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DESIGNING A PROGRAM FOR MATHEMATICALLY GIFTED STUDENT

Wan Kang(Seoul National University of Education)

INTRODUCTION

Although many educators emphasize the importance of education for gifted students, we still don't know much about how to teach or guide them effectively. Especially for the gifted students within a certain subject area such as mathematics, the situation is more serious. One possible reason for such a situation is that we have rare opportunity to contact mathematically gifted students. In fact, discriminating one's gift in mathematics itself is a very complicated process. It may take a little long time to observe and distinguish one's mathematical behavior to be superior innately. Sometimes, actual processes of teaching mathematics may be necessary to discover one's innate superiority in mathematics.

In Korea, several science high schools were established for the gifted students who wish to be scientists. Because groups of gifted students are already gathered in those science high schools, one may think that there is little difficulty for researchers to study on the way of teaching mathematics to gifted students. It would be better, however, for researchers and educators if we have some program to help the mathematically gifted students earlier than when they enter high schools.

The Gifted Education Institute of Korea (GEIK) has been conducting a special program, Mathematics Acceleration Project (MAP) for mathematically gifted students since 1987. The purpose of this study was to diagnose the project to detect substantial problems in it. This study also intended to suggest alternative ways of supporting the mathematically gifted children.

A BRIEF LOOK-IN AT MAP

The First Stage of MAP

The predecessor of the present GEIK was a research institute attached to Korean Association for the Gifted Children (KAGC). In 1985, the late Dr. Yun-Tai Chung started, the KAGC program (cf. Chung, Moon, & Kim, 1990), a thinking ability program for gifted children at his house in Bongchon-dong, Seoul. The KAGC program was a kind of weekly meeting of two hours with a teacher and 3 to 6 gifted children to have activities developed by Dr. Chung and his staffs. With staffs of the program, parents of the children who participated in the program, and some influential persons, Dr. Chung organized the KAGC in 1987. KAGC moved its office to Daechi-Dong, Seoul in July 1989, and opened its own research institute.

In May 1987, in process of the thinking ability program, KAGC began to support individual programs for the children who are talented in specific areas such as mathematics. At first, there were three students whose mathematical talent were recognized. In that fall, the number of students who participated in the individual program in mathematics increased to 4, and it became 18 in August 1988. At that time, many students in the individual program were studying mathematics of levels far over their own school grades. For example, a fourth grader was studying mathematics of the 11th grade level.

The Second Stage of MAP

Recognizing the need of more systematic ways of helping the gifted childrens' study of mathematics, instead of the individual program, KAGC started a separate mathematics program by referring to the Johns Hopkins University's project, the Study of Mathematical Precocious Youth (SMPY).

At the beginning, KAGC's new mathematics program was provided in two ways. The first way was a continuation of the individual program for the early graders with the contents of elementary school mathematics. In this way, three to five children formed a team. Each team studied mathematics in 40 to 50 minutes per a weekly meeting of 2 hours, as a part of the KAGC's thinking ability program. Using elementary school mathematics textbooks, a teacher of the team gave the children homework, and discussed about the children's work at the next meeting. After studying each mathematics textbooks, children were given test. Only the children who shows 80% completion for the test were allowed to study mathematics of the next level. Other children had to take a test again after studying more about mathematics of the same level.

The second way was a separate study of mathematics for junior and senior high school level (7th to 12th grade level mathematics), not including activities of the KAGC's thinking ability program. At a two hours meeting weekly, a team of 2 or 3 students of grade 3rd to 8th solved mathematics problems with their teacher's guide. Most students of this level in the mathematics program showed that they were interested in problems requiring logical inference more than simple calculation. They used mathematics problem books which were known popularly among Korean high school students.

KMO and MAP

In summer 1989, the Korean Mathematical Olympiad (KMO) was held at the Seoul National University by the Korean Mathematical Society (KMS). As a rule, KMS allowed only the 10th and 11th graders who had been recommended by their high school principals to participate the KMO. KMS, however, gave consideration for MAP students to participate in the KMO. Thus, the KAGC recommended five MAP students who were 5th to 9th graders to the 1989 KMO. All five MAP students participated in the KMO made good records, and one of them received an encouragement award from the KMS. Being inspired with this fact, KMS allowed MAP students to enter the KMO winter workshop in January 1990, and four of the five KMO participants entered the workshop.

