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ROUGH MEMBERSHIP FUNCTION

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1. Instroduction

Let X be a subset of the universe U, and let R be an equivalence relation on U, call an indiscernibility relation. We define lower and upper rough set of X in the pair $P = (U, R)$, denoted $\underline{P}(X)$ and $\overline{P}(X)$ respectively, as follows:

$$\underline{P}(X) = \{x \in U : [x]_R \subset X\},$$

$$\overline{P}(X) = \{x \in U : [x]_R \cap X \neq \emptyset\}$$

where $[x]_R$ denotes the equivalence class of the relation R containing x. We also denote the boundary of X in P by

$$B_P(X) = \overline{P}(X) - \underline{P}(X).$$

Thus we may define two membership functions $\underline{\in}_P$ and $\overline{\in}_P$, called strong and weak membership, respectively, as follows:

$$x \ \underline{\in}_P X \text{ if and only if } x \in \underline{P}(X),$$

$$x \ \overline{\in}_P X \text{ if and only if } x \in \overline{P}(X).$$

If $x \ \underline{\in}_P X$ we say that "x surely belongs to X in P" and $x \overline{\in}_P X$ means "x possibly belongs to x in P".

2. Fuzzy sets

Zadeh in his monumental paper [4th], defined a fuzzy set as follows: "A fuzzy set X in U is characterized by a membership function $f_X(x)$ which associates with each point in U a real number in the interval $[0, 1]$, with the value of $f_X(x)$ at x representing the 'grade of membership' of x in X." The union and intersection of fuzzy sets X and Y are defined as follows:

$$f_{X\cup Y}(X) = Max(f_X(x), f_Y(x)),$$
$$f_{X\cap Y}(X) = Min(f_X(x), f_Y(x)).$$

for every $x \in U$.

The complement $-X$ of a fuzzy set X is defined by the membershep function

$$f_{-X}(x) = 1 - f_X(x)$$

for every $x \in X$.

3. Rough membership function

In this section, we will briefly review the properties of lower and upper rough sets, and compare the notions of rough membership function and the membership function of a fuuzy set. From the definition of lower and upper rough set in P we have the following properties of $\underline{P}(X)$ and $\overline{P}(X)$:

(1) $\underline{P}(U) = \overline{P}(U) = U$ and $\underline{P}(\emptyset) = \overline{P}(\emptyset) = \emptyset$,
(2) $\underline{P}(x) \subset X \subset \overline{P}(X)$,
(3) $\overline{P}(X \cup Y) = \overline{P}(X) \cup \overline{P}(Y)$,
(4) $\underline{P}(X \cup Y) \supset \underline{P} \cup \underline{P}(Y)$,
(5) $\overline{P}(X \cap Y) \subset \overline{P}(X) \cap \overline{P}(Y)$,
(6) $\underline{P}(X \cap Y) = \underline{P}(X) \cap \underline{P}(Y)$,
(7) $\overline{P}(-X) = -\underline{P}(X)$ and $\underline{P}(-X) = -\overline{P}(X)$.

Moreover we have

(8) $\underline{\underline{P}}\,\underline{\underline{P}}(X) = \overline{\overline{P}}\,\underline{\underline{P}}(X) = \underline{\underline{P}}(X)$,
(9) $\overline{\overline{P}}\,\overline{\overline{P}}(X) = \underline{\underline{P}}\,\overline{\overline{P}}(X) = \overline{\overline{P}}(X)$.

Now we may define two rough membership functions $\underline{f}$ and $\overline{f}$, called lower and upper rough membership function respectively.

DEFINITION 3.1. *The lower rough membership function $\underline{f}$ is defined by the membership function $f[\underline{P}(X)]$ from a lower rough set $\underline{P}(X)$ into the unit interval* $[0, 1]$.

PROPOSITION 3.2. *Let $\underline{f}$ be a lower rough membership function. Then*

(i) $\underline{f}_{X\cup Y}(x) \geq Max(f[\underline{P}(X)], f[\underline{P}(Y)])$,
(ii) $\underline{f}_{X\cap Y}(X) = Min(f[\underline{P}(X)], f[\underline{P}(Y)])$,
(iii) $\underline{f}_{-X}(x) = 1 - f[\underline{P}(X)]$.

Proof From the definition $\underline{f}$ and the properties of $\underline{P}(X)$ we have

(i) $\underline{f}_{X\cup Y}(x) = f[\underline{P}(X \cup Y)] \geq f[\underline{P}(X)\underline{P}(Y)] = Max(f[\underline{P}(X)], f[\underline{P}(Y)])$
(ii) $\underline{f}_{X\cap Y}(x) = f[\underline{P}(X \cap Y) = f[\underline{P}(X), \underline{P}(Y)] = Min(f[\underline{P}(X)], f[\underline{P}(Y)])$
(iii) $\underline{f}_{-X}(x) = f[\underline{P}(-X)] = f[-\overline{P}(X)] = 1 - f[\underline{P}(X)]$.

DEFINITION 3.3. *The upper rough membership function $\overline{f}$ is defined by the membership function $f[\overline{P}(X)]$ from a upper rough set $\overline{P}(X)$ into the unit interval* $[0, 1]$.

PROPOSITION 3.4. *Let $\overline{f}$ be an upper rough membership function. Then*

(i) $\overline{f}_{X\cup Y}(x) = Max(f[\overline{P}(X), f[\overline{P}(Y)])$,
(ii) $\overline{f}_{X\cap Y}(x) \leq Min(f[\overline{P}(X)], f[\overline{P}(Y)])$,
(iii) $\overline{f}_{-X}(x) = 1 - f[\overline{P}(X)]$.

Proof The proof of Proposition 3.4 is similar to Proposition 3.2.

From Proposition 3.2 and Proposition 3.4 we see that, although the rough membership function has some resemblance to the membership function of a fuzzy set, the notion of rough set cannot be reduced to the notion of fuzzy set.

In next works, we shall try to establish a rough realation R on a rough set $P(X)$ and shall investigate the various types of reflexivities in $P(X)$

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