A Deformation Model of a Bag-Finger Skirt and the Motion Response of an ACV in Waves

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Abstract

In this paper, the effect of a skirt deformation on the responses of an Air Cushion Vehicle in waves is investigated. The air in the bag and plenum chamber is assumed to be compressible and to have a uniform pressure distribution in each volume. The free surface deformation is determined in the framework of a linear potential theory by replacing the cushion pressure with the pressure patch which is oscillating and translating uniformly. And the bag-finger skirt assumed to be deformed due to the pressure disturbance while its surface area remained constant. The restoring force and moment due to the deformation of bag-finger skirt from equilibrium shape is incorporated with the equations of heave and pitch motions. The numerical results of motion responses due to various ratios of the bag and cushion pressure or bag-to-finger depth ratios are shown.

1. INTRODUCTION

Since the first Air Cushion Vehicle applying the idea of Christopher Cockerell, the SR.N1, was launched in England at 1959, many ACV's have been built and used to various applications because they have many advantages such as high speed and amphibiousness etc.. And theoretical and experimental works to analyze a craft ride quality have been performed by many researchers throughout the world. But theoretical models for computing the response of hovercraft have not yet been developed sufficiently to use as a design tool owing to the absence of adequate experimental data on some of the mechanism present[1-3].

The analysis of responses of an ACV over regular waves was started by Reynolds [4]. He developed a linearized equation of motion by considering a single-plenum craft with single-degree of freedom in heave. A quadratic expression for the fan characteristics, the incompressible Bernoulli equation and the usual equations of continuity were used in this analysis. Later Reynolds et al.[5] extended this work to include pitch motion in addition to heave by adopting a craft with a transverse skirt. The pressure deviations of the fore and aft compartment from

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their equilibrium values were used to formulate a pitch equation of motion. The important assumptions included in both papers were that the skirt hemline makes no contact with the water surface and the wavy surface is rigid.

The effect of the presence of the water surface upon the perturbations of pressure in the plenum in a surface effect ship was examined by Breslin[6], Kim and Tsakonas[7], and in an ACV by Doctors[8,9]. Breslin assumed that the deformation of the water surface participates in the generation of the bubble pressure in conjunction with the actions of seals, fans, etc., and the deformation of the water surface under the oscillatory rectangular pressure patch, having an infinite beam, in uniform translation be used to display the way in which the motion of the water surface participates in the determination of the pressure variations in the plenum air. His work was later extended to three dimensions by Kim and Tsakonas. They evaluated wave elevation, escape area at the stern and volume induced by an oscillatory rectangular patch in uniform translation for the entire range of the speed frequency parameter τ of practical interest, from very low to considerably high. Earlier than the Kim and Tsakonas, Doctors had developed the same analysis to evaluate the hydrodynamic influence, and he applied this result to the motion of Air Cushion Vehicle which was taken by Reynolds. The hydrodynamic influence was felt through the alteration of the air gap under the skirt due to water deflection and a change in the effective flux balance of air in the cushion, which was assumed to be incompressible. Also he evaluated the non-linear effect on the motion responses of the craft for different wave heights. He extended his previous work to higher Froude numbers and encounter frequencies of practical interest[10], and to include the effect of compressibility of the air by considering only the accumulation term in the continuity equations for the chambers.

Rhee and Lee[10] made a similar analysis to evaluate the responses of an ACV in uniform translation over regular waves, in which the effects of the height and inclination of the skirt on the motion responses were examined. They evaluated the hydrodynamic influence due to cushion pressure in line with Doctors[9], but developed a numerical approach that is valid for the entire range of the parameter τ and for a polygonal pressure patch by use of Stoke's theorem. In the dynamic analysis of the air flow in chamber and duct, the adiabatic and isentropic flow law was applied directly to the equations of the mass conservation.

The object of this paper is to present a method for analyzing the skirt deformation due to pressure changes and surface elevations, using this method, the heave and pitch responses of an ACV with bag and finger skirt in uniform translation over regular waves are evaluated. A model for the deformation of the bag-finger skirt is proposed. The hydrodynamic influence and the air flow are formulated in line with Rhee and Lee[10], but the skirt deformation and its effects on the air flow and skirt forces are included.

The heave and pitch responses of an ACV with bag-finger skirt to regular waves are calculated for different ratios of the bag and cushion pressure and for different shapes of bag-finger skirt by use of linear equations of motion. The shape of bag-finger skirt has an important effect on the motion responses at somewhat high frequencies.

2. MODEL OF SKIRT DEFORMATION

A bag and finger type skirt, which was shown in Figure 1, is considered for this research. The skirt deformation is assumed to depend on the restoring coefficient, the pressure in the

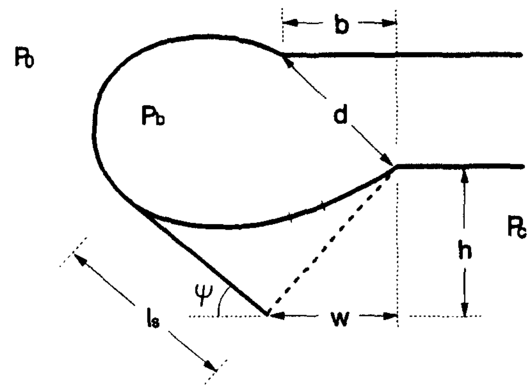

Figure 1: Schematic view of a bag-finger skirt

bag and the plenum chamber, and on the free surface elevation. The analysis of the restoring force of the bag for a simplied model is given in Appendix.

In this section, the skirt deformations due to the pressures and the free surface elevation are examined.

