \pm\left\{-2\delta\left(x - \frac{1}{2}\right)^2 + \frac{\delta}{2}\right\} - h, \quad (15)$$

which may include an error of $O(\delta^2)$, and + (−) corresponds to the upper(lower) surface of the body. Then using Eq.(12), we can obtain q_S, θ_S easily. However, in order to get Q_i exactly, we need to go through an iteration procedure because of the last term including T in Eq.(11). As mentioned before, T is of $O(\delta^3)$, and as a first approximation, we neglect this term along with the second term of [ ] in Eq.(10), which also includes the function T. Formally, then this is a second order theory, but as vindicated by Tulin [2] it is notable that this approximate theory gives the included angle at the crest of a limiting wave 120° as in the exact theory. Thus neglecting the effect of T, we have a very straightforward procedure, and in Fig.2, for K=6.25, δ=0.2, h=1.0, we show $Q_i(\phi)$, and $\alpha(\phi)$ defined as

$$\alpha(\phi) = \int_{-\infty}^{\phi} Q_i(\hat{\phi})d\hat{\phi}. \quad (16)$$

with which Eq.(10) can be rewritten as

$$H(\phi) = ie^{-i[K\phi-\alpha(\phi)]}\int_{-\infty}^{\phi} Q_i(\hat{\phi})e^{i[K\hat{\phi}-\alpha(\hat{\phi})]}d\hat{\phi}, \quad (17)$$

This form is preferred because of its handiness in numerical coding.

We show the surface elevation in Fig. 3, for F=0.3, 0.4, 0.5, when δ=0.2, h=0.8. We observe that as F increases, relatively speaking more energy is given to the free wave than to the local wave, and that the first hollow and the first hump immediately following the disturbance is the biggest in its magnitude.

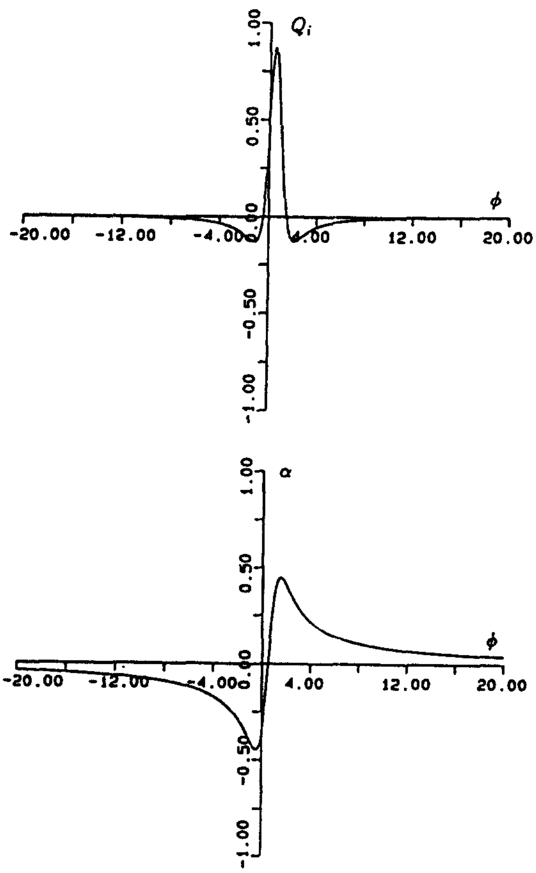

Fig. 2 a) $Q_i(\phi)$ and b) $\alpha(\phi)$, for K=6.25, δ = 0.2 h=1.0, and the body shape is given by Eq. (14)

To see the behaviour of the solution for the strong disturbance for which the biggest elevation is close to the upper bound given by Eq.(1), in Fig. 4 we demonstrate the wave elevation for δ=0.2, 0.23, 0.24, when F=0.4, h=0.6. As the disturbance gets thicker, i.e. stronger, the modulation of wave amplitude becomes more conspicuous. We note that the first hump just behind the body is near the upper bound(=0.08) given by Eq.(1). In this regard, we also