Next year, KAGC recommended four MAP students of grade 4 to 9 to participate in the 1990 KMO in which 1076 students in total participated. Two of the four MAP students received awards: one who was an 8th grader received a bronze medal, and the other who was only a 6th grader received an honorary award from KMS.

Since the 1991 KMO, the sponsorship of KMO was transferred from KMS to the Ministry of Education. Because the policy of KMO was also changed not to allow students under 10th grade to participate KMO, participation of MAP students has been not granted since then. Participation of MAP students in KMO winter workshop sponsored by KMS, however, has been being continued.

The Third Stage of MAP

Increasing the number of participants in the mathematics program, KAGC decided to launch a more systematic program for the mathematically gifted students. Consequently, in June 1991, the mathematics program was renamed Mathematics Acceleration Project (MAP), and the author of this article joined to MAP as a chief staff. The main enterprises that MAP pursued originally were as follows: (1) To pick out gifted children who were motivated to study mathematics as early as possible; (2) To check the width and depth of the gifted children's understanding of mathematics they have learned, and to modulate the pace of their studying mathematics in order for them to study mathematics effectively; (3) To hold workshops for the gifted children to develop their ability of solving mathematical problems; (4) To develop materials and methods of teaching mathematics to the gifted students especially in the level of high school mathematics.

One of the features that distinguish MAP activities after 1991 from those before 1991 was a test system that confirm each MAP student's understanding of mathematics he or she had learned. At present, about 30 student and 10 teachers are participating in MAP. Two or three students make a team, and each team has a 2 hours meeting weekly with a teacher.

RESEARCH DESIGN

Observation and Interview

Because most students in MAP were very busy to have many activities besides the MAP weekly meetings, it was very hard to have them to attend at extra meetings for investigating their behavior. Thus, as a sort of teaching experiment, this study was done by collecting and analyzing data from observations, interviews and MAP test results.

Observed were behaviors of the students when they learn and interact with their teachers at the weekly meetings. Sometimes, if the student's teacher allowed the researcher to join with their studying, the researcher could have chances to ask the students about what they were learning and how they understand it.

Interviews with the students and their parents formed one of the major part of source data for this research. Some contents of the dialogue were recorded during the interview. Mostly, the mother of the student was an interviewee. Each interview proceeded about 30 minutes. In the beginning of each interview, the student sat together with his or her parent (or parents) about 5 minutes and talked about how he or she likes and studies mathematics. The parent and the interviewer talked about the growth process of the student and about possible difficulties that the student encountered or may encounter in his or her studying mathematics. Sometimes, the student's teacher participated in the interview.

MAP Test

To test the students' understanding of mathematics they had learned in MAP, the level and major content of mathematics the students should learn were set previously as follows:

E4, E5, E6: elementary school mathematics for the 4th, 5th and 6th grade respectively

M1: middle school mathematics (7th grade)

M1-A (sets, numbers, variables, expressions, linear equations)

M1-B (functions, plane figures, solid figures, Euler's formula)

M2: middle school mathematics (8th grade)

M2-A (operations of numbers and expressions, equalities and inequalities, linear functions)

M2-B (probabilities, congruence of figures, similar figures)

M3: middle school mathematics (9th grade)

M3-A (operations of numbers and expressions, quadratic equations, quadratic functions)

M3-B (statistics, Pythagorean theorem, sine, cosine, tangent, properties of circles)

HG: high school mathematic for general studies

HG-A (sets, propositions, number systems, mathematical expressions, equations, inequalities)

HG-B (exponential, logarithms, rational functions, irrational functions, exponential functions, logarithmic functions)

HG-C (trigonometry, analytic geometry, region of inequalities)

HS: high school mathematics for science studies

HS-A (equations, inequalities, matrices, number sequences, complex numbers, trigonometry)

HS-B (limits of sequences and functions, differentials, integral)

HS-C (figures in space, analytic geometry, vectors, probabilities)

Tests were given for all MAP students in every 4 months. After rating the answers of the students, the MAP staffs decided the pass for each student to the next level of mathematics. If a student pass a level, he or she get an index of progress according to the following:

E4 (4.0)	E5 (5.0)	E6 (6.0)
M1-A (6.5)	M1-B (7.0)	
M2-A (7.5)	M2-B (8.0)	
M3-A (8.5)	M3-B (9.0)	
HG-A (9.3)	HG-B (9.7)	HG-C (10.0)
HS-A (10.3)	HS-B (10.7)	HS-C (11.0)

An index of accelerated progress was calculated as followed:

(index of progress) - (school grade year the student finished)

For example, if a 5th grade student passes the M3-A level mathematics test, the student gets 8.5 - 4 = 4.5 points as an index of accelerated progress.

