# APPROXIMATE QUEUE LENGTH DISTRIBUTION OF MMPP/D/1 IN AN ATM MULTIPLEXER

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# ATM 다중화기의 MMPP/D/1큐잉 모델의 큐길이 분포에 대한 근사방법

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요 약

본 논문에서는 필자들이 이전 논문에서 제안한 근사방법을 ATM다중화에 대한 모델인 MMPP/D/1큐의 큐길이 분포 계산에 적용하였다. 도착하는 셀과 서비스 하기 전에 서버가 관측한 큐길이 분포들간의 관계식을 유도하여 계산하는데 이용하였다. MMPP/D/1큐에 대해 제안된 근사공식을 이용하여 큐길이 분포를 계산한 결과와 이 큐잉 시스템을 시뮬레이션하여 얻은 결과와 비교하여 일치함을 확인하였다. 더우기 제안된 방법은 일반적인 큐잉 모델에 대한 큐길이 분포계산을 신속히 수행할 수 있으며 ATM망의 트래픽 분석을 신속하고 정확하게 계산하는 데 유용할 것이다.

ABSTRACT

Our previously proposed method is further applied to find the queue length distribution of MMPP/D1 in an ATM multiplexer. We derive some useful relationship between the queue length distribution seen by arriving cells and for a server befor each service.

The relations were used to improve our approximation. For MMPP/D1 the calculated results show a good agreement with those obtained by a simulation of the system. Furthermore, our approximation provides fast numerical algorithms for general traffic models.

These advantages demomstrate that our approximation method is useful for a fast and accurate traffic analysis in ATM networks.

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## I. Introduction

In Broadband Integrated Services Digital Networks (BISDN), a wide variety of traffic will be carried by a link of Asynchronous Transfer Mode (ATM) system. In this system, the real-time analysis of some statistical distribution such as queue length and delay is extremely important to utillize network resources as efficiently as possible and increase the quality of services(QoS) of all existing services as well as those with yet-unknown characteristics that will emerge in the future. In this paper, considering the queue length which represents the number of cells waiting in the queue of a multiplexer, we define $Q(\gamma)$ as the probability that the queue length is larger than $\gamma$ and refer it to the queue length distribution (QLD). The tail of QLD is closely related with the loss rate at a multiplexer with a finite size buffer.[1,2] In BISDN, the maximum loss rates allowed for various BISDN applications are diverse. For instance, the cell loss rate is $10^{-8}$ for videophone and $10^{-10}$ for TV distribution.[3] To satisfy such requirements in various traffic conditions, it is desirable to have tools to analyze QLD in real time for general traffic models.

In previous papers[4,5] we proposed an approximation approach on the computation of QLD of single server queues. The formalism was based on two steps of mixed bound technique using the probability generating funtion(PGF) of the number of arriving customers and service capacity. For an arbitrary superposition of general traffic sources, our approximate formula for the QLD is given in simple form. The calculation of QLD using our approach is to find the global saddle point in two parametric dimensions consisting of the time interval and the real parameter used for PGF's of the arrival and service processes.

For $M+\sum N_jD_j/M/1$ and $M+\sum N_jD_j/D/1$, we found a good agreement between our approximation and other calculations.[4] Especially, for M/M/1, the proposed approach gave the exact solution.[4],[5] It is noted that Nakagawa[6] made a similar approach using the PGF of arrival process and the Chernoff upper bound technique. His approximation had an upper bound chracteristic, but for some simple models such as M/M/1 and M/D/1 his results showed a considerable deviation from the exact solution even after a less rigorous modification that the calculated results were further divided by a constant to gain the correct result for Q(0).

In this paper, we apply our approximation method to an ATM multiplexer. Especially, we will focus on the QLD of the queue MMPP/D/1 which is modeled by feeding the Markov modulated Poisson process(MMPP) into a single server queue with fixed service time. In section II, we discuss some useful properties of QLD in these queueing models and derive relations between the QLD for arriving customers and that for the server before service. These relations are useful to improve our approximation on QLD in Section III. In Section III, we review our approximation formalism for the QLD of general queues[4],[5] and propose a slightly modified formula using the relations in Section II. In Section IV, our theory is applied to MMPP/D/1. The calculated results are compared with those obtained by a direct simulation of the system. As a special case of MMPP/D/1, we also discuss M/D/1 in this section. Finally, the conclusion of this paper is provided in section V.