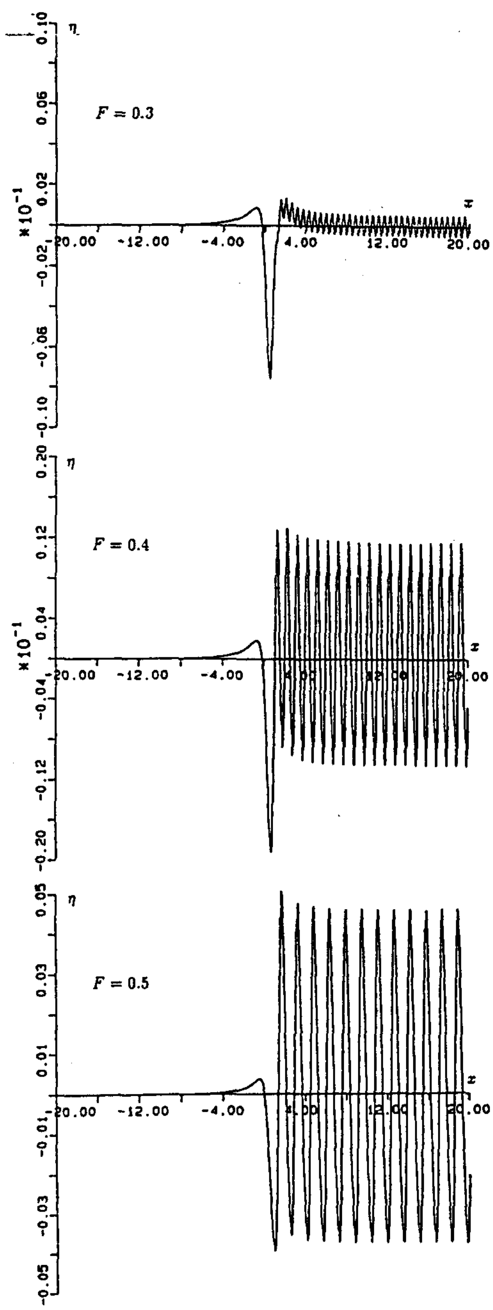

Fig. 3 Surface elevations obtained by using the Tulin[2]' s theory, for three F' s when δ = 0.2, h=0.8, for the body given by Eq.(14)

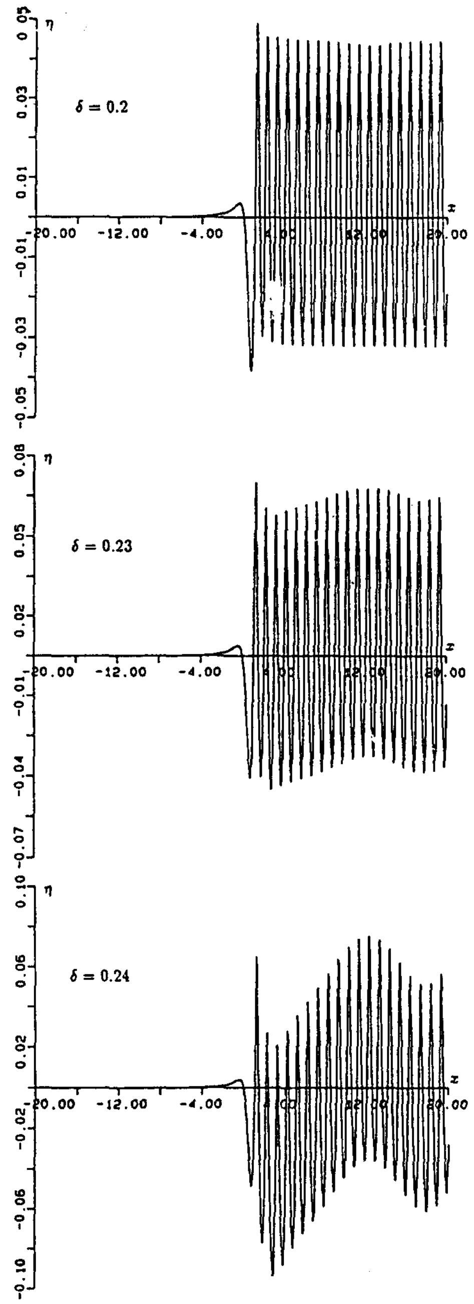

Fig. 4 Surface elevations obtained by using the Tulin's theory, for three δ's when F= 0.4, h=0.6, for the body given by Eq.(14)

