

## 반복학습기법을 이용한 서보모터용 위치센서오차의 자동 보정

## (Automatic Error Correction of Position Sensors for Servo Motors via Iterative Learning)

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## 要 約

이 논문에서는 서보모터용 위치센서의 오차를 자동으로 보정할 수 있는 반복학습 알고리즘을 제시한다. 기존의 비학습적인 방법들과 달리, 제시된 알고리즘은 이상적인 위치센서나 위치센서 오차의 완벽한 모델에 관한 사전정보를 필요로 하지 않는다. 저자가 아는 한, 이 논문에 의하여 반복학습기법이 위치센서의 오차보정 문제에 처음으로 적용된 듯하다. 더욱이, 제안된 알고리즘은 정적인 미지의 함수를 학습할 수 있고, 측정신호의 미분을 이용하지 않으며, 시스템의 초기조건이 모든 반복단계에서 동일하기를 요구하지 않는다는 점에서, 기존의 반복학습기법과 차이점을 갖는다. 제시된 알고리즘의 보편성과 실용성을 입증하기 위하여, 엄밀한 수렴성의 증명과 실험결과를 제시한다.

## Abstract

In this paper, we present an iterative learning method of compensating for position sensor error. The previously known compensation algorithms need a special perfect position sensor or a priori information about error sources, while ours does not. To our best knowledge, any iterative learning approach has not been taken for sensor error compensation. Furthermore, our iterative learning algorithm does not have the drawbacks of the existing iterative learning control theories. To be more specific, our algorithm learns an uncertain function itself rather than its special time-trajectory and does not request the derivatives of measurement signals. Moreover, it does not require the learning system to start with the same initial condition for all iterations. To illuminate the generality and practical use of our algorithm, we give the rigorous proof for its convergence and some experimental results.

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## I. Introduction

The control accuracy of servo systems driven by electric motors depends highly on the accuracy of position sensors. Unfortunately, the output of a practical position sensor is distorted to some extent by its nonideal dynamic characteristics. Therefore, the outputs of practical position sensors need to be compensated for in order to meet the high accuracy specifications of servo systems.

In the prior literature, not much attention has been paid to the compensation method for position sensor error. Hung and Hung<sup>[6]</sup> measured directly and compensated digitally for the position sensor error. To do this, they needed a special ideal sensor. On the other hand, Hanselman<sup>[5]</sup> analyzed the various sources that could give rise to the position error. Based on this error analysis, he proposed in<sup>[6]</sup> a method of reducing or eliminating the position sensor error. The error analysis in<sup>[5]</sup>, however, considered only the individual effect of each error source on position error.

In this paper, we present a learning algorithm which compensates for the deterministic error of a position sensor by estimating the inverse of its input-output mapping iteratively. Our compensation method can correct directly the position sensing error resulting from a combination of all error sources. Recently, considerable research effort has been devoted to high performance control of AC servo motors.

The torque control algorithms in<sup>[4,7,13]</sup> can force AC servo motors to behave like DC servo motors, provided that the true values of rotor position are available. The key idea of our learning algorithm for position sensor error compensation lies in that inaccurate information of rotor position causes torque ripple. Our compensation method does not require a priori information of torque-controller, position sensor model, or motor parameters, but is based only on the steady-

state responses of the servo system with a nonideal position sensor. Hence, it does not need a special ideal sensor. To our best knowledge, any iterative learning approach has not been taken for sensor error compensation. Furthermore, our iterative learning algorithm for position sensor error does not have the drawbacks of the existing iterative learning control theories<sup>[1,2,3,9,12]</sup> in the following respects.

Our algorithm can learn the inverse of the input-output mapping of the nonideal position sensor, but not just its special time-history. In our sensor compensation problem, only the output signal of the nonideal position sensor is accessible. Therefore, its differentiation can produce significant noise and hence is not desirable. For this reason, our algorithm does not use the derivatives of the sensor output signal. In addition, we show that the sensor output signal tends to be a stable limit cycle at each iteration. As the result, our algorithm does not require an initial position reset mechanism in order to assure that the learning system starts with the same initial conditions for all iterations. The paper is organized as follows. In Section II, we state precisely our problem of sensor error compensation and discuss the typical servo motor drive system with a nonideal position sensor. In Section III, we present our learning algorithm to compensate for the position sensor error with the rigorous proof for its uniform convergence. We introduce two lemmas to describe the dynamic behavior of the torque control system with a nonideal position sensor. In Section IV, we show that a wide class of servo motors satisfies the convergence condition of our learning algorithm. In order to illuminate further the practical significance of our learning algorithm, we perform some experiments through use of an NSK VR type DD motor with a VR type resolver as the position sensor. Finally, Section V contains our concluding remarks.

