

On the Hydrodynamic Forces Acting on a Partially Submerged Bag

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Abstract

The hydrodynamic problem is treated here when a pressurized bag is submerged partially in the water and the end points of it oscillate. SES(Surface Effect Ship) has a bag filled with pressurized air at the stern in order to prevent the air leakage, and the pitch motion of SES is largely affected by the hydrodynamic force of the bag. The shape of a bag can be determined with the pressure difference between inside and outside. Once the hydrodynamic pressure is given, the shape of a bag can be obtained, however in order to calculate the hydrodynamic pressure we should know the shape change of the bag, and vice versa. Therefore the type of boundary condition on the surface of a bag is a moving boundary like a free surface boundary.

The present paper describes the formulation of this problem and treats a linearized problem. The computations of the radiation problem for an oscillating bag are shown in comparison with the case that the bag is treated as a rigid body. The hydrodynamic forces are calculated for various values of the pressure inside the bag and the submerged depth.

1. INTRODUCTION

The SES which is widely used as a high speed passenger ship has a pressurized bag at the stern to prevent the air leakage. The stern bag not only prevent air leakage but also has a large effect on the pitch motion of the craft. The pitch motion is affected by the side hulls, bow skirt and the stern bag. But the effect of bow skirt is small because of the mechanism of it, and that of side hulls is also small because of a small displacement in comparison with that of the craft. The pitch damping due to the stern bag is not all the damping of the craft of course, but the effect of the stern bag is considerably large. However, the hydrodynamic problem of the pressurized bag seems not to have been treated.

The problem when a pressurized bag submerges partially into water and oscillates has a different nature compared with that of a rigid body. In this paper, the problem is formulated. On the surface of a bag, the kinematic boundary condition is not sufficient to set up the

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boundary value problem. The reason is that the shape of a bag can be obtained provided that the hydrodynamic pressure is given. However in order to calculate the hydrodynamic pressure the shape change of the bag should be known, and vice versa. Therefore both the dynamic and kinematic boundary conditions should be satisfied on the unknown surface of the bag. This is a typical free boundary problem. Furthermore the boundary condition is more complex than that of a free surface, in which the condition has a point-wise form. If the pressure changes on a certain portion of a bag, the whole shape of the bag changes. Thus the pressure change in one point has an effect on the boundary condition in the whole surface, and the boundary condition can not be represented as a point-wise form, and becomes more complicated.

The formulation is made in the framework of a potential theory with additional assumptions that the mass of the bag is negligible and so does the tangential force variation on the bag, and the bag has no elongation in the girthwise length. And the bag is assumed to be fed with a constant pressure and the air flow in the bag is neglected. The solution of a static problem was obtained by iteration method, and the dynamic problem was solved by using Green's identity defined in the fluid domain.

2. STATIC PROBLEM

In this section, the static problem is treated when the bag is partially submerged in water. The assumptions are made so that the mass of the bag is negligible and the elongation of the bag is negligible also. Because most bags are made of flexible fibers, the above assumptions are reasonable. In this section, the shape of a bag and its change due to pressure change will be treated.

2.1 Shape of a Bag

The shape of a bag is obtained in this section when the pressure inside and outside of the bag differ from each other, under the assumption that the tension is constant along the perimeter of the bag unless the bag undergoes tangential forces. The pressure, tension and curvature of a bag are related by the following Laplace's formula.[1]

$$p_b - p = \frac{T}{R} \quad (1)$$

where p_b is the pressure inside the bag and p outside. Here, the pressure is understood as a gage pressure hereafter. T is tensile force and R the radius of curvature of the bag, where the sign convention is that R is positive when the origin of the radius is located toward the inside of the bag. We introduce a parameter l , the arc length of the perimeter along the bag, as shown in Fig.1 and defied as

$$\left(\frac{dx}{dl}\right)^2 + \left(\frac{dy}{dl}\right)^2 = 1 \quad (2)$$

The bag is attached to a structure at the two points, A and B . The positions of A and B are represented as (x_A, y_A) and (x_B, y_B) . The angles θ_A, θ_B are defined as the one between the

positive x -axis and direction tangential to the bag increasing l . l increases from point A to B . The total length of the perimeter is L . Consider the equation that represents the shape of a bag. The radius of curvature is the reciprocal of the derivative of a tangential angle with respect to arc length, so the tangential angle can be written as follows.

$$\begin{aligned}\theta(l) &= \int_0^l \frac{1}{R} du + \theta_A \\ &= \int_0^l \frac{p_b - p}{T} du + \theta_A\end{aligned}\quad (3)$$

The shape of a bag can be obtained from Eq.(2) and Eq.(3).

$$\begin{aligned}x(l) &= \int_0^l \cos(\theta(u)) du + x_A \\ y(l) &= \int_0^l \sin(\theta(u)) du + y_A\end{aligned}\quad (4)$$

Once the positions of two points A, B , the perimeter L , and the pressure difference $p_b - p$ are given, we can obtain the shape of the bag from the above equations. Two unknowns T and θ_A (the angle can be one of θ_A and θ_B , here θ_A was chosen) should be obtained from the condition that the point $(x(L), y(L))$ must be (x_B, y_B) .