note that one of the Salvesen[5]s concluding remarks was that if the submergence is smaller than three times the thickness of the body the perturbation theory should not be used for low speeds due to the wave breaking. It is conjectured that the amplitude modulation is related to the overprediction of the surface elevation by the approximate theory.

In Fig. 5 we present the surface elevation obtained by using the Poisson and the Dawson FSBC for the same case with $\delta=0.24$. The Poisson model underpredicts, while the Dawson model overpredicts and much of its elevation is greater than the upper bound given by Eq.(1). This trend of underestimate by the Poisson model, and of overestimate by the Dawsom was also observed in VL, and was described in the discussion after the Eq.(3) too.

5. Newly Proposed FSBC

In VL, it was pointed out that the Dawson FSBC is not based upon a perturbation theory, and that rather it can be easily derived if we look at it from the numerical point of view. Kim [14] also noted that the Dawson FSBC is the same as the first iteration of his iterative scheme for solving the exact nonlinear FSBC. Hence, it can be thought that purely numerical approach may be the way how we derive a new FSBC, which gives us reasonable results for the cases treated in the previous chapters. To be 'reasonable' we demand two conditions. We require that the new FSBC give greater result for the cases shown than the Poisson model, and that it observe the upper bound given by Eq.(1). The exact kinematic and the dynamic FSBC for two dimensional flows are

$$\Phi_x \eta' - \Phi_y = 0 \quad \text{on} \quad y=\eta(x), \tag{18}$$

$$\eta = \frac{F^2}{2}\left(1-\Phi_x^2-\Phi_y^2\right) \quad \text{on} \quad y=\eta(x) \tag{19}$$

respectively, and the combined form obtained by

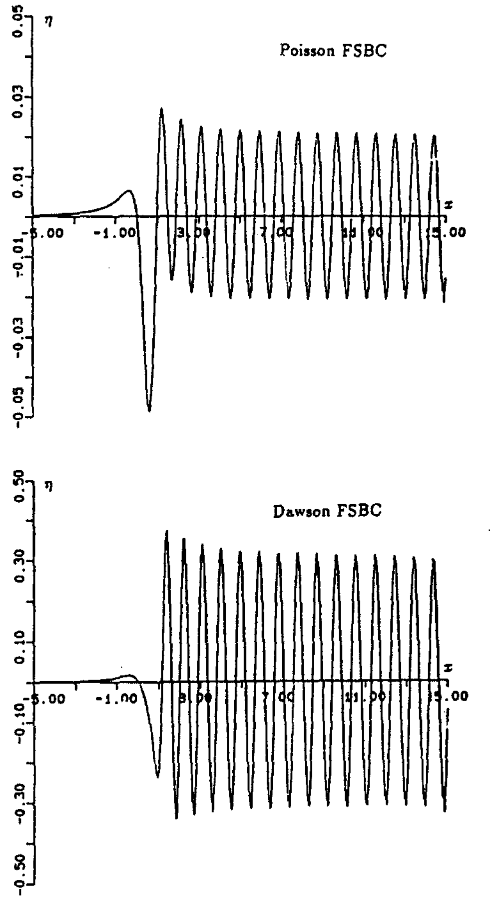

Fig. 5 Surface elevations obtained by using the Poisson and the Dawson FSBC, for the body given by Eq.(14) when $F=0.4$, $\delta=0.24$, $h=0.6$

eliminating η is

$$\Phi_x^2\Phi_{xx} + 2\Phi_x\Phi_y\Phi_{xy} + \Phi_y^2\Phi_{yy} + K\Phi_y = 0 \ , \quad \text{on} \quad y=\eta(x). \tag{20}$$