II. Preliminaries

The dynamics of the electric actuators such as DC motors, Brushless DC motors (BLDCM), and step motors can be described by

$$J\ddot{\theta} + B\dot{\theta} + C\text{sgn}(\dot{\theta}) = \tau_c - \tau_L. \tag{1}$$

Here, the constants  $J$ ,  $B$ , and  $C$  are, respectively, moment of inertia, viscous friction coefficient, and Coulomb friction coefficient. The variables  $\theta$  and  $\tau_L$  represent rotor position and load torque, respectively. On the other hand,  $\tau_c$  represents motor torque and is a continuous function of rotor position  $\theta$  and phase current  $i \in R^q$ , where  $q$  denotes the number of phases. Let  $i^* \in R^q$  be the reference phase current. If the phase current is directly controlled<sup>[4,10]</sup>, we can assume that  $i = i^*$  and hence that the motor torque is a function of  $\theta$  and  $i^*$ . That is,

$$\tau_c = T(\theta, i^*). \tag{2}$$

It is a natural property of rotating machines that the above function  $T : R \times R^q \rightarrow R$  is periodic such that

$$T(\theta + 2\pi, i^*) = T(\theta, i^*), \forall \theta \in R, i^* \in R^q. \tag{3}$$

Now, suppose that we have a function  $I : R \times R^q \rightarrow R^q$  satisfying

$$T(\theta, I(\theta, \tau^*)) = \tau^*, \forall \theta \in R, \tau^* \in R. \tag{4}$$

Then, choose the reference phase current  $i^*$  by

$$i^* = I(\theta, \tau^*), \tag{5}$$

where  $\tau^* \in R$  represents the reference torque. Then,

$$\tau_c = T(\theta, i^*) = T(\theta, I(\theta, \tau^*)) = \tau^*, \forall \theta \in R, \tau^* \in R. \tag{6}$$

Hence, the torque controller in (5) linearizes the nonlinear dynamic of an electric motor in

(1) and (2) as follows

$$J\ddot{\theta} + B\dot{\theta} + C\text{sgn}(\dot{\theta}) = \tau^* - \tau_L. \tag{7}$$

On the other hand, it follows from (3) and (4) that the inverse of  $T$  is also periodic, i.e.,

$$I(\theta + 2\pi, \tau^*) = I(\theta, \tau^*), \forall \theta \in R, \tau^* \in R. \tag{8}$$

The above feedback linearizing approach is currently popular in the area of motor control<sup>[4,7,13]</sup>

Unfortunately, the output of a practical position sensor is distorted to some extent due to its nonideal dynamic characteristics and hence the error-free information of rotor position is not available to the torque controller  $T$ . Since the dynamics of a rotor position sensor is fast enough to be neglected, the input-output dynamic characteristics of a nonideal position sensor can be modeled by a continuous nonlinear function  $g$ . That is,

$$\hat{\theta} = g(\theta). \tag{9}$$

We define the error function  $n$  of a nonideal position sensor by

$$n(\theta) \triangleq g(\theta) - \theta. \tag{10}$$

Then, the block diagram representing the function of a nonideal position sensor can be depicted as in Fig.1. (a). It is obvious that the error function  $n$  is periodic and is upper bounded.