In the case that the pressure difference $p_b - p$ is constant along the perimeter of the bag, the shape of the bag is a circular arc and T, θ_A can be easily obtained, but otherwise it is difficult to obtain a closed solution so that the numerical method must be used to do it. The numerical solution is obtained by using the modified Newton method.[2]

$$\begin{aligned}f_1 &= x(L) - x_B = 0 \\ f_2 &= y(L) - y_B = 0,\end{aligned}\quad (5)$$

$$\begin{aligned}\begin{Bmatrix} T^{k+1} \\ \theta_A^{k+1} \end{Bmatrix} &= \begin{Bmatrix} T^k \\ \theta_A^k \end{Bmatrix} - \begin{Bmatrix} \Delta T^k \\ \Delta \theta_A^k \end{Bmatrix} \\ \begin{Bmatrix} \Delta T^k \\ \Delta \theta_A^k \end{Bmatrix} &= \left[\lambda_k \mathbf{I} + \begin{bmatrix} \frac{\partial f_1^k}{\partial T} & \frac{\partial f_1^k}{\partial \theta_A} \\ \frac{\partial f_2^k}{\partial T} & \frac{\partial f_2^k}{\partial \theta_A} \end{bmatrix} \right]^{-1} \begin{Bmatrix} f_1^k \\ f_2^k \end{Bmatrix},\end{aligned}\quad (6)$$

$$\begin{aligned}f_1 + if_2 &= \int_0^L \exp \left[i \int_0^v \frac{p_b - p}{T^k} du + i\theta_A^k \right] dv \\ &\quad + (x_A + iy_A) - (x_B + iy_B),\end{aligned}\quad (7)$$

where the superscript k means the iteration step, and the pressure outside is a function of $(x(l), y(l))$. The modified Newton method is a modification of a Newton method in which both the Newton and the steepest descent method are used. When $\lambda_k = 0$ the method becomes the Newton method, and when $\lambda \gg 1$, the steepest descent method. Near the true solution the Newton method is very effective, however in the region little away from the true solution the convergence of the Newton method is poor, and sometimes for the initial guess too far from the true solution the iteration may not converge. So the steepest descent method is required, which converges slow but confirms a better convergence for a wide range of the initial guesses.

The value ranged from 1% to 10% of the largest absolute value of $(\frac{\partial f_1}{\partial T}, \frac{\partial f_1}{\partial \theta_A}, \frac{\partial f_1}{\partial T}, \frac{\partial f_1}{\partial \theta_A})$ was used for λ_k . The initial guess can be chosen as follows, from Eq.(3),

$$\begin{aligned}\theta_B &= \frac{1}{T} \int_0^L (p_b - p) du + \theta_A \\ &= \frac{L}{T} \langle p_b - p \rangle + \theta_A,\end{aligned}\quad (8)$$

where $\langle p_b - p \rangle$ is the mean value along the perimeter. If the perimeter L is given we can draw the circular arc whose end points are point A and B , in such shape θ_A, θ_B can be chosen as an initial guess. And an initial guess for the tension is from the above equation. ($\theta_B - \theta_A < 2\pi$ must be hold)

As the bag moves down, the buoyancy force becomes larger. If the buoyancy force is greater than a certain value, the static stability problem can arise, and the shape of the bag will be changed abruptly. Suppose the case that the heights of points A and B are equal. The force equilibrium in the upward direction is

$$p_b d - B_{ouy} = T(-\sin \theta_A + \sin \theta_B),$$

and in the x -direction,

$$T(\cos \theta_A - \cos \theta_B) = 0.$$

If there is no external force in the x -direction, the angles must satisfy $\theta_B = -\theta_A$ because of the geometrical symmetry. Further if the buoyancy B_{ouy} becomes larger and reaches $p_b \cdot d$, the angle will be $\theta_A = -\pi$ from the force equilibrium in the upward direction. At this moment, if the external force in the x -direction is given infinitesimally, θ_A, θ_B will be changed by an amount of positive $\Delta\theta$. Then the force in the x -direction will be

$$\begin{aligned}&T(\cos(\theta_A + \Delta\theta) - \cos \theta_A - \cos(\theta_B + \Delta\theta) + \cos \theta_B) \\ &= T(-\sin \theta_A + \sin \theta_B)\Delta\theta,\end{aligned}$$

in which the force equals to zero if $\theta_A = -\pi$. That is, there is no restoring force in x -direction. And if $\theta_A < -\pi$, the restoring becomes negative, and the static instability takes place.

If we want to analyze the problem of the case $B_{ouy} > p_b \cdot d$, the wall is needed to block the ‘fling around’ of the bag. This static instability has an effect on Eq.(6) to calculate the shape of a bag. Thus the static instability is likely to occur, the under-relaxation of $\Delta T, \Delta\theta_A$ is recommended.

2.2 Shape Change due to Pressure Change

Suppose that the shape of the bag and the tensile force are given. The shape and the tension will be changed if the pressure changes in a certain portion of the bag. If the solution will be sought by using the prescribed method when the pressure is changed, the computational burden will be larger because the iteration must be performed in each case. If we want to solve non-linear problem, the prescribed method has to be used. However when the amount of the pressure change is small, the linearization of the problem is useful.