Here, $\eta' = \frac{d\eta}{dx}$, and the subscripts x, y represent the partial derivatives. Decomposing the total velocity potential Φ as follows,

$$\Phi(x,y) = x + \phi(x,y), \tag{21}$$

where $\phi(x,y)$ is the perturbed velocity potential, then substituting this into Eq (20), we get

$$\left(1+\phi_x\right)^2\phi_{xx}+2\left(1+\phi_x\right)\phi_y\phi_{xy}+\phi_y^2\phi_{yy}+K\phi_y=0\ ,$$

$$\text{on}\quad y=\eta(x)\ . \tag{22}$$

Now, neglecting the third order terms, we obtain

$$\left(1+2\phi_x\right)\phi_{xx}+\left(K+2\phi_{xy}\right)\phi_y=0\ ,\ \text{on}\quad y=\eta(x). \tag{23}$$

Transferring the surface $y=\eta(x)$ to $y=0$, we need consider the Taylor expansion of the leading order terms only, and in the expansion η can be replaced by $-F^2\phi_x$. However, when this was done, the computational result did not look promising, so we decided to neglect this effect due to the transfer of the surface on which the BC is applied. This point may require further study. Since Eq.(23) is still nonlinear; it is needed to somehow linearize it unless we want to solve a nonlinear equation numerically. Employing the concept of predictor-corrector method, we approximate ϕ_x and ϕ_{xy} in ( ) of Eq.(23) by the corresponding linear solutions given by the Poisson model, and apply the resulting equation on the surface y = 0 . Consequently, we have

$$\left(1+2\hat{\phi}_x\right)\phi_{xx}+\left(K+2\hat{\phi}_{xy}\right)\phi_y=0\ ,\ \text{on}\quad y=0, \tag{24}$$

where now $\hat{\phi}$ is the velocity potential for the Poisson model. Eq.(24) can be rewritten for the numerical coding as

$$\left(1+2\hat{u}\right)u_x+\left(K+2\hat{v}_x\right)v=0\ ,\ \text{on}\quad y=0 \tag{25}$$

where $\nabla\phi=(u,v)$ and $\nabla\hat{\phi}=(\hat{u},\hat{v})$. Once $u(x)$ on the surface y=0 is known, the free surface elevation can be obtained from

$$\eta=-F^2u,\quad \text{on}\quad y=0. \tag{26}$$

Eq.s(25,26) will be denoted as the NP FSBC hereafter. It is very simple to implement Eq.(25) in a panel code, once one has the solution for the Poisson model, since one can make use of the influence coefficients already prepared for the Poisson solver, and because the only difference is the coefficient of u_x and v in the FSBC.

Using the so developed panel code for the body used for the Tulin's theory, extensive computations were carried out. As a sample result, we show in Fig. 6 the surface elevations for F = 0.4 , δ = 0.2 , h = 0.8 , predicted by the Poisson, by the Dawson and by the NP FSBC. Comparing these with the Tulin's result given in Fig. 3, we see a surprisingly good improvements of the prediction by the NP model over other linear theories. We note that the solution by the NP model satisfies the two conditions required at the beginning of this chapter.

We also computed the surface wave generated by the Salvesen's hydrofoil, when F=0.422 , h=0.9174 , and the results predicted by the Poisson, by the Dawson, and by the NP model along with the Salvesen's measurements are shown in Fig. 7. We note that the maximum of the surface elevation measured by Salvesen is close to the upper bound, 0.089 for F = 0.422 , given by Eq.(1). For such strong disturbance none of the models tested gives a satisfactory result. But the change of the NP model is in the right direction i.e., closest to the experimental result among the tested.