2.1 Deformation due to the Pressure in the Bag

When the pressure P_b in the bag increases, the bag is going to move downward. The pressure is understood as a gage pressure hereafter. Suppose the pressure P_b increases by an amount of ΔP_b and the bag deforms by an amount of the angular displacement $\Delta\psi$. The upward force acting on the bag is

$$-b\Delta P_b,$$

where b is the lateral distance of two points to which the bag is attached. And the force on the finger is

$$-P_c l_s \cos(\psi + \Delta\psi) + P_c l_s \cos\psi \cong P_c l_s \sin\psi\Delta\psi,$$

where P_c is the pressure in the plenum chamber, and l_s the length of finger. The restoring force of the bag is as follows.

$$P_b d\Delta\psi,$$

where d is the distance of two points to which the bag is attached as in Fig.1.

From the equilibrium condition of forces, we can obtain the angular deformation $\Delta\psi$ as follows,

$$\Delta\psi = \frac{-b\Delta P_b}{P_b d - P_c l_s \sin\psi}. \tag{1}$$

Since the force acting on the bag and its counterpart on the structure cancel each other, the force acting on the skirt system due to the pressure increase, ΔP_b, becomes

$$P_b d\Delta\psi + b\Delta P_b = -b\left(\frac{1}{D} - 1\right)\Delta P_b, \tag{2}$$

where D is defined as,

$$D \equiv 1 - \frac{P_c l_s \sin\psi}{P_b d}. \tag{3}$$

From the angular displacement $\Delta\psi$ we obtain the vertical displacement of the lowest point of the finger,

$$w\Delta\psi = -\frac{wb}{P_b dD}\Delta P_b, \tag{4}$$

and the horizontal displacement of the point,

$$h\Delta\psi = -\frac{hb}{P_b dD}\Delta P_b. \tag{5}$$

where w is the lateral distance from the edge of plenum chamber to the lowest point of finger, and h is the height of the ceil in the plenum chamber.

2.2 Deformation due to the Pressure in the Plenum Chamber

As the pressure in the plenum chamber P_c increases, the bag will deform in the upward direction. Suppose the pressure P_c increases by an amount of ΔP_c and the bag deforms by $\Delta\psi$. The upward force acting on the bag is

$$\Delta P_c(w + l_s\cos\psi),$$

and the force on the finger is

$$-\Delta P_c l_s \cos\psi + P_c l_s \sin\psi\Delta\psi.$$

And the restoring force of the bag is as follows,

$$P_b d\Delta\psi.$$

Similarly, we can obtain the angular displacement $\Delta\psi$ as follows,

$$\Delta\psi = \frac{w}{P_b dD}\Delta P_c. \tag{6}$$

Among the forces acting on the bag, $w\Delta P_c$ will be included in the force on the pressurized support area, so we omit the term $w\Delta P_c$ here. Therefore the upward force acting on the skirt system due to the pressure increase ΔP_c in the plenum chamber can be represented as follows,

$$P_b d\Delta\psi - w\Delta P_c = w\left(\frac{1}{D} - 1\right)\Delta P_c. \tag{7}$$

The vertical displacement of the lowest point of the finger is

$$w\Delta\psi = \frac{w^2}{P_b dD}\Delta P_c, \tag{8}$$

and the horizontal displacement of the point,

$$h\Delta\psi = \frac{hw}{P_b dD}\Delta P_c. \tag{9}$$

(Positive sign means the outward direction.)

2.3 Deformation due to the Free Surface Elevation

As the surface below the skirt system moves upward, the finger touches the surface and the downward force acting on the finger decreases. In consequence, the bag experiences upward force and the bag deforms upward and lift up the finger. If the skirt system does not move, the upward force on the skirt due to the surface elevation can be represented as

$$P_c \frac{h_w}{\tan\psi}, \tag{10}$$

where h_w is the elevation of the surface. Suppose the bag deforms by an amount of angular deformation $\Delta\psi$, the restoring force of the bag is

$$P_b d \Delta\psi.$$

Furthermore when the lower part of the finger Δl touches the surface, the upward force acting on the finger is as follows,

$$P_c(l_s \sin\psi\Delta\psi + \cos\psi\Delta l).$$

We know that Δl and $\Delta\psi$ have the following relation by inspecting the skirt geometry.

$$\Delta l = \frac{1}{\sin\psi}(h_w - w\Delta\psi), \tag{11}$$

where Δl must be positive. Thus $\Delta\psi$ must satisfy the following condition,

$$\Delta\psi \leq \frac{h_w}{w}. \tag{12}$$

From the equilibrium condition of forces, we can obtain $\Delta\psi$ as follows,

$$\Delta\psi = \frac{P_c \frac{h_w}{\tan\psi}}{P_b d - P_c l_s \sin\psi + P_c \frac{w}{\tan\psi}}. \tag{13}$$

In order to satisfy the condition (12), the following condition must be satisfied,

$$P_b d - P_c l_s \sin\psi \geq 0.$$

Above condition may be rewritten as

$$D = 1 - \frac{P_c l_s \sin\psi}{P_b d} \geq 0. \tag{14}$$

If the above condition is not satisfied, there is no way to satisfy the equilibrium condition of forces, so the bag continues to undergo a deformation. However, this will not happen in real situation, this just means that the skirt is in its unstable equilibrium state. When D equals to unity, the deformation of skirt due to the surface elevation will be small. And when D equals to zero, the skirt deforms in the way in which the lowest point of the finger always remains on the surface. As D becomes smaller, the skirt will deform more easily due to the surface elevation. One may choose D as the criterion of the skirt responsiveness.