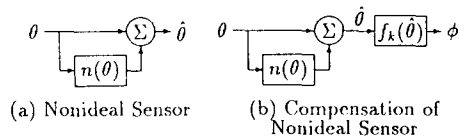


Fig. 1. Sensor Signal Error and its Compensation.  
 (a) Nonideal Sensor  
 (b) Compensation of Nonideal Sensor.

$$n(\theta + 2\pi) = n(\theta) \quad (11)$$

$$|n(\theta)| \leq \mu \stackrel{\Delta}{=} \sup_{\theta \in [0, 2\pi]} |n(\theta)| \quad (12)$$

In general, the reference point of rotor position does not coincide with that of the position sensor. This reference offset corresponds to  $g(0)$  or  $n(0)$  here. Fortunately, it is practically simple to detect  $g(0)$ . In the case of VR type DD motors, for example, we turn on one phase and turn off the other phases. Then, one of stator teeth is aligned with one of rotor teeth. We choose this rotor position as the reference point of rotor position. Then,  $g(0)$  is just the output of the nonideal position sensor at this rotor position. From now, we assume that the reference offset is removed, i.e.,

$$g(0) = n(0) = 0. \quad (13)$$

As can be seen from (9), the true value  $\theta$  of rotor position can be recovered from the output  $\hat{\theta}$  of a nonideal position sensor, if  $g^{-1}$  is known. The function  $f_k$  in Fig. 1.(b) stands for the  $k$ -th estimate of  $g^{-1}$ . In this paper, we attempt to find a learning algorithm which can make the sequence  $\{f_k\}_{k=1}^{\infty}$  converge uniformly to  $g^{-1}$ . Throughout this paper, the subscript  $k$  denotes the iteration number.

From Fig. 1.(b), we can see that the compensated value  $\phi$  of the sensor output  $\hat{\theta}$  can be represented by

$$\phi = f_k \circ g(\theta) \quad (14)$$

and that the  $k$ -th compensation error  $e_k$  can be defined by

$$e_k(\theta) \stackrel{\Delta}{=} \phi - \theta = f_k \circ g(\theta) - \theta. \quad (15)$$

More specifically speaking, our objective is to find an update rule for  $f_k$ ,  $k=1, 2, 3, \dots$  which can make  $\overline{e_k} \rightarrow 0$  as  $k \rightarrow \infty$ , where  $\overline{e_k}$  is defined as follows

$$\overline{e_k}(\theta) \stackrel{\Delta}{=} \sup_{\theta \in (-\infty, \infty)} |\overline{e_k}(\theta)| \quad (16)$$

Let us investigate the effect of the position sensor error on torque response. Suppose that we have a torque controller  $I$  which satisfies (5). Suppose further that the position sensor is nonideal but is compensated as is shown in Fig. 1.(b). Instead of (5) and (6), we then have

$$i^* = I(\phi, \tau^*) \quad (5')$$

and

$$\tau_r = T(\theta, I(\phi, \tau^*)). \quad (6')$$

As the result, (6) will not necessarily hold, since  $f_k \neq g^{-1}$  in general. For each  $\tau^* \in R$  and  $\mu \in [0, \infty]$ , however, there exists a constant  $\alpha$  such that

$$\left| \frac{\partial}{\partial \phi} T(\theta, I(\phi, \tau^*)) \right| \leq \alpha, \text{ if } |\phi - \theta| < \mu. \quad (17)$$

This is due to continuity and periodicity of  $T$  and  $I$ .

The block diagram representation of the torque control system given by (1), (2), and (5') is given in Fig.2. And its dynamic equations can be written as follows

$$J\ddot{\theta}_k + B\dot{\theta}_k + C \operatorname{sgn}(\dot{\theta}_k) = T(\theta_k, I(\phi_k, \tau^*)) - \tau_k, \quad \phi_k = f_k(g(\theta_k)) \quad (7')$$

Here and in what follows, we insert the subscript  $k$  into all variables that are affected by the  $k$ -th estimate  $f_k$  of  $g^{-1}$ . In the next section, we present a learning algorithm for  $g^{-1}$  which does not depend on motor parameters ( $J, B, C$ ), torque model ( $T: R \times R^2 \rightarrow R$ ), or torque control strategy ( $I: R \times R \rightarrow R^2$ )

### III. Main Results

Before presenting our learning rule to update  $f_k$ ,  $k = 1, 2, \dots$  iteratively, we need

to make some assumptions on the system in Fig. 2. First, we assume that

$$(C1) \quad \left| \frac{d}{d\theta} n(\theta) \right| < 1, \forall \theta \in R.$$