MAP staffs used the index of accelerated progress only for the purpose of adjusting the student's pace of studying mathematics at MAP and not for any other purposes such as to measure the student's ability to study mathematics. The researcher, on the other hand, used the indexes collectively for the purpose of interpreting the appropriateness of the mathematical activities in the program, which was also a part of this research.

RESULTS

From the Interviews

In most cases, parent informed that the child's mathematical talent were shown in the early stage through his or her strong interest in counting or operating numbers. This feature of mathematically gifted children in MAP coincides with the Krutetskii's findings (cf. Krutetskii,1976, p. 180-223). The following illustration shows how a gifted can calculate big numbers.

Student C was born in 1986. When he was interviewed, he was 7 years old and a 1st grader. He said he liked to read a book titled "An Interesting Mathematical Voyage." He also said he was studying mathematical facts such as the number of diagonals in a polygon, proportional expressions, similarity of triangles. When he was asked what he would be in the future, he said he wanted to be an astronomer and study about the Black Hole. Being asked to explain more about the Black Hole, he immediately picked up a pencil and talked about it with a figure (Figure 1).

In Figure 1, you can find the name of a mathematician, Schwartsilt and a technical term "a singular point," written in Korean. What drew the researcher's attention was the number 9×10^{10}, a scientific notation shown in the bottom right corner of the figure. The following protocol of the talking between the researcher and the student C is about the scientific notation.

Researcher: How do you draw that figure?

Student C: The speed of light is three hundred thousand [km/sec].

This [figure] should be squared.

A square of three hundred thousand?

Wow!

We should know the power of 10.

Researcher: What power of 10 is in three hundred thousand?

Student C: It is the 5th power of 10.

So, it is 9 times 10 raised to the 10th power [9×10^{10}].

. . .

Some of the gifted children in MAP showed that they like to make mathematical formulas. An illustration of this is shown in the Figure 2.

The fact that students like to make their own mathematical formulas indicates that they have the mathematical creativity. Although the formulas they made were those others already had made or those that are not so useful, one can say that they do their own mathematics.

Most MAP students replied that they satisfied with the project because it provides an opportunity for them to join to a peer group that consists of a similar level of mathematical abilities. They seemed even to get certain self-confidence and to be encouraged to their life through the MAP activities.

From the MAP Tests

In a mathematics acceleration program, acceleration without checking may be meaningless. Many MAP students were showing very fast progress externally in studying mathematics far ahead of their school grade level just before the first MAP test in July 1991. After the test, however, it is found that a few MAP students could not pass the test of even 1 or 2 levels below the level of mathematics they were studying in MAP at

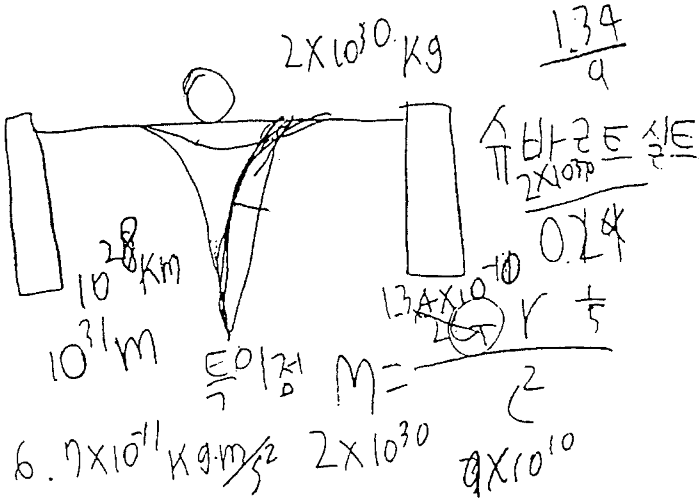

Figure 1: Student C's drawing explaining the Black Hole.