We consider the changes of T and θ_A due to the pressure change. The point $(x(L), y(L))$ must be (x_B, y_B) even if the pressure changes.

$$\begin{aligned}x(L) - x_B &= 0 \\ y(L) - y_B &= 0,\end{aligned}$$

in which $x(L), y(L)$ are the function of p, T and θ_A . The differentials of the above equations are

$$\begin{aligned}\frac{\partial x(L)}{\partial p} dp + \frac{\partial x(L)}{\partial T} dT + \frac{\partial x(L)}{\partial \theta_A} d\theta_A &= 0 \\ \frac{\partial y(L)}{\partial p} dp + \frac{\partial y(L)}{\partial T} dT + \frac{\partial y(L)}{\partial \theta_A} d\theta_A &= 0.\end{aligned}\quad (9)$$

From the above equations, we can obtain dT and $d\theta_A$ as follows,

$$\begin{Bmatrix} dT \\ d\theta_A \end{Bmatrix} = - \begin{bmatrix} \frac{\partial x(L)}{\partial T} & \frac{\partial x(L)}{\partial \theta_A} \\ \frac{\partial y(L)}{\partial T} & \frac{\partial y(L)}{\partial \theta_A} \end{bmatrix}^{-1} \begin{Bmatrix} \frac{\partial x(L)}{\partial p} dp \\ \frac{\partial y(L)}{\partial p} dp \end{Bmatrix} \quad (10)$$

Eq.(9) and Eq.(10) were written formally, and it is not so simple because dp is a function of l . Let's represent the equations more precisely. Because $dp(l)$ is a function, $x(l), y(l)$ are functionals, so the derivatives of functionals with respect to a function can be obtained in distribution sense. The shape changes due to $dp(l)$ can be obtained by looking for the changes due to the pressure change by an amount of $\epsilon_p \delta(l-s)$ at one point $l=s$, and multiplying the amplitude of pressure change $dp(s)$, and by integrating them from 0 to L ,

$$\frac{\partial x(L)}{\partial p} dp = \int_0^L \lim_{\epsilon_p \rightarrow 0} \frac{\partial x(L)_{p=p+\epsilon_p \delta(l-s)}}{\partial \epsilon_p} dp(s) ds.$$

For $y(L)$, the same scheme can be used. And from Eq.(3) and Eq.(4),

$$\begin{aligned}\frac{\partial(x(l) + iy(l))}{\partial p} dp &= \int_0^L \left\{ \int_0^l e^{i[\frac{1}{T} \int_0^v (p_b - p) du + \theta_A]} \left(\frac{i}{T} \int_0^v -\delta(u-s) du \right) dv \right\} dp(s) ds \\ &= \int_0^l \left\{ \int_s^l e^{i[\frac{1}{T} \int_0^v (p_b - p(u)) du + \theta_A]} \frac{-i}{T} dv \right\} dp(s) ds.\end{aligned}\quad (11)$$

The partial differentials with respect to T and θ_A can be written as follows by using Eq.(3),(4).

$$\begin{aligned}\frac{\partial(x(l) + iy(l))}{\partial T} &= \int_0^l e^{i[\frac{1}{T} \int_0^v (p_b - p(u)) du + \theta_A]} \times \\ &\quad i \int_0^v (p_b - p(u)) du \left(-\frac{1}{T^2} \right) dv \\ \frac{\partial(x(l) + iy(l))}{\partial \theta_A} &= \int_0^l e^{i[\frac{1}{T} \int_0^v (p_b - p(u)) du + \theta_A]} i dv.\end{aligned}\quad (12)$$

Because $x(l), y(l)$ are the functions of p, T, θ_A , the change of the bag's shape becomes

$$\begin{aligned}dx(l) &= \left(\frac{\partial x}{\partial T} dT + \frac{\partial x}{\partial \theta_A} d\theta_A + \frac{\partial x}{\partial p} dp \right), \\ dy(l) &= \left(\frac{\partial y}{\partial T} dT + \frac{\partial y}{\partial \theta_A} d\theta_A + \frac{\partial y}{\partial p} dp \right).\end{aligned}$$

The third term in the above equation would be sought as explained above. The above equations can be written by using the result of Eq.(10).

$$\begin{aligned}
 dx &= \frac{\partial x}{\partial p} dp - \left\{ \frac{\partial x}{\partial T} \quad \frac{\partial x}{\partial \theta_A} \right\} \left[\begin{array}{cc} \frac{\partial x(L)}{\partial T} & \frac{\partial x(L)}{\partial \theta_A} \\ \frac{\partial y(L)}{\partial T} & \frac{\partial y(L)}{\partial \theta_A} \end{array} \right]^{-1} \left\{ \begin{array}{c} \frac{\partial x(L)}{\partial p} dp \\ \frac{\partial y(L)}{\partial p} dp \end{array} \right\} \\
 &= L_x \cdot dp, \\
 dy &= \frac{\partial y}{\partial p} dp - \left\{ \frac{\partial y}{\partial T} \quad \frac{\partial y}{\partial \theta_A} \right\} \left[\begin{array}{cc} \frac{\partial x(L)}{\partial T} & \frac{\partial x(L)}{\partial \theta_A} \\ \frac{\partial y(L)}{\partial T} & \frac{\partial y(L)}{\partial \theta_A} \end{array} \right]^{-1} \left\{ \begin{array}{c} \frac{\partial x(L)}{\partial p} dp \\ \frac{\partial y(L)}{\partial p} dp \end{array} \right\} \\
 &= L_y \cdot dp,
 \end{aligned} \tag{13}$$

where L_x, L_y are the linear operators.

3. FORMULATION

The boundary condition on the surface of a rigid body is well known and simple, but on the surface of a flexible body, like a pressurized bag, the boundary condition has a complicated nature. The hydrodynamic pressure makes the bag change its shape, and the change of a bag makes the hydrodynamic pressure changed, and vice versa. So the boundary type is a moving boundary like the free surface boundary, and both the dynamic and the kinematic conditions must be applied.