We may rewrite the angular displacement of the bag due to the surface elevation,

$$\Delta\psi = \frac{P_c \frac{h_w}{\tan\psi}}{P_b d D + P_c \frac{w}{\tan\psi}}.$$

The upward force acting on the skirt system is

$$\begin{aligned} P_b d \Delta\psi &= P_c \frac{h_w}{\tan\psi} \frac{P_b d}{P_b d D + P_c \frac{w}{\tan\psi}} \\ &= P_c \frac{h_w}{\tan\psi}[1 + D'], \end{aligned} \tag{15}$$

where D' is defined as follows,

$$D' = \frac{1 - D - \frac{P_c}{P_b}\frac{w}{d}\frac{1}{\tan\psi}}{D + \frac{P_c}{P_b}\frac{w}{d}\frac{1}{\tan\psi}}. \tag{16}$$

Comparing this with (10), the skirt force is increased by the skirt response-force factor D' in the present model.

3. MATHEMATICAL MODEL

The heave and pitch responses of an Air Cushion Vehicle with a bag-finger skirt travelling at a speed of advance U in regular waves are examined. The coordinate system and the craft are shown in Figure 2.

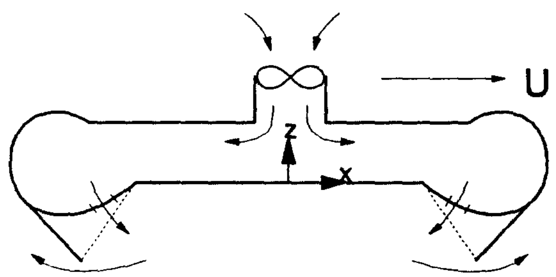

Figure 2: Model of a craft and coordinate system

The origin of coordinate lies at midship and vertically at the top of plenum chamber.

3.1 Air Flows

The air pressure and density changes are assumed to have the adiabatic isentropic relationship, *i.e.*,

$$\rho_c = \rho_a \left(1 + \frac{P_c}{P_a}\right)^{1/\gamma}, \tag{17}$$

where ρ_a and P_a are density and pressure of air in atmospheric condition respectively, and ρ_c and P_c are in the plenum chamber, γ is the ratio of specific heat and is taken as 1.4.

Under the assumption that the air is compressible, and the pressure is uniform throughout the volume at any instant in time, the conservation of mass in the plenum chamber may be written as

$$\frac{d}{dt}(\rho_c V_c) = \dot{\rho}_c V_c + \rho_c \dot{V}_c = \rho_c Q_i - \rho_c Q_e, \tag{18}$$

where V_c is the volume of the plenum chamber, Q_i and Q_e the volumetric flow rate entering into and exiting from the plenum chamber respectively. And the dot above a variable means a derivative with respect to time. With equation (17), the above equation may be rewritten as

$$\frac{V_c}{\gamma(P_c + P_a)} \dot{P}_c = Q_i - Q_e - \dot{V}_c. \tag{19}$$

In a similar way, the conservation of mass in the bag and duct may be written as

$$\frac{V_d}{\gamma(P_b + P_a)} \dot{P}_b = -\left(\frac{1 + P_c/P_a}{1 + P_b/P_a}\right)^{\delta} Q_i + Q_f, \tag{20}$$

where V_d and P_b are the volume and pressure in the bag and duct, respectively. Q_f denotes the inlet volume flow rate into the bag and duct, and δ is a reciprocal of γ.

The flow rate though the fan Q_f and the pressure difference across the fan P_f (this is normally P_b in the absence of the secondary duct, hence P_f will be the same as P_b hereafter) are assumed to have the relation below.

$$P_f = C_1 + C_2 Q_f + C_3 Q_f^2, \tag{21}$$

where C's are constants given from experiments of the fan characteristics.

The volumetric flow rates are assumed to be governed by the steady orifice flow law, then Q_i and Q_e can be represented as follows,

$$Q_i = \left(\frac{1 + P_b/P_a}{1 + P_c/P_a}\right)^{\delta} k A_i \sqrt{2(P_b - P_c)/\rho_a(1 + P_b/P_a)^{\delta}}, \tag{22}$$

$$Q_e = k A_e \sqrt{2P_c/\rho_a(1 + P_c/P_a)^{\delta}}. \tag{23}$$

where k is an orifice flow discharge coefficient, A_i the inlet orifice area into the plenum chamber and A_e the escape area under the skirt.

The rate of change of the plenum chamber is assumed to be

$$\dot{V}_c = A_c\{\dot{z} - x_c\dot{\theta}\} - \dot{V}_{cpw} + \dot{V}_w + \dot{V}_s, \tag{24}$$

where A_c is the pressurized support area, x_c the centroid of A_c, and z and θ denote the heave and pitch displacements. And V_{cpw} is the volume change due to the free surface deformation (will be given in the next section), V_w due to incident waves and V_s due to skirt deformations. The escape area under the skirt may be written as

$$A_e = A_{eo} + \eta_s \int_l (z - x\theta - \zeta_{pw} - \zeta_w)\, dl + \eta_s A_{es}, \tag{25}$$

where A_{eo} denotes the escape area at the equilibrium state, ζ_{pw} and ζ_w are the free surface elevation due to the cushion pressure and the incident waves, respectively and where A_{es} is the escape area due to the skirt deformation. The integral has to be carried out along the cushion perimeter. Since the escape area does not change proportionally to the relative motion responses, η_s is introduced to evaluate the escape area properly.