## II. USEFUL PROPERTIES OF QLD

In ATM networks, user information is transmitted between communication entities using fixed size ATM cells. Queueing processes in an ATM multiplexer can therefore be modeled by G/D/1 because the service time is the same for all cells. For an ATM multiplexer we observe the following policy of service: A server measures the queue length in a deterministic manner whether the queue is occupied or not. Of course, it will measure the queue length to be zero, if the queue is empty. The interval between two measurements is called as the time slot and can be set to be equal to the service time. Without loss of generality, we assume that services take place at the beginning of slots, and arrivals during slots. Under this policy of service in the continuous time domain, an arriving cell has to wait in the queue a fraction of one time slot before a server in the idle state starts its service. In this section, we will derive some relations between the QLD for arriving cells(Scheme Ⅰ) and that for the server before services(Scheme Ⅱ) in a stationary queueing system.

Our discussion starts with the definition of a stable queue as follow: A queue is said to be stable if for all $0 < e < 1$ and Schemes $X = I$ and $II$, there exists $T$ such that $P\,[N(n,t;X) > 1] > 1-e$ for $t > T$, where $N(n,t;X)$ is the number of events that the queue length is equal to n in interval (0, t) in Scheme $X$, and $P$ refers to probability. For a correct statistical analysis on the QLD of a stable queue, the time interval of an ensemble process must be longer than $T$.

Let $A(t)$ and $S(t)$ be the number of arribing cells and the full service capacity of a server, respectively, in time interval(0, t). As a server in an ATM multiplexer checks the queue at every time slot periodically, the service process is independent of the arrival process. Then, the utilization factor(or the service load) may be represented by

$$\rho = \left\langle \frac{A(t)}{S(t)} \right\rangle_{t>T} \tag{1}$$

where $\langle Z \rangle_c$ denotes the ensemble average of $Z$ with condition c. A stable queue is guaranteed if $\rho < 1$.

Let $q_t$ be the queue length at time t, then $N(n,t,II)$ may be expressed by

$$N(n,t,II) = \begin{cases} S(t) - A(t) - q_o + q_t, & \text{if } n = 0; \\ N(n-1,t,I), & \text{if } elsewhere. \end{cases} \tag{2}$$

Obviously, we have

$$A(t) = \sum_{n=0}^{\infty} N(n,t,I) \tag{3}$$

and

$$S(t) = \sum_{n=0}^{\infty} N(n,t,II). \tag{3}$$

With a series of cell arrivals producing a stable queue, let us define Q(r, t; X) as follows:

$$Q(r,t,X) = \frac{1}{Y(t)} \sum_{n=r+1}^{\infty} N(n,t,X), \tag{4}$$

where Y(t) = A(t) for X = $I$ and Y(t) = S(t) for X = $II$. Then the QLD in Scheme X = $I$ and $II$ is the ensemble average of Q(r,t:X), denoted by Q(r;X):

$$Q(r,X) = \langle Q(r,t,X) \rangle_{t>T}. \tag{5}$$

Equations (1), (4) and (5) give the following relation between QLD in two schemes $I$ and $II$,

$$\begin{aligned} Q(r,II) &= \langle Q(r,t,II) \rangle_{t>T} \\ &= \left\langle \frac{1}{S(t)} \sum_{n=r+1}^{\infty} N(n,t,II) \right\rangle_{t>T} \\ &= \rho \left\langle \frac{1}{A(t)} \sum_{n=r+1}^{\infty} N(n-1,t,I) \right\rangle_{t>T} \\ &= \rho Q(r-1;I). \end{aligned} \tag{6a}$$

This relation is independent of $q_0$ and $q_t$ in (2), is valid for general queueing models in an ATM multiplexer, and holds for all $\rho$. As $Q(-1;X) = 1$ for X = $I$ and $II$, we have

$$Q(0;II) = \rho. \tag{6b}$$

It is noted that (6b) is a special case of the following well-Known formula[7] for $P_{loss}$ the probability that an arbitrary cell is lost from the queue of a finite size buffer:

$$P_{loss} = \frac{1 - Q(0;II)}{\rho}$$

If the buffer size is infinite, $P_{loss} = 0$ and $Q(0:II) = \rho$. Furthermore, it is easy to show that all the following conditions Q(r; *I* ) ≥ pQ(r-1; *I* ) and Q(r+1; *II*) ≥ pQ(r;*II*) are equibalent.

To characterize a queueing system under this service policy, we define $k(0)$ as

$$\kappa(0)=\frac{Q(0;\,I\,)}{Q(0;\,II\,)}=\frac{Q(0;\,I\,)}{p} \tag{7}$$

For queueing systems with k(0) > 1, the number of events of zero queue length found by the server of the system is larger than that measured by the arriving customers. As the burstiness of an arrival process increases, $k(0)$ increases. Hence, $k(0)$ is an important characteristic parameter of a queueing system in the present service policy. In this paper, we will use two formulas in (6a) and (6b) to improve our approximation of the QLD of single serve queueing models in an ATM multiplexer.