3.1 Linear Radiation Problem

We treat the problem when the end points of a bag oscillate. As the end points move, the bag tends to move like a rigid body, but actually the bag cannot move as a rigid body because the pressure outside varies and the surface of the bag deforms. Denote the displacement of the bag from the static equilibrium position as dx, dy , and the displacement of a rigid body which initially has the same shape with the bag as dX_E, dY_E . When the end points move, the displacement of the surface is related with the dynamic condition.

$$p = -\rho g y - \rho \phi_t \quad \text{for } y < 0.$$

The displacement of the surface of the bag depends upon the following two factors: the effect of the movement of the end points, and the one due to pressure changes. The displacement dy can be written as

$$\begin{aligned}
 dy &= L_y \cdot dp + dY_E \\
 &= -\rho g L_y^* \cdot dy - \rho L_y^* \cdot d\phi_t + dY_E
 \end{aligned}$$

$$dy = [I + \rho g L_y^*]^{-1} [-\rho L_y^* \cdot d\phi_t + dY_E], \tag{14}$$

in which I is the identity operator and L_y^* is an operator which is reduced from L_y by ignoring the part of $y > 0$. Similarly dx can be obtained as follows,

$$\begin{aligned} dx &= L_x \cdot dp + dX_E \\ &= -\rho g L_x^* dy - \rho L_x^* d\phi_t + dX_E \\ &= -\rho L_x^* [I + \rho g L_y^*]^{-1} d\phi_t \\ &\quad - \rho g L_x^* [I + \rho g L_y^*]^{-1} dY_E + dX_E. \end{aligned} \quad (15)$$

Only the portion $y < 0$ is required to solve the boundary value problem. Examining the above two equations closely, we know that the portion $y > 0$ has no effect on the portion $y < 0$, because the pressure remains constant over the portion $y > 0$. Thus we can rewrite the equations only for $y < 0$.

$$\begin{aligned} dx^* &= -\rho \mathbb{L}_x [I + \rho g \mathbb{L}_y]^{-1} d\phi_t - \rho g \mathbb{L}_x [I + \rho g \mathbb{L}_y]^{-1} dY_E^* + dX_E^* \\ dy^* &= [I + \rho g \mathbb{L}_y]^{-1} [-\rho \mathbb{L}_y d\phi_t + dY_E^*], \end{aligned} \quad (16)$$

where $\mathbb{L}_x, \mathbb{L}_y$ are L_x, L_y defined only on the portion $y < 0$. Explaining the operators L^*, \mathbb{L} using Eq.(11) and Eq.(13), L^* is the operator in which dp has a non-zero value for $y < 0$ and zero for $y > 0$, and \mathbb{L}^* performs the operation only for $y < 0$ among the operation of L^* .

The kinematic condition is the condition that the normal component of the velocity of a bag's surface must be the same as the normal component of the fluid velocity. Thus,

$$\begin{aligned} \phi_n &= V_n \cdot \vec{n} = n_x x_t^* + n_y y_t^* \\ &= n_x X_{Et}^* + [n_y - n_x \rho g \mathbb{L}_x] [I + \rho g \mathbb{L}_y]^{-1} Y_{Et}^* \\ &\quad - [n_x \rho \mathbb{L}_x [I + \rho g \mathbb{L}_y]^{-1} + n_y \rho [I + \rho g \mathbb{L}_y]^{-1} \mathbb{L}_y] \phi_{tt} \\ &= n_x X_{Et}^* + C_y Y_{Et}^* - C_\phi \phi_{tt}, \end{aligned} \quad (17)$$

where

$$\begin{aligned} C_y &= [n_y - n_x \rho g \mathbb{L}_x] [I + \rho g \mathbb{L}_y]^{-1} \\ C_\phi &= [n_x \rho \mathbb{L}_x [I + \rho g \mathbb{L}_y]^{-1} + n_y \rho [I + \rho g \mathbb{L}_y]^{-1} \mathbb{L}_y] \end{aligned}$$

As shown above, the boundary condition can be represented not point-wisely but globally.

The boundary value problem is summarized as follows.

$$\begin{aligned} \nabla^2 \phi &= 0 && \text{in fluid domain} \\ \phi_y + 1/g \phi_{tt} &= 0 && \text{on } y = 0 \\ \phi_n + C_\phi \phi_{tt} &= n_x X_{Et}^* + C_y Y_{Et}^* && \text{on the surface of the bag,} \end{aligned} \quad (18)$$

and an appropriate radiation condition. Once the solution of the above problem is obtained, we can calculate the pressure on the surface of the bag from the following equation.

$$\begin{aligned} dp &= -\rho g dy - \rho d\phi_t \\ &= -\rho [I + \rho g \mathbb{L}_y]^{-1} \{g dY_E + d\phi_t\} \quad \text{for } y < 0. \end{aligned} \quad (19)$$

The first term of the above equation is static pressure and the second term is hydrodynamic pressure. Substituting the above equation into Eq.(13), we can obtain the shape change of the bag, and into Eq.(10), the tension and θ_A . The change of θ_B can be calculated by differentiating Eq.(3) and substituting Eq.(10) into this resulting equation.

The dynamic force acting on the point A is, in x -direction

$$\begin{aligned} f_{xA} &= (T + dT) \cos(\theta_A + d\theta_A) - T \cos \theta_A \\ &= -T \sin \theta_A d\theta_A + \cos \theta_A dT, \end{aligned} \quad (20)$$

and in y -direction

$$\begin{aligned} f_{yA} &= (T + dT) \sin(\theta_A + d\theta_A) - T \sin \theta_A \\ &= T \cos \theta_A d\theta_A + \sin \theta_A dT. \end{aligned} \quad (21)$$

The force on the point B can be calculated similarly, and so the heave and sway forces and the roll moment acting on the bag are obtained.