The deviations of the variables from their equilibrium values are assumed to be small, the equilibrium values are denoted by placing subscript 'o', otherwise deviations henceforth. We linearize the inlet flow as follows,

$$Q_i = Q_{ipc}P_c + Q_{ipb}P_b, \tag{26}$$

where

$$\begin{aligned} Q_{ipc} &= -\frac{\rho_{do}}{\rho_{co}} kA_i \sqrt{\frac{2(P_{bo} - P_{co})}{\rho_{do}}} \left(\frac{\delta}{P_{co} + P_a} + \frac{1}{2(P_{bo} - P_{co})} \right), \\ Q_{ipb} &= \frac{\rho_{do}}{\rho_{co}} kA_i \sqrt{\frac{2(P_{bo} - P_{co})}{\rho_{do}}} \left(\frac{\delta}{2(P_{bo} + P_a)} + \frac{1}{2(P_{bo} - P_{co})} \right). \end{aligned}$$

And,

$$\begin{aligned} Q_i' &= kA_i \sqrt{\frac{2(P_b - P_c)}{\rho_d}} \\ &= Q_{ipc}' P_c + Q_{ipb}' P_b, \end{aligned} \tag{27}$$

where

$$\begin{aligned} Q_{ipc}' &= -kA_i \sqrt{\frac{2(P_{bo} - P_{co})}{\rho_{do}}} \frac{1}{2(P_{bo} - P_{co})}, \\ Q_{ipb}' &= kA_i \sqrt{\frac{2(P_{bo} - P_{co})}{\rho_{do}}} \left(\frac{1}{2(P_{bo} - P_{co})} - \frac{\delta}{2(P_{bo} + P_a)} \right). \end{aligned}$$

And the escape area,

$$A_e = A_{ez}z + A_{e\theta}\theta + A_{epc}P_c + A_{epb}P_b + A_{ew}\zeta, \tag{28}$$

where ζ is the amplitude of the incident wave and

$$\begin{aligned} A_{ez} &= \eta_s \int_l dl, \\ A_{e\theta} &= -\eta_s \int_l x dl, \end{aligned}$$

$$\begin{aligned} A_{epc} &= \eta_s \int_l \frac{w^2}{P_{bo} dD} dl - \eta_s \int_l \zeta_{pw} dl, \\ A_{epb} &= -\eta_s \int_l \frac{wb}{P_{bo} dD} dl, \\ A_{ew} &= -\eta_s \int_l \zeta_w dl. \end{aligned}$$

ζ_{pw} and ζ_w are the surface elevations due to the cushion pressure and of the incident wave, respectively. And the outlet flow becomes

$$Q_e = Q_{ez} z + Q_{e\theta}\theta + Q_{epc} P_c + Q_{epb} P_b + Q_{ew}\zeta, \tag{29}$$

where

$$\begin{aligned} Q_{ez} &= k\sqrt{2P_{co}/\rho_{co}} A_{ez}, \\ Q_{e\theta} &= k\sqrt{2P_{co}/\rho_{co}} A_{e\theta}, \\ Q_{epc} &= k\sqrt{2P_{co}/\rho_{co}} \left(A_{epc} + A_{eo} \left[\frac{1}{2P_{co}} - \frac{\delta}{2(P_{co} + P_a)} \right] \right), \\ Q_{epb} &= k\sqrt{2P_{co}/\rho_{co}} A_{epb}, \\ Q_{ew} &= k\sqrt{2P_{co}/\rho_{co}} A_{ew}. \end{aligned}$$

And the time rate of volume change is

$$\dot{V}_c = V_z \dot{z} + V_\theta \dot{\theta} + V_{pc} \dot{P}_c + V_{pb} \dot{P}_b + V_w \dot{\zeta}, \tag{30}$$

where

$$\begin{aligned} V_z &= A_c - \eta_s \int_l (w^2 + h^2) \frac{P_{co}(1 + D')}{P_{bo} d \tan\psi} dl, \\ V_\theta &= -A_c x_c + \eta_s \int_l (w^2 + h^2) \frac{P_{co}(1 + D')}{P_{bo} d \tan\psi} x dl, \\ V_{pc} &= -V_{cpw} + \int_l (w^2 + h^2) \frac{w}{P_{bo} dD} dl + \eta_s \int_l (w^2 + h^2) \frac{P_{co}(1 + D')}{P_{bo} d \tan\psi} \zeta_{pw} dl, \\ V_{pb} &= -\int_l (w^2 + h^2) \frac{b}{P_{bo} dD} dl, \\ V_w &= -V_{cw} + \eta_s \int_l (w^2 + h^2) \frac{P_{co}(1 + D')}{P_{bo} d \tan\psi} \zeta_w dl, \end{aligned}$$

where V_{cpw} is the volume change due to the free surface deformation excited by the cushion pressure and V_{cw} due to the incident wave (both will be given in the next section).

3.2 Free Surface Deformation

The deformation of free surface due to the cushion pressure is obtained by Rhee and Lee[10]. We use their results, and just write down only the assumptions and methods of calculation.

The water is assumed to be incompressible and inviscid, and the depth of water is infinite. The cushion pressure is replaced with the pressure patch which oscillates and translates with a constant speed on the otherwise calm water. The shape of the pressure patch is restricted within a polygonal one. The free surface deformation is obtained in the framework of linear potential theory. The resulting free surface deformation is proportional to the pressure acting, *i.e.*

$$\zeta_{pw} \cdot P_c$$

The free surface elevation of the regular head waves incoming from the positive x-axis is

$$\zeta_a e^{i(k[x+Ut]+\omega t)} = \zeta_a e^{i(kx+\omega_e t)} = \zeta_w \cdot \zeta, \tag{31}$$

where

$$\omega_e = \omega + Uk \quad , \quad k = \omega^2/g$$
$$\zeta_w = e^{ikx} \quad , \quad \zeta = \zeta_a e^{i\omega_e t}$$

Here ζ_a is a wave amplitude, k is the wave number, ω a circular frequency of incoming wave and ω_e an encountering frequency.