This assures the existence of  $g^{-1}$ . In turn, this implies that the true value of rotor position can be recovered from the output of nonideal position sensor, if  $g^{-1}$  is known. Most of good industrial position sensors are expected to satisfy (C1)

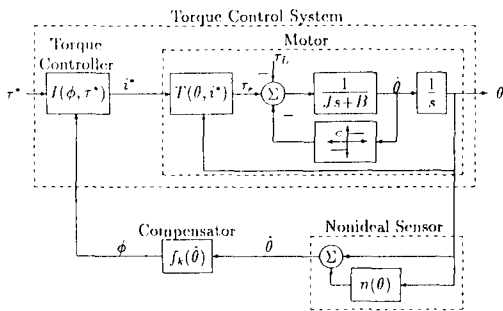


Fig. 2. Torque Control System with Nonideal Position Sensor.

We also make an assumption on the initial conditions of the system in Fig. 2. as follows.

$$(C2) \quad |\hat{\theta}_k(0)| < \tau_M / B, \text{ for } k = 1, 2, \dots$$

Here,  $\tau_M$  denotes the maximum allowable torque of the electric motor on which a nonideal position sensor is mounted. The quantity  $\tau_M/B$  in (C2) corresponds to the rotor speed that the electric motor driven by the maximum allowable torque  $\tau_M$  achieves in the steady state. Note that (C2) is much less restrictive than the conditions that the existing iterative learning control theories<sup>11,2, 13,12</sup> impose on initial states.

Finally, we assume that

$$(C3) \quad \tau_L \text{ is constant during learning process.}$$

In practice, the load torque  $\tau_L$  is not necessarily constant throughout learning process. However, it is reasonable to assume that  $\tau_L$  is constant in the case when learning

period is short.

(C4)  $\tau^*$  is set constant during learning process such that

$$\tau^* - |\tau_L| - C > 6\mu\alpha, \quad \tau^* + |\tau_L| + C \leq \tau_M - \mu\alpha.$$

As will be shown later, the condition (C4) guarantees that the rotating direction of the electric motor is determined only by the sign of  $\tau^*$  but is not affected by other sources such as position sensing error, Coulomb friction torque, and load torque.

Now, we describe the dynamic behaviors of the system in Fig. 2. under the prescribed assumptions. Speaking specifically, we show that the time functions  $\hat{\theta}_k, \hat{\theta}_k, k = 1, 2, \dots$  are invertible and periodic.

**Lemma 3.1** : Suppose that

(i)  $f_k$  is chosen so that

$$f_k(x + 2\pi) = f_k(x) + 2\pi, \quad \forall x \in R. \quad (18)$$

(ii)  $e_k$  is sufficiently small so that

$$\bar{e}_k \leq \mu. \quad (19)$$

Then, the system in Fig. 2. has the following properties in the steady state.

(a)  $\theta_k$  and  $\hat{\theta}_k$  are strictly increasing in  $t$ .

(b) There exists a positive constant  $T_k$  such that

$$\theta_k(t + T_k) = \theta_k(t) + 2\pi,$$

$$\hat{\theta}_k(t + T_k) = \hat{\theta}_k(t) + 2\pi,$$

$$\phi_k(t + T_k) = \phi_k(t) + 2\pi.$$

*Proof* : The proof is quite elaborate and lengthy. Hence, it is omitted because of limited space.  $\square$

Suppose that the hypotheses (i), (ii) of Lemma 3.1 are satisfied and that the system in Fig. 2. with  $f_k$  as the  $k$ -th estimate of  $g^{-1}$  reaches the steady state. Then, the properties (a), (b) in Lemma 3.1 justify the following procedure to define the function  $f_{k+1} : R \rightarrow R$  on the basis of the responses of the

system in Fig. 2. with the function  $f_k$  as the  $k$ -th estimate of  $g^{-1}$ .

*Step 1* : Pick up  $t_k$  and  $t'_k$  such that  $\hat{\theta}_k(t_k) = 2\pi m_k$  and  $\hat{\theta}_k(t'_k) = 2\pi(m_k+1)$  for a positive integer  $m_k$ .

*Step 2* : Compute  $T_k \triangleq t'_k - t_k$ .