3.2 Numerical Implementation

The original boundary-value problem is replaced by the following integral equation using Green's identity

$$\phi(P) = \int_S \{G_{nQ}(P, Q)\phi(Q) - G(P, Q)\phi_n(Q)\} dS(Q),$$

where P is the field point and Q source point, and $G(P, Q)$ is the fundamental solution of Laplace equation which satisfies the free surface boundary condition.[3] First we discretize the boundary surface and assume that the values of ϕ, ϕ_n are constant over each segment and equal to the values at midpoint. Next by performing the integrations over each segment analytically, the above integral equation reduces to the following matrix equation.

$$\{\phi\} = [G_n]\{\phi\} - [G]\{\phi_n\} \quad (22)$$

When we assume the motion is time harmonic, we obtain the following equation by substituting the boundary condition into the above equation.

$$[I - G_n + \omega^2 G C_\phi]\{\phi\} = -[G]\{n_x X_{Et}^* + C_y Y_{Et}^*\}. \quad (23)$$

After discretization, the operator C_ϕ and C_y turn into matrices and n_x, n_y diagonal matrices. Once the solution of the above equation is found, the pressure on the surface of a bag can be calculated by Eq.(19).

4. NUMERICAL RESULTS

All calculations were carried out with a single precision on the i386 based PC. The length of the bag was divided into 100 elements. The dimensionless parameters are defined as follows: perimeter length L/d , submerged depth $depth/d$, volume inside $V' = V/d^2$, submerged area

$A' = A/d^2$, pressure inside $p' = p_b/\rho g d$, tension $T' = T/p d$, frequency $\omega/\sqrt{g/d}$, added mass $a/\rho d^2$, damping coefficient $b/\rho d^2 \sqrt{g/d}$.

In Fig.2, the shapes of a bag with various submerged depth are shown. The submerged depth is defined such that the depth is zero when the bag touches the free surface and then the depth is defined as the length the point A goes down. As expected, the bag is more flexible as the pressure inside the bag becomes smaller.

In Fig.3, the static properties are shown. As the depth grows, the volume inside the bag decreases, the angle increases and the tension decreases. The angle must have the value less than π in this scheme, so the depth cannot become larger than the value shown in the figure.

In Fig.4, the restoring force is shown in comparison with the result for the rigid body. Restoring ratio '1' means that the restoring of a bag is the same with the restoring of the rigid body which has the same shape with the bag. The restoring ratio decreases as the pressure decreases and the depth increases.

In Fig.5 through Fig.8, the added mass and the damping coefficient of a bag are shown. In calculation of the hydrodynamic forces, the forces are obtained by direct integration of the hydrodynamic pressure on the surface of the bag, because the force directly integrated and by Eq.(20) have the same value, and there is some kind of numerical error in the force from Eq.(20). The added mass of a bag is so small except in low frequency region, and in some region becomes negative. There are many cases in which the negative added mass can be obtained, such as the motions in restricted water and near the wall, and the multi-body motions. So this negative added mass is not surprising and the terminology, the added mass, is not appropriate in those cases. When the pressure is high, the damping is close to that of rigid body, but as the pressure decreases, it decreases except in low frequency sway mode and have a different behavior. As seen in Eq.(23), the added mass and the damping coefficient vanishes as the frequency goes to infinity.

The hydrodynamic force can be represented with added mass and damping coefficient for a sinusoidal motion with frequency ω as follows,

$$\text{Force} = -\omega^2 a + \omega b i$$

Therefore if the added mass or the damping coefficient has non-zero value at infinite frequency, the force at infinite frequency also has an infinite value. So in the case of non-zero added mass or damping at infinite frequency, the body must give infinite energy to a fluid to maintain oscillation at infinite frequency. This is the case of a rigid body. But the bag has a different mechanism from a rigid body. The pressure acting on a fluid is limited and this may be several times of the pressure inside a bag. Even if it may have a large value, the pressure and the force are limited by a certain finite value. Thus the added mass and damping cannot have a non-zero value at infinite frequency, and decrease to zero as the frequency increases to infinity.

5. CONCLUSIONS

In this paper, the hydrodynamic problem was treated when a bag filled with pressurized air submerges partially in the water and the end points of it oscillate. Different from the case

of a rigid body, this problem has some distinct hydrodynamic features: both the kinematic and dynamic conditions are required for the boundary condition on the surface of a bag because the surface of a bag can be deformed by the pressure acting on it, and furthermore the boundary condition cannot be represented locally but globally.

In this paper, this problem was formulated and the numerical calculations were carried out. Through this work, the following conclusions are drawn.

The boundary condition on the bag is a mixed type and represented globally.

The static restoring force becomes smaller than that of rigid body as the pressure inside goes down or the submerged depth becomes larger.

In the low frequency region, the added mass and the damping coefficient of a bag have values close to those of rigid body. As the frequency becomes larger they have the different behavior compared with those of rigid body, and the added mass becomes negative in some high frequency region. And the added mass and the damping coefficient vanish as the frequency goes to infinity.

The added mass and the damping coefficient are smaller than those of rigid body except the sway damping coefficient at low frequency.

Author hopes that the further study will be taken including the compressibility of the air, the wall to block 'fling around' of the bag and a forward speed effect.