The escape area due to the free surface deformation is obtained by integrating the free surface elevations along the skirt perimeter, and the volume changes V_{cpw}, V_{cw} are obtained by integrating the free surface elevation over the cushion area. The integration over the cushion area is transformed to the integral along the skirt perimeter by use of Stoke's theorem.

3.3 Skirt Forces

In section 2, the force acting on the skirt is analyzed locally. Each element of the skirt is assumed to move independently, and the frictional force due to the contact of the finger with the water surface is neglected.

Forces and moment of the skirt system can be obtained by integrating the local forces as

$$F_s = \int_l dF_V,$$
$$M_s = -\int_l x dF_V + \int_l (z_G + h)\sin\beta dF_H, \tag{32}$$

where dF_V and dF_H are the vertical and horizontal force component of the skirt system, respectively and β is the angle of skirt hemline to the positive x-direction.

From the results in section 2, the vertical force of the skirt system can be obtained as follows,

$$F_s = F_z z + F_\theta \theta + F_{pc} P_c + F_{pb} P_b + F_w \zeta, \tag{33}$$

where

$$F_z = -\eta_s P_{co} \int_l \frac{1+D'}{\tan\psi} dl$$
$$F_\theta = \eta_s P_{co} \int_l \frac{1+D'}{\tan\psi} x dl$$

$$\begin{aligned} F_{pc} &= \int_l \frac{w}{D} dl + \eta_s P_{co} \int_l \frac{1+D'}{\tan\psi} \zeta_{pw} dl \\ F_{pb} &= -\int_l \frac{b}{D} dl \\ F_w &= \eta_s P_{co} \int_l \frac{1+D'}{\tan\psi} \zeta_w dl. \end{aligned}$$

The bow-down pitching moment is

$$M_s = M_z z + M_\theta \theta + M_{pc} P_c + M_{pb} P_b + M_w \zeta, \tag{34}$$

where

$$\begin{aligned} M_z &= \eta_s P_{co} \int_l \frac{1+D'}{\tan\psi} x dl - \eta_s P_{co} \int_l (z_G + h) \sin\beta dl \\ M_\theta &= -\eta_s P_{co} \int_l \frac{1+D'}{\tan\psi} x^2 dl - \eta_s P_{co} \int_l (z_G + h) \sin\beta x dl \\ M_{pc} &= -\int_l \frac{w}{D} x dl - \eta_s P_{co} \int_l \frac{1+D'}{\tan\psi} \zeta_{pw} x dl + \eta_s P_{co} \int_l (z_G + h) \sin\beta \zeta_{pw} dl \\ M_{pb} &= \int_l \frac{b}{D} x dl \\ M_w &= -\eta_s P_{co} \int_l \frac{1+D'}{\tan\psi} \zeta_w x dl + \eta_s P_{co} \int_l (z_G + h) \sin\beta \zeta_w dl, \end{aligned}$$

where z_G is the vertical position of the center of gravity.

3.4 Equations of Motions

The heave and pitch equations of motion about the origin of coordinate is

$$m\ddot{z} - m x_G \ddot{\theta} = A_c P_c + F_s, \tag{35}$$
$$I\ddot{\theta} - m x_G \ddot{z} = -A_c x_c P_c + M_s, \tag{36}$$

where m is the mass of the craft and I the moment of inertia, x_G the longitudinal position of the center of gravity.

The equations of conservation of mass in the plenum chamber and the bag and duct are

$$C_{pc} \dot{P}_c = Q_i - Q_e - \dot{V}_c, \tag{37}$$
$$C_{pb} \dot{P}_b = -Q_i' + Q_f, \tag{38}$$

where

$$C_{pc} = \frac{\delta V_c}{P_{co} + P_a} \quad , \quad C_{pb} = \frac{\delta V_d}{P_{bo} + P_a}. \tag{39}$$

The craft motions can be obtained by solving these four equations simultaneously. Using the variables appeared in preceding sections, we may rewrite the equations of motions as follows,

$$V_z\dot{z} + Q_{ez}z + V_\theta\dot{\theta} + Q_{e\theta}\theta + (C_{pc} + V_{pc})\dot{P}_c + (Q_{epc} - Q_{ipc})P_c$$
$$+V_{pb}\dot{P}_b + (Q_{epb} - Q_{ipb})P_b = -V_w\dot{\zeta} - Q_{ew}\zeta, \quad (40)$$

$$Q'_{ipc}P_c + C_{pb}\dot{P}_b + \left(Q'_{ipb} - \frac{1}{C_2 + 2C_3Q_{fo}}\right)P_b = 0, \quad (41)$$

$$m\ddot{z} - F_z z - mx_G\ddot{\theta} - F_\theta\theta - (A_c + F_{pc})P_c - F_{pb}P_b = F_w\zeta, \quad (42)$$

$$-mx_G\ddot{z} - M_z z + I\ddot{\theta} - M_\theta\theta + (A_c x_c - M_{pc})P_c - M_{pb}P_b = M_w\zeta. \quad (43)$$

Since the incident wave is sinusoidal in time, we assume the motions are sinusoidal also, thus the above equations turn into the simultaneous algebraic equations.