*Step 3* : Determine the time function  $\psi_k : [t_k, t_k + T_k] \mapsto [2\pi m_k, 2\pi(m_k + 1)]$  by 
$$\psi_k(t) \triangleq \frac{2\pi}{T_k}(t - t_k) + 2\pi m_k, \quad t_k \leq t \leq t_k + T_k. \quad (20)$$

*Step 4* : Define

$$f_{k+1}^0 : [2\pi m_k, 2\pi(m_k + 1)] \mapsto [2\pi m_k, 2\pi(m_k + 1)] \text{ by}$$

$$f_{k+1}^0(x) \triangleq \psi_k \circ \hat{\theta}_k^{-1}(x), \quad \forall x \in [2\pi m_k, 2\pi(m_k + 1)]. \quad (21)$$

*Step 5* : Define  $f_{k+1} : R \rightarrow R$  by periodically expanding  $f_{k+1}^0$  as follows. For all  $j = 0, \pm 1, \pm 2, \dots$  and  $x \in [2\pi m_k, 2\pi(m_k+1)]$ ,

$$f_{k+1}(x + 2\pi j) \triangleq f_{k+1}^0(x) + 2\pi j. \quad (22)$$

It should be clear from the definition of  $\psi_k$  in (20) and  $f_{k+1}^0$  in (21) with the properties {a}, {b} in Lemma 3.1 that (22) determines the function  $f_{k+1}$  uniquely regardless of different choices of  $m_k$ . On the other hand, (9)-(11), (13), and (C1) imply that, for  $J = 0, \pm 1, \pm 2, \dots$ ,

$$\hat{\theta}_k = 2\pi j \text{ iff } \theta_k = 2\pi j. \quad (23)$$

Therefore, we see that the function  $\psi_k$  defined by (20) is just the "time-average signal" of  $\theta_k$

Using the function  $\psi_k$ , we now define the position ripple  $\bar{\epsilon}_k$  and its upperbound as follows

$$\epsilon_k(t) \triangleq \psi_k(t) - \theta_k(t), \quad \forall t \in [t_k, t_k + T_k] \quad (24)$$

$$\bar{\epsilon}_k \triangleq \sup_{t \in [t_k, t_k + T_k]} |\epsilon_k(t)| \quad (25)$$

The following Lemma 3.2 clarifies the relationship between the upper bound  $\bar{\epsilon}_k$  of compensation error and the upper bound  $\bar{\epsilon}_k^*$  of position ripple.

**Lemma 3.2** : Suppose that all the

hypotheses of Lemma 3.1 are satisfied. Then,

$$\bar{\epsilon}_k \leq \eta \bar{\epsilon}_k, \quad (26)$$

where

$$\eta \triangleq \frac{8\pi\alpha}{\tau^* - \tau_l - C - 2\mu\alpha}. \quad (27)$$

*Proof* : The proof is omitted because of limited space.  $\square$

Now, we are ready to present our learning rule that can update  $f_k$ ,  $k=1,2,\dots$  iteratively so that the sequence  $\{f_k\}_{k=1}^\infty$  converges uniformly to  $g^{-1}$ .

**Theorem 3.1** : Define the update rule for  $f_k$ ,  $k=1,2,\dots$  by (20)-(22), where  $f_1$  is chosen as the identity mapping, i.e.,

$$f_1(x) \triangleq x, \quad \forall x \in R. \quad (28)$$

Then, the update rule is well defined for all  $k = 1, 2, \dots$  if

$$\eta < 1. \quad (29)$$

Futhermore, it holds that

$$\bar{\epsilon}_k \leq \eta^{k-1} \mu, \text{ for } k = 1, 2, \dots \quad (30)$$

and hence that

$$\bar{\epsilon}_k \rightarrow 0 \text{ as } k \rightarrow \infty. \quad (31)$$

*Proof* : We show by induction that our assertion is true. Clearly,  $f_1$  in(28) is well defined and satisfies (18). On the other hand, we can see from (10), (12), and (15) that (30) holds with  $k = 1$ .