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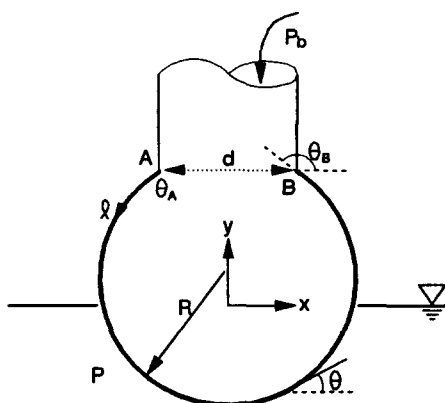


Figure 1: Coordinate system and the bag

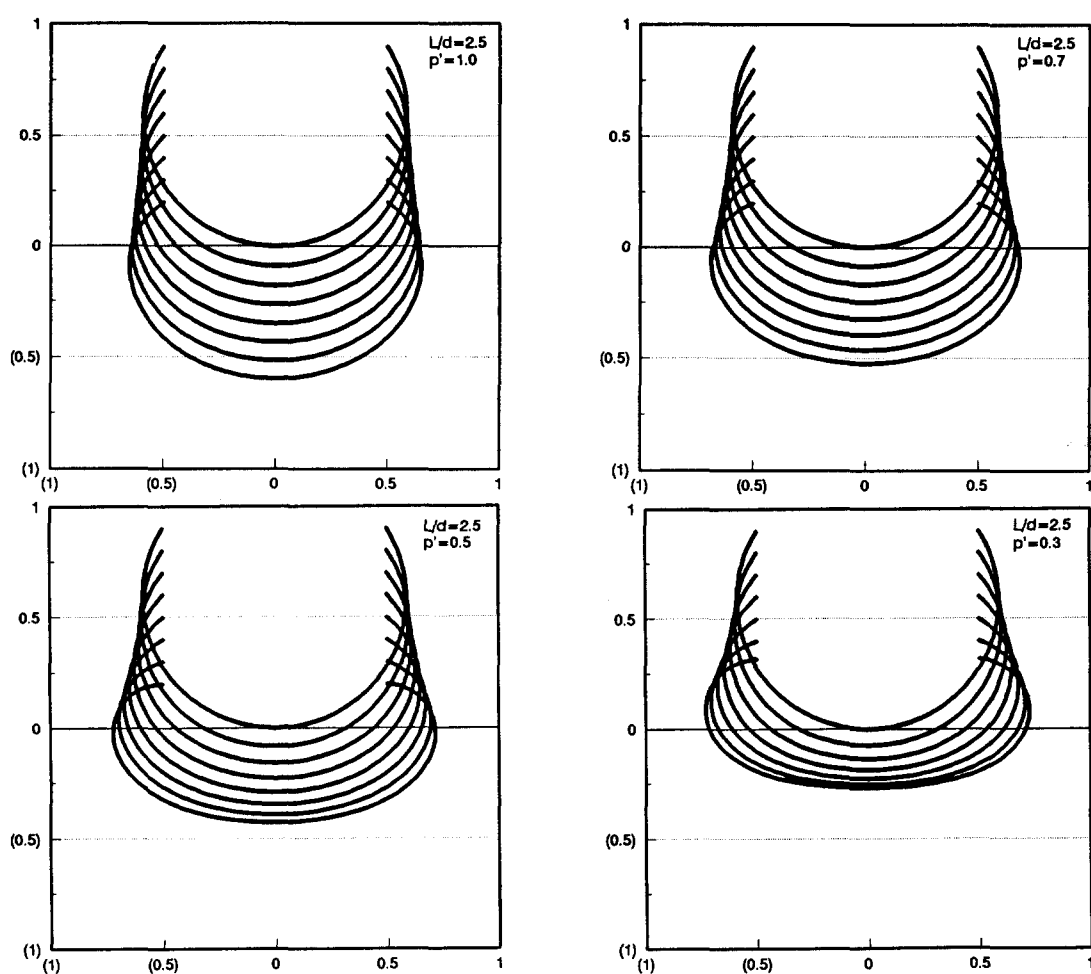


Figure 2: The shape of a bag with various submerged depths

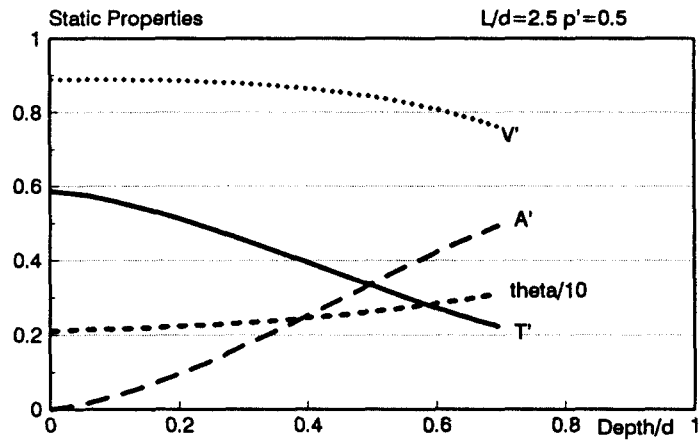


Figure 3: The static properties of a bag(p' pressure inside a bag, V' volume inside a bag, A' submerged area, T' tension, All are nondimensional values)