4. NUMERICAL RESULTS

To investigate the effects of a skirt deformation on the motion response of an ACV in waves, the heave and pitch response of a Plenum-Chamber Type ACV with a bag-finger type skirt in regular head waves have been calculated by using a linear theory. The schematic views of the craft and the bag-finger skirt adopted for the present computations are shown in Figure 1 and 2, and the principal particulars are shown in Table 1 and 2. The craft is assumed to have a constant speed in waves, and the motion responses are calculated in a frequency domain at cushion length based Froude numbers of 1.0 and 1.5. The cushion length of the craft is 20 *m*. In all figures, the motion responses of the craft having same principal particulars in Table 1, but with a rigid skirt, are shown as a solid line for a reference. The heave responses were nondimensionalized by the incident wave amplitude and the pitch responses by the maximum slope of the incident wave.

Table 1. Principal particulars of the craft and coefficients

B/L	0.5	A_i/L^2	0.01
$m/\rho_w L^3$	0.006	V_d/L^3	0.0125
$I/\rho_w L^5$	3.25×10^{-4}	k	0.6
z_G/L	0.05	$C_1/\rho_w g L$	0.04
x_G/L	0	$C_2\sqrt{L^3/g\rho_w^2}$	0.0
h/B	0.1	C_3L^4/ρ_w	-30.

Table 2. Particulars of the bag-finger skirt

l_s/B	0.07	w/B	0.02
b/B	0.05	ψ	45°
d/B	0.1	η_s	1

In Figures 3 and 4, the effect of a ratio of bag to cushion pressure on the motion response was shown. In the calculation, the ratio of bag to cushion pressure was obtained by changing the inlet area. The pressure in the bag, P_b, 6257, 4369, 3484 (N/m^2) were corresponded to the nondinmensionalized inlet area, A_i/L^2, 0.005, 0.01, 0.015, respectively, while the cushion pressure was kept a constant value of 2412 (N/m^2). And the skirt responsiveness factor

D were 0.8092, 0.7267, 0.6573 and the skirt response-force factor D' 0.1283, 0.1946, 0.2566 respectively. The motion response increased as the pressure in the bag increased and as D' decreased.

In Figures 5 and 6, the motion responses were calculated for several values of d/B. The calculations were carried out for the values of 0.07, 0.10, 0.13, and D and D' could be obtained as 0.6096, 0.7267, 0.7898 and 0.3032, 0.1946, 0.1432 respectively. The motion responses did not change significantly for this case.

In Figures 7 and 8, the motion responses were calculated with various b/B's. For the values b/B, 0.02, 0.05, 0.08 were used, and in this case D and D' were not changed and their values were 0.7267 and 0.1946 respectively. It can be known that the motion response increases as b/B decreases. However, since D and D' is not affected by b/B, the new formulation for D and D' needed to include this effect.

Figures 9 and 10 showed the effect of w/B on the motion responses. In this case, D could not be changed and D' had the values of 0.3761, 0.1946, 0.05534 according to the different values of w/B of 0.0, 0.02,0.04, respectively. It can be known that the motion response increases as w/B increases.

Above results may be summarized as follows: The skirt response-force factor D' can be used to predict the craft motion. The motion response grows large as D' decreases, and the values of D' can be easily changed by changing the values of w/B.

On the other hand, the skirt responsiveness factor D can be used to predict the skirt deformation, as D decreases, the skirt deformation becomes large, and a negative value of D indicates that the unstable skirt deformation may happen.

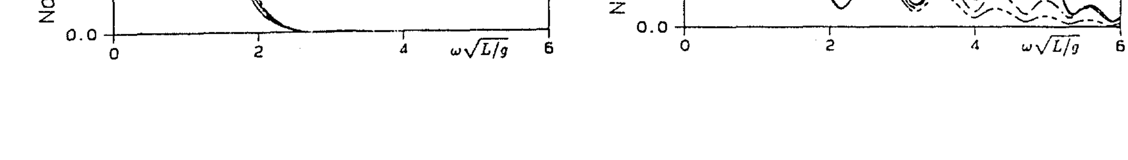

Figure 3: Motion response with various A_i/L^2's at $F_n = 1.0$. Solide line is for the rigid skirt

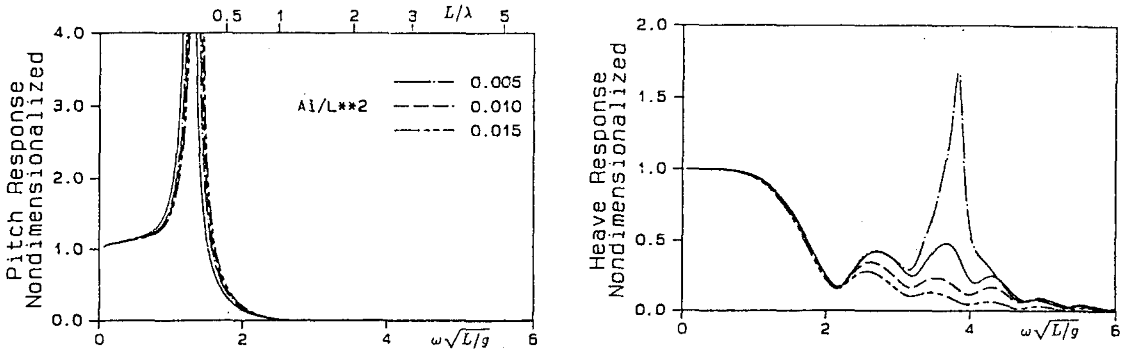

Figure 4: Motion response with various A_i/L^2's at $F_n = 1.5$. Solide line is for the rigid skirt