Next, suppose that  $f_r$  is well-defined and

$$f_r(x + 2\pi) = f_r(x) + 2\pi, \quad \forall x \in R \quad (32)$$

$$\bar{\epsilon}_r \leq \eta^{r-1} \mu. \quad (33)$$

By (29), (32), and (33), Lemma 3.1 holds for  $k = r$ . Consequently,  $f_{r+1}$  via *step 1-step 5* is

well defined and

$$f_{r+1}(x+2\pi) = f_{r+1}(x) + 2\pi, \forall x \in R. \quad (34)$$

By (34) with (10), (11), and (15).

$$e_{r+1}(x+2\pi) = e_{r+1}(x), \forall x \in R. \quad (35)$$

By (15), *step 1-step 5*, (23), and (35).

$$\begin{aligned} \bar{e}_{r+1} &= \sup_{x \in [2\pi m, 2\pi(m+1)]} |f_{r+1}(g(x)) - x| \\ &= \sup_{t \in [t_r, t_r + T_r]} |f_{r+1}(g(\theta_r(t))) - \theta_r(t)| \\ &= \sup_{t \in [t_r, t_r + T_r]} |f_{r+1}(\hat{\theta}_r(t)) - \theta_r(t)| \\ &= \sup_{t \in [t_r, t_r + T_r]} |\psi_r(t) - \theta_r(t)| \\ &= \bar{E}_r. \end{aligned}$$

By Lemma 3.2, this and (33) imply that (30) holds with  $k = r+1$ .

Hence, by induction, we can conclude that the update rule is well defined for all  $k = 1, 2, \dots$  and (30) holds for all  $k = 1, 2, \dots$ . Finally, (31) is the immediate consequence of (30) under the condition in (29). And this completes the proof.  $\square$

In our update rule, we have chosen  $f_1$  as the identity mapping. This means that, at the first iteration, we feedback the output of the nonideal position sensor directly into the torque controller without compensation.

The direct application of the existing iterative learning control theories<sup>[1,2,3,9,12]</sup> to our compensation problem does not seem possible since the true information of system states (here,  $\theta$  and  $\dot{\theta}$ ) is not available. Moreover, our iterative learning approach differs from the prior approaches in the following respects. First, ours uses the time-averaged estimate  $\psi_k$  but not the true data of rotor position, as can be seen from (20). Second, ours uses only the available information  $\hat{\theta}$  but not its derivatives, as can be seen from (21) and (22). Third, ours estimates the function  $g^{-1}$  rather than its special time-histories. Finally, the

convergence of our update rule does not impose any restriction on the initial conditions  $\theta_k(0)$ ,  $\dot{\theta}_k(0)$ .

In our iterative learning approach, the reference torque  $\tau^*$  is chosen large enough to ensure that  $\theta_k$  is strictly increasing for all iterations. The asymptotic periodicity of  $\theta_k$  shown in Lemma 3.1 is guaranteed purely by the natural properties of a rotating machine. After the torque control system in Fig.2 reaches the steady state at each iteration, our learning scheme updates the estimate of  $g^{-1}$  based on the waveform of  $\hat{\theta}$  for one period only.

#### IV. Practical Examples

Our learning algorithm for position sensor error compensation presented in Section III can be applied to a wide class of electric motors. In this section, we show that AC syncro motors and VR type DD motors satisfy the conditions for the convergence of our learning algorithm. Especially, for the case of VR type DD motors, we present some experimental results in order to illuminate further the practical significance of our compensation method.

##### Example 4.1 : (AC Syncro motor)

The torque model of a three-phase AC syncro motor takes the following form.

$$T(\theta, i^*) = k_r \left[ i_1^* \sin(\theta) + i_2^* \sin\left(\theta - \frac{2}{3}\pi\right) + i_3^* \sin\left(\theta - \frac{4}{3}\pi\right) \right] \quad (36)$$

A choice of the torque controller  $I$  satisfying (4) is

$$I(\theta, \tau^*) = \left( \frac{2\tau^*}{3k_r} \right) \begin{bmatrix} \sin(\theta) \\ \sin\left(\theta - \frac{2}{3}\pi\right) \\ \sin\left(\theta - \frac{4}{3}\pi\right) \end{bmatrix} \quad (37)$$

Then, simple calculation yields the following fact

$$T(\theta, I(\phi, \tau^*)) = \tau^* \cos(\phi - \theta), \forall \theta, \phi \in R \quad (38)$$