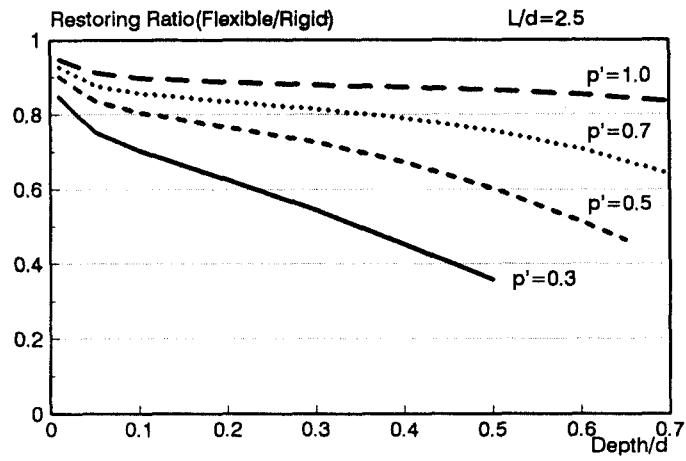


Figure 4: The restoring force of a bag, non-dimensionalized with the restoring force of the rigid body which is the same in shape

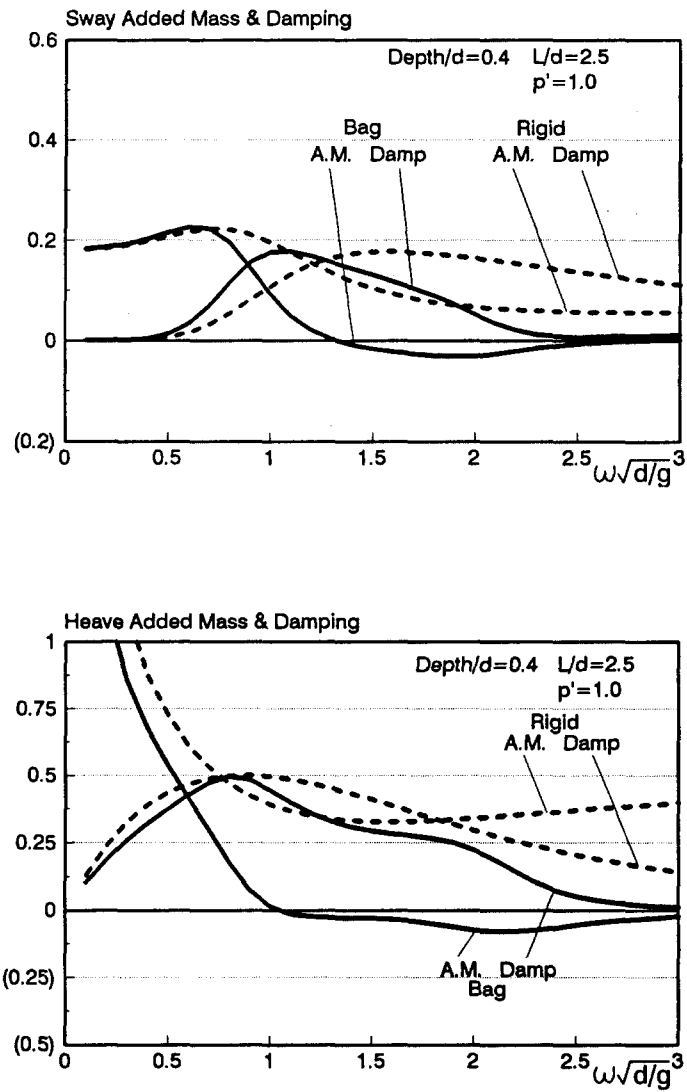


Figure 5: Added mass and damping of a bag when the non-dimensionalized pressure is 1.0

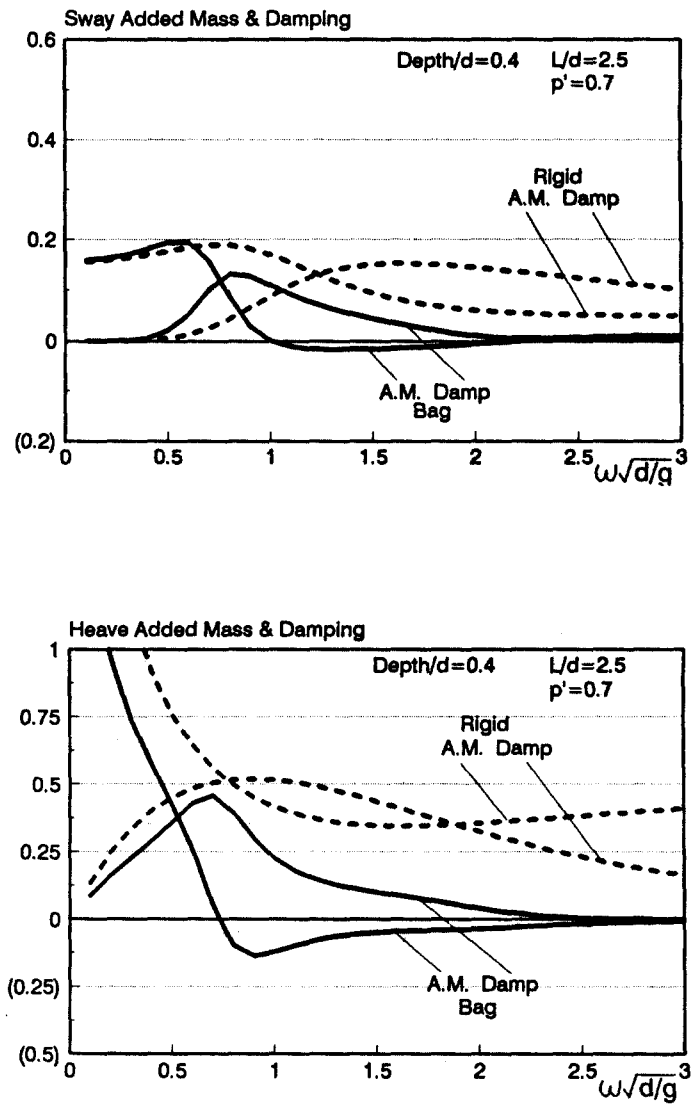


Figure 6: Added mass and damping of a bag when the non-dimensionalized pressure is 0.7