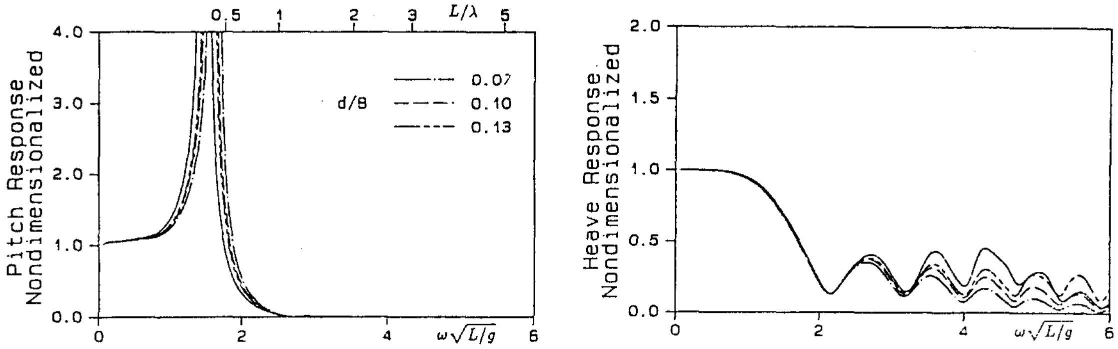

Figure 5: Motion response with various d/B's at $F_n = 1.0$. Solide line is for the rigid skirt

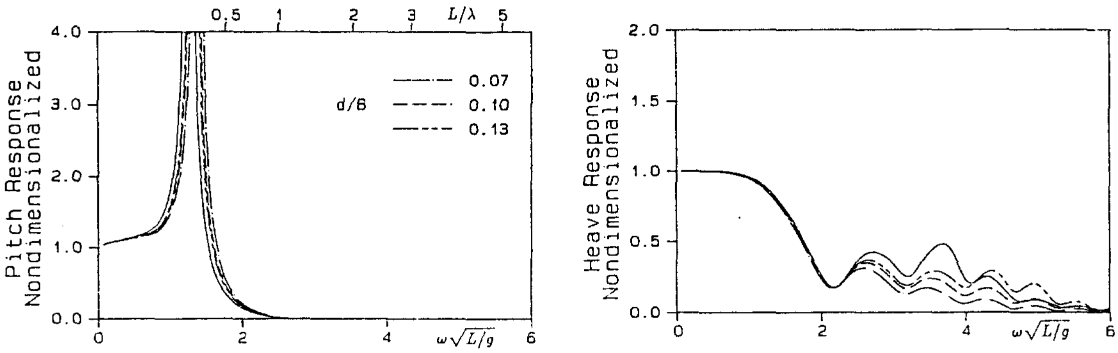

Figure 6: Motion response with various d/B's at $F_n = 1.5$. Solide line is for the rigid skirt

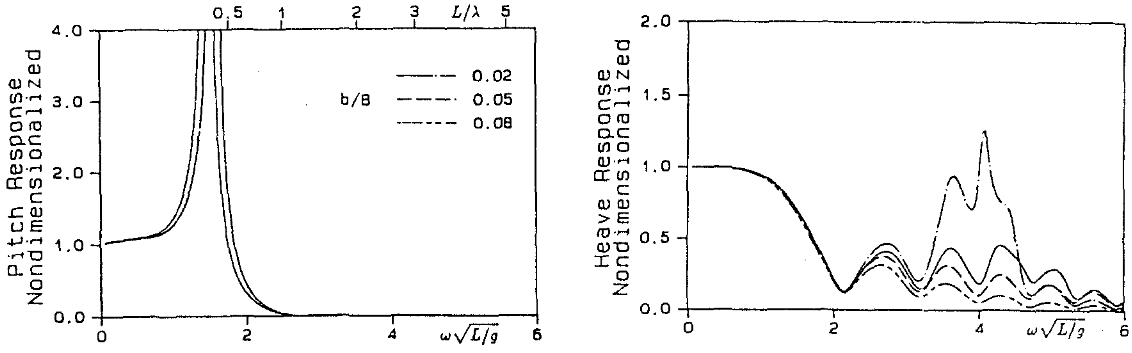

Figure 7: Motion response with various b/B's at $F_n = 1.0$. Solide line is for the rigid skirt

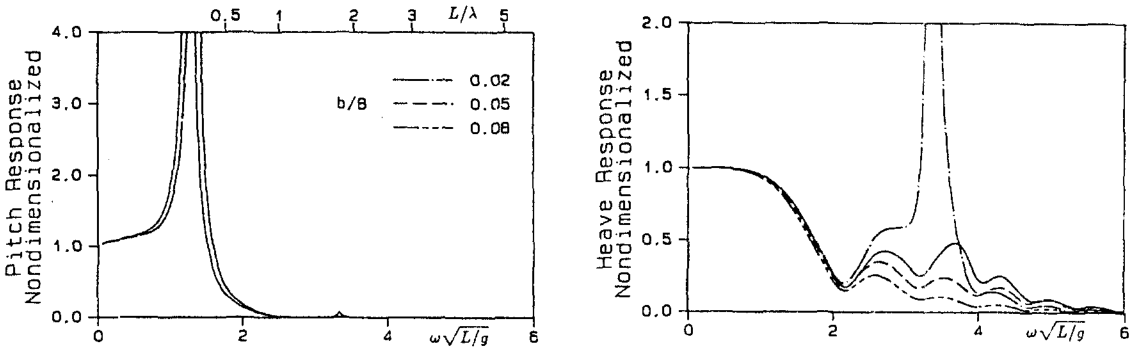

Figure 8: Motion response with various b/B's at $F_n = 1.5$. Solide line is for the rigid skirt