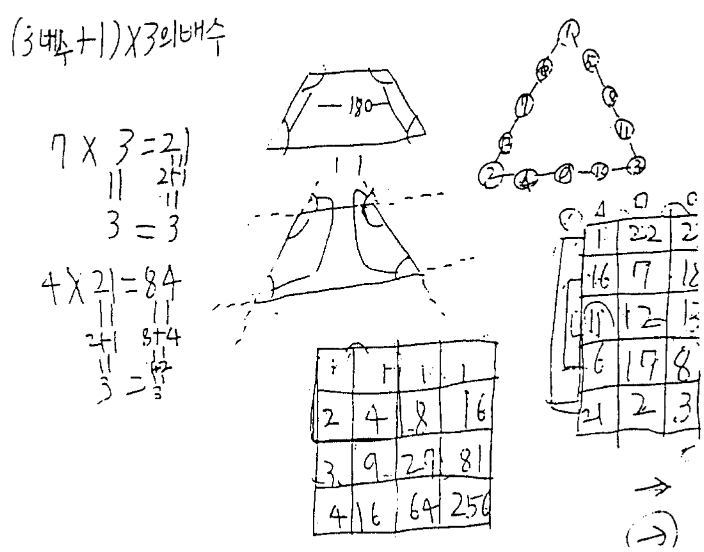

Figure 2: Mathematical formulas that student M (a 5th grader) made immediately after the researcher asked.

that time. For those students, it was inevitable to adjust their pace of studying mathematics in MAP.

Another result from the MAP tests was that the progress of studying mathematics that MAP students showed was not in a constant rate. Although all the MAP students were eager to fulfill the task activities of the project, progress was different from student to student. Especially, setback occurred at a specific juncture from elementary school mathematics to secondary school mathematics or from junior high school mathematics to senior high school mathematics.

Above all, such an aspect in setback periods is clearly related to the school system of Korea: the elementary school - the junior high school - the senior high school. The difference of contents between mathematics courses of different school levels, however, depends not merely on the sequence of the mathematics curriculum but on the degree of abstractness and formality in mathematical activities. Analyzing into a deeper phase, therefore, it means that the level of mathematics that gifted students study in a mathematics acceleration program should be adjusted according to the degrees of abstractness and formality in mathematics.

DISCUSSION

The concept of "acceleration" seems to have been the only focus of the MAP activities. Overemphasizing the concept of acceleration in a mathematics program for the gifted may yield some trouble. Above all, the accelerated study of mathematics may prove fruitless. In fact, the accelerated study itself is not so a hard task for mathematically gifted students. Even in advanced mathematics, if only mathematical skills are emphasized restrictively, most students over the average ability level could hardly feel any difficulty.

Another possible trouble is that students may lose various mathematical experiences which are rarely provided in school mathematics. Helping mathematically gifted students is not to stop at raising their records in mathematics courses but to be extended to provide rich environment of mathematical activities for them. A mathematics program in the gifted education should be provided for those who want to use mathematical experiences in various fields including mathematics rather than for those who just want to be mathematicians.

Thus, in order to improve a mathematics program for the gifted, one should consider the attribute of students as well as that of mathematics itself. The results of this research show that the students in MAP should be categorized in two groups according to their characteristics shown in the program:

Type A: Students who have high abilities in general but need to develop more mathematical abilities (this can happen because it is quite well for the general ability to appear in the literacy in which the numeration takes a certain portion), or students who are capable in mathematics but not reached to the level of formalized mathematics.

Type B: Students who have reached to the level of formalized mathematics early in their ages. (They have basic mathematical skills fully and are interested in mathematics very eagerly.)

Students in one group (type A) need an

alternative program that enhance their mathematical abilities to think abstractly in formal mathematics, whereas students in the other group (type B) need a modified program of mathematical activities given in a module structure.

For the development of a mathematics program for the type A, the Krutetskii's study on mathematical abilities gives significant implications. Although the study was to investigate the structure of mathematical abilities in school children, one can draw an educational idea. Confrey & Lanier (1980, pp. 549-556) suggests that, until the student's mathematical ability reaches to a level of logical mathematics, a practice in thinking itself considering the psychological characteristics of mathematical abilities is needed ahead of a logical and systematic study of mathematical concepts.

On the other hand, an effective mathematical activity is related not only to mathematical skills but also to the body of accumulated mathematical knowledge. Generally, the scope and sequence of school mathematics depends on the systematic and hierarchic aspects of mathematics. Such a structure of the mathematics curriculum may not fit the gifted students' studying mathematics.

An alternative way of restructuring activities for the mathematically gifted students of type B is the local mathematization (cf. Freudenthal, 1973, p. 131-146). In the local mathematization, knowledge to learn is organized according to the genetic principle rather than the curricular hierarchy. As a consequence, activities provided in the program are comparted into modules.

At present, a research team in GEIK is developing two sets of mathematics program for types A and B respectively. The study of verifying the effectiveness of those programs will the successive tasks of this research.

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