By (38), we have the following inequality.

$$\left| \frac{\partial}{\partial \phi} T(\theta, I(\phi, \tau^*)) \right| = |\tau^* \sin(\phi - \theta)| \leq |\tau^*| |\phi - \theta| \quad (39)$$

This shows that (17) can be satisfied by

$$\alpha = |\tau^*| \mu. \quad (40)$$

This choice gives

$$\eta = \frac{8\pi |\tau^*| \mu}{\tau^* - \tau_L - C - 2\mu^2 |\tau^*|}. \quad (41)$$

In the particular case of  $\tau_L = C = 0.1 \tau^*$  and  $\mu = 0.02$ , we have  $\eta = 0.629$ . Note that  $\mu = 0.02$  corresponds to about  $1.146^\circ$ . This large sensor error can be compensated for at the fast convergence rate of  $\eta = 0.629$ .

**Example 4.2 : (VR type DD Motor)**

The torque model of a VR type DD motor with 3 phases can be written as

$$T(\theta, i^*) = T_0(\theta, i_1^*) + T_0\left(\theta - \frac{2}{3}\pi, i_2^*\right) + T_0\left(\theta - \frac{4}{3}\pi, i_3^*\right), \quad (42)$$

where the function  $T_0 : R \times R \rightarrow R$  has the properties

$$T_0(\theta, y) = T_0(\theta, -y), T_0(-\theta, y) = -T_0(\theta, y), \quad \forall \theta, y \in R. \quad (43)$$

In order to maximize torque/mass ratio, VR type DD motors usually operate with magnetic saturation. As the result,  $T_0$  is a highly nonlinear function and hence is hard to be described in explicit form.

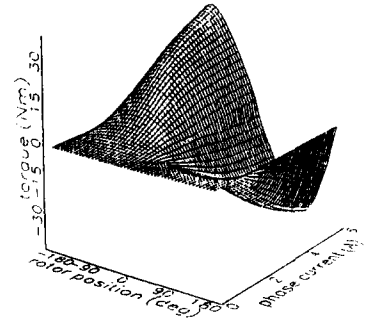
On the other hand, Wallace and Taylor<sup>[13]</sup> proposed recently an inverse function technique, which corresponds here to finding the torque controller  $I$  which satisfies (4). The torque controller  $I$  for VR type DD motors can be described by

$$I(\theta, \tau^*) = \left( I_0(\theta, \tau^*), I_0\left(\theta - \frac{2}{3}\pi, \tau^*\right), I_0\left(\theta - \frac{4}{3}\pi, \tau^*\right) \right)^T, \quad (44)$$

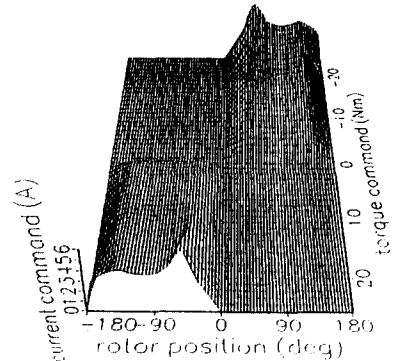
where the function  $I_0 : R \times R \rightarrow R$  has the property.

$$I_0(\theta, \tau^*) = 0, \quad \forall \theta \in [0, \pi], \tau^* \in R^* \quad (45)$$

For our experimental study, we used a VR type DD motor (Ref. No. RS0608FN001) with 150 poles, maximum stator current 6 [A], maximum speed 1 [rps], and  $\tau_M = 25$  [Nm], which is manufactured by NSK Co., Japan. The position sensor mounted on the NSK VR type DD motor is a VR type resolver. VR type resolvers are cheap and rugged, but are not accurate enough for high-precision control. Therefore, we have to compensate for the nonideal characteristics of VR type resolvers if we want to use them for high-precision control. The functions  $T_0$ ,  $I_0$  of the NSK VR type DD motor we obtained in look-up table form by using a method similar to those in<sup>[11,13]</sup> are depicted graphically in Fig. 3. (a) and Fig.3. (b), respectively



(a)  $T_0(\theta, i_1^*)$



(b)  $I_0(\theta, \tau^*)$

Fig. 3. Graphic representation of  $T_0$  and  $I_0$  for NSK VR Type DD Motor.