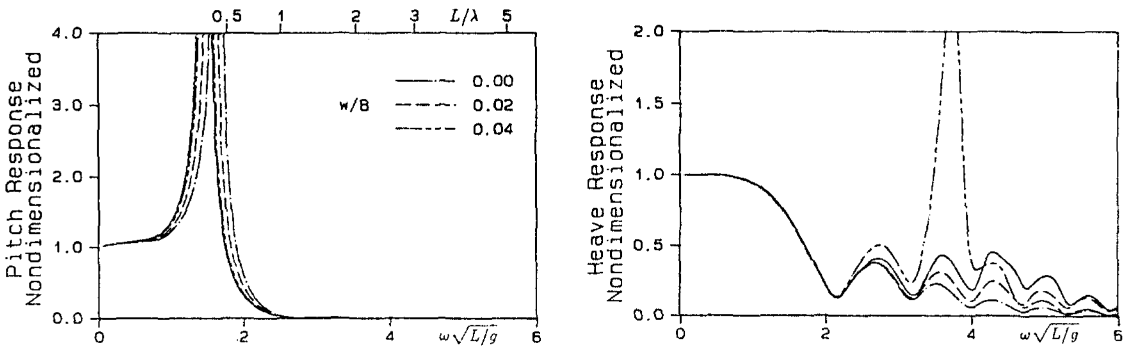

Figure 9: Motion response with various w/B's at $F_n = 1.0$. Solide line is for the rigid skirt

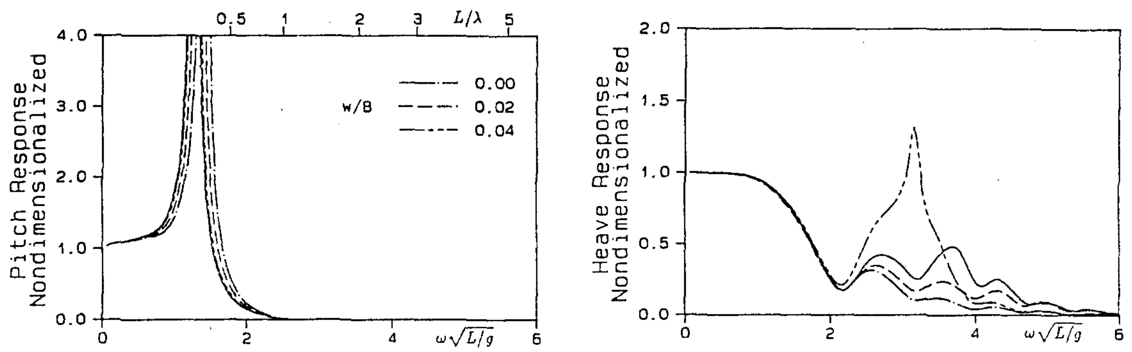

Figure 10: Motion response with various w/B's at $F_n = 1.5$. Solide line is for the rigid skirt

5. CONCLUSIONS

A simplified model has been developed to explain the skirt deformation due to the pressure change in the bag and plenum chamber and due to the water surface elevation. This model can also predict the skirt force.

A series of numerical calculations for the craft motion has been carried out for the heave and pitch response with different values of inlet area and skirt parameters. Through this investigation, our findings are:

1. The motion response grows large as the skirt response-force factor D' decreases.

2. To reduce the motion response by increasing D', one should increase the inlet area, or decrease d/B or w/B.

3. The skirt deforms more easily as the skirt responsiveness factor D decreases. To reduce D, the inlet area should be increased or d/B should be decreased. is needed.

It is concluded that the skirt responsiveness factor D and the skirt response-force factor D' can be used to predict of the skirt responsiveness and the craft motion respectively. It is desirable to compare the present results with experimental measurements in the future. It is also desirable to investigate on additional factors including the effect of b/B.

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APPENDIX: The Restoring Coefficient of a Bag

Consider the simplified model of a bag shown below.

Since the bag is attached to the structure with hinge, only the force on the bag is transmitted to the structure. The lateral force acting on the bag can be represented as follows,

$$T(\cos\psi_2 + \cos\psi_1) = d(P_b - P_0). \tag{A.1}$$

where T is the tensile force of the bag and can be considered constant along the bag, and d is the distance between two points that meet the structure. The upward force acting on the bag can be represented as follows,

$$T(\sin\psi_2 - \sin\psi_1) = F_0. \tag{A.2}$$

When the upward force changes with an amount of ΔF, and in consequence ψ_1, ψ_2 change with an amount of $\Delta\psi_1, \Delta\psi_2$ respectively, then the upward force becomes

$$F_0 + \Delta F = T(\sin(\psi_2 + \Delta\psi_2) - \sin(\psi_1 + \Delta\psi_1)). \tag{A.3}$$

Thus ΔF becomes

$$\begin{aligned} \Delta F &= T\left(\sin(\psi_2 + \Delta\psi_2) - \sin(\psi_1 + \Delta\psi_1) - \sin\psi_2 + \sin\psi_1\right) \\ &= T\left(\cos\psi_2\Delta\psi_2 - \cos\psi_1\Delta\psi_1\right). \end{aligned} \tag{A.4}$$

If we assume $\Delta\psi_2 = -\Delta\psi_1 = \Delta\psi$, then

$$\Delta F = T(\cos\psi_2 + \cos\psi_1)\Delta\psi. \tag{A.5}$$

Substituting (A.1) into (A.5), we obtain

$$\Delta F = d(P_b - P_0)\Delta\psi \tag{A.6}$$

Thus the restoring coefficient of a bag with respect to the angular deformation can be written as follows,

$$\frac{dF}{d\psi} = d(P_b - P_0). \tag{A.7}$$