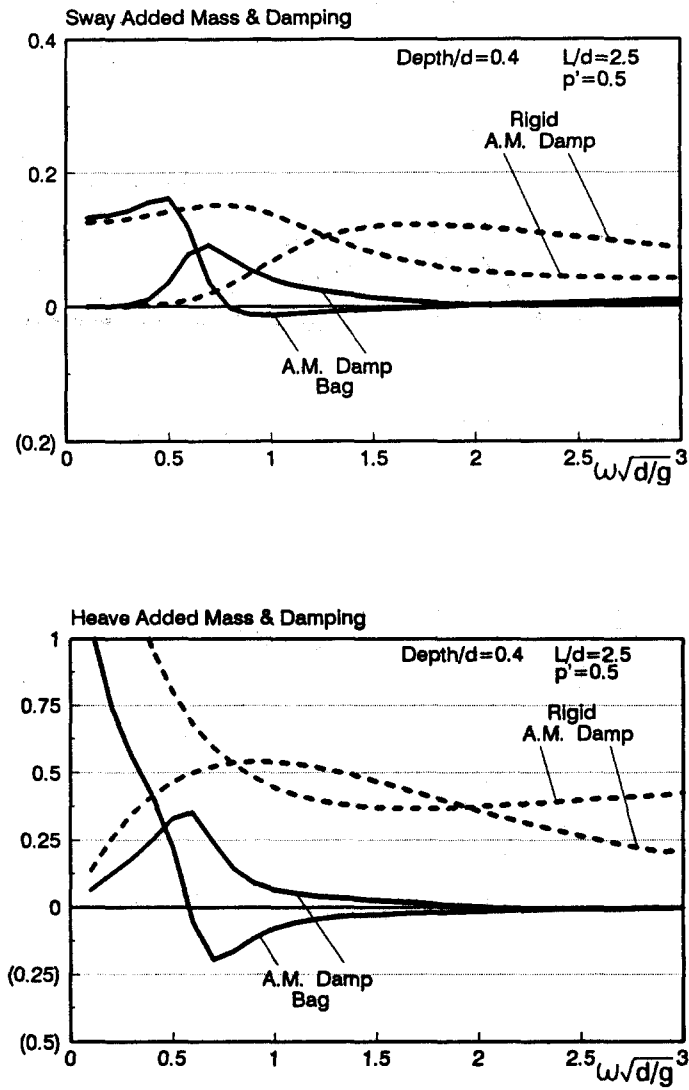


Figure 7: Added mass and damping of a bag when the non-dimensionalized pressure is 0.5

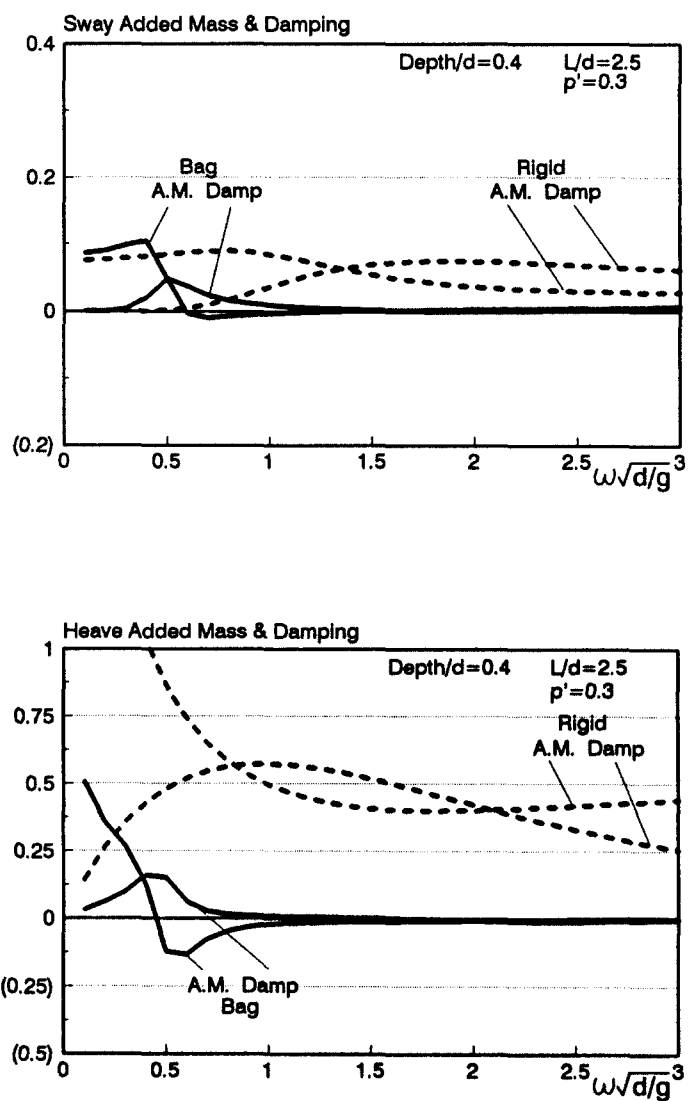


Figure 8: Added mass and damping of a bag when the non-dimensionalized pressure is 0.3