In our experiment, the torque command  $\tau^*$  was set to 1.25 [Nm] , which corresponds to 5% of  $\tau_M$  . The experimental results are summarized in Fig.4.(a)-(c) and Fig.5. The time-histories of  $\dot{\phi}_k$  for  $k = 1,3,5$  in the steady state are depicted in Fig.4. Fig.5 presents the graphic plots of  $n_k(x) = f_k^{-1}(x)x, k=1, \dots, 5$ . Just in four iterations, the sequence  $\{n_k\}$  converges and velocity ripple is reduced one-tenth or less, as can be seen from Fig.5 and Fig.4. Our experimentation firmly demonstrates the practical use of our learning algorithm.  $\square$

Fig. 4. Time-History of  $\dot{\phi}_k, k = 1,3,5$  in the Steady State.

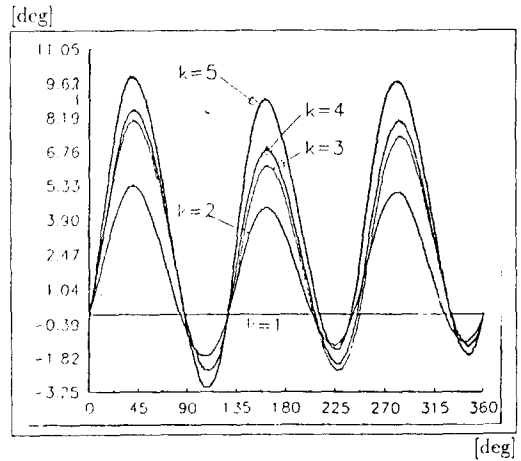
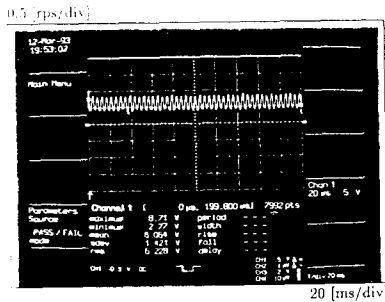
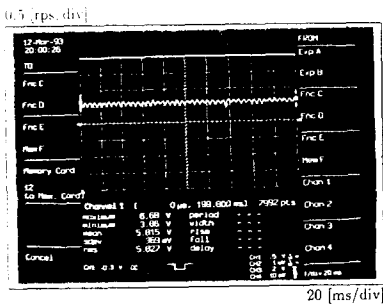


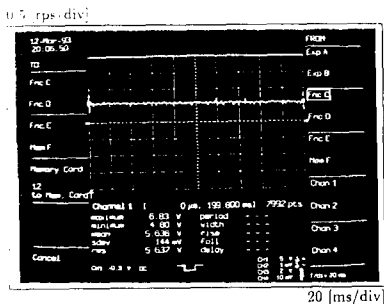
Fig. 5. Graphic Plot of  $n_k, k = 1,2,3,4,5$ .



(a)  $\dot{\phi}_k$ (ripple is 23.4%)



(b)  $\dot{\phi}_k$ (ripple is 6.35%)



(c)  $\dot{\phi}_k$ (ripple is 2.56%)

### V. Conclusion

In this paper, we have presented a learning algorithm which can compensate automatically for position sensing error without using any other special sensors. In order to justify the generality and practical significance of our learning algorithm, we have provided the rigorous proof for its uniform convergence under reasonable assumptions. And, we have shown that typical electric motors satisfy the conditions for the convergence of our learning algorithm. Also, we have demonstrated its practicality through some experiments using a VR type DD motor with a VR type resolver as the position sensor. To our best knowledge, this is the first result to consider an iterative learning approach for sensor error compensation. Moreover, our result suggests that the concept of iterative learning can be used to learn not only the special time-history of an uncertain function but also the uncertain function itself.

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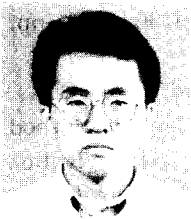
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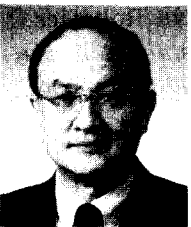
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