6. Conclusions

In this work first we sorted out the unresolved findings of Van & Lee[1] by noting that there is an upper bound for the surface elevation produced by a moving submerged body. Then, with the help of the Tulin's[2] theory, we

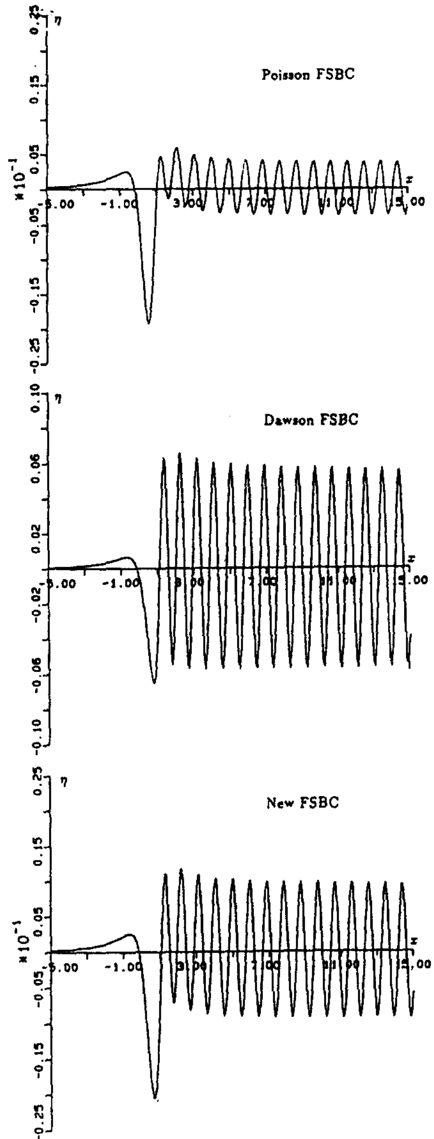

Fig. 6 Surface elevations obtained by using the Poisson, the Dawson and the new[Eq. (25)] FSBC, for the body given by Eq.(14) when $F=0.4$, $\delta=0.2$, $h=0.8$

made it certain that the Poisson model underpredicts, and that the Dawson overpredicts the surface waves generated by a submerged body moving with low speed (F≤0.5). Dawson model has another problem in that it gives bigger sur-

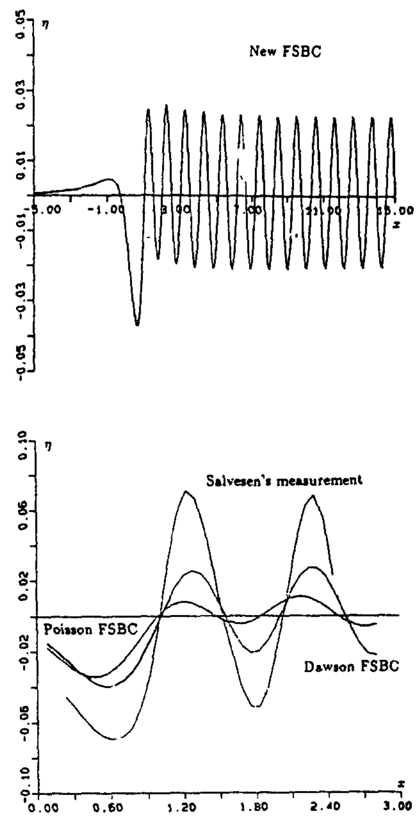

Fig. 7 Surface elevations obtained by using a) the new [Eq. (25)] FSBC,b) the Poisson, the Dawson FSBC, and the Salvesen's measurement, for the Salvesen[5]'s foil when $F=0.422$, $\delta=0.343$, $h=0.9174$

face elevations than the upper bound required by the Bernuolli's equation for some cases. To find a new FSBC without such deficiencies, we tried to derive a FSBC purely in numerical way. With some luck, the newly proposed BC, which may be called an improved Poisson model, showed a much better performance than other linear theories. It should be emphasized that one can try to derive a new FSBC, and to get some results, but that without a sound theory, like that of Tulin[2]'s, it is very hard to judge how good the new results are. Another point to stress is that there is still a good possibility for anyone to show the existence of an even better-working FSBC by, say, including higher order terms and by introducing ingenious concepts for solving it. Thus, one may say that there is much yet to learn about the free surface wave problems, and that it will be more and more so due to the ever fast advancement of the high speed computer and to our constant desire to make best use of it.

Acknowledgments

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