## Ⅲ. APPROXIMATE QUEUE LENGTH DISTRIBUTION

We consider a queueing system with a single server and cell arrivals producing a stable queue. Let B(t) be the number of cells that received the service in time interval (0, t). As the full service capacity of a server S(t)≥B(t) for all t>0, we have the following relation for $q_t$ the queue length at t:

$$q_t=q_0+A(t)-B(t)\geq q_0+A(t)-S(t)\geq A(t)-S(t),\ t>0. \quad (8) \tag{8}$$

As (8) is true for all t>0 in all ensembles, we have

$$Q(r)\geq \max_{t>0}\{P\ [A(t)-S(t)-r-1\geq 0\ ]\ \}. \tag{9}$$

Now we use Chernoff bound technique,[1],[2] which may be stated briefly as follows : For the random variable U taking on integers n = 0, ±1, ±2..., and all real parameters z > 1, we have

$$P\,[\ U(t)\geq 0]\ \leq\ \min_{z>1}\ \Psi_U(z,t), \tag{10}$$

where $\Psi_U(z)$ is the probability generating function (PGF) of U and is defined as the mean value of $z^U$.

$$\Psi_U(z,t)=E[\ z^U]\ =\sum_{n=-\infty}^{\infty}P[\ U(t)=n]\ z^n. \tag{11}$$

It is noted that PGF's of two independent variables A(t) and S(t) satisfy the following relation,

$$\Psi_{A-S}(z,t)=\Psi_A(z,t)\,\Psi_S(z^{-1},t). \tag{12}$$

Using (9-12), we obtain an approximate QLD, Q'(r) for general queueing models in the continuous time process as follows ;

$$Q'(r)=\max_{t>0}\left\{\min_{n\geq 1}\{\Psi_A(z,t)\Psi_s(z^{-1},t)z^{-(r+1)}\}\right\}. \tag{13a}$$

On the other hand, for the discrete time process, our approximate QLD is given by

$$Q'(r)=\max_{s>0}\left\{\min_{z\geq 1}\{\Psi_A(z,s)\Psi_S(z^{-1},s)\,z^{-(r+1)}\}\right\},\ s=1,2,3\cdots \tag{13b}$$

Using (13), we cannot determine bound characteristics of our approximation, because both the lover and upper bound techniques are utilized in the formalism. However, we believe that for most queueing models this approach gives a better approximation than other methods using multiple bounds of the same bound characteristics.

To further improve our approximation of QLD, we introduce a parameter $r_0$ in (13) and denote the formula with Q''(r).

$$Q''(r)=\max_{t>0}\left\{\min_{z\geq 1}\{\Psi_A(z,t)\,\Psi_S(z^{-1},t)\,z^{-(r+1+r_0)}\}\right\}.(14a) \tag{14a}$$

Utilizing (6b), $r_0$ may be obtained from the requirement $Q^{\sim}(0;II) = \rho$. For queueing models in the discrete time domain, a similar formula is given by

$$Q^{\sim}(r) = \max_{s \geq 1} \left\{ \min_{z \geq 1} \{ \Psi_A(z,s) \Psi_S(z^{-1}, s) \right. \\ \left. z^{-(r+1+r_0)} \} \right\}. \tag{14b}$$

It is noted that the PGF of a traffic model is dependent on boundary conditions at the start and the end of a time interval. For instance, in the derivation of the PGF of the number of arrivals using Scheme $I$, the type of cells arriving at the start and the end of an interval must be taken into account statistically to determine the boundary conditions of arrival process, while these boundary conditions are not so important in Scheme $II$. Hence, the QLD in Scheme $II$ is easier to calculate than that in Scheme $I$. To calculate the QLD is Scheme $I$, one may first calculate the QLD in Scheme $II$ and subsequently use (6a),

$$Q^{\sim}(r; I) = \frac{1}{\rho} Q^{\sim}(r+1; II). \tag{15}$$

The calculated results using (13) in Scheme I are generally not equal to those obtained from (15). However, for most probability values and utilization factors, the difference between two results is small enough to be negligible.

Both (13) and (14) are quite simple in form and favorable to a fast numerical calculation. Furthermore, they can be easily applicable to a superposition of independent arriving process, since the PGF of a superposition of independent arrival process is given by the product of PGF of individual sources.

Let the functions in the right term of (9) and (13) for given $r \geq 0$ have the maximum values at $t = \tau(r)$ and $\tau'(r)$, respectively. As $\tau(r)$ is approximately equal to $\tau'(r)$, let us ignore the difference between them. It is noted that $A(t)-B(t) = q_t - q_0$ in (8). Hence, $\tau(r)$ may be understood as the most probable time inteval that an increment in queue length is greater thn r under the condition of $S(\tau) = B(\tau)$ or, equivalently, $N(0, \tau(r)) = 0$. $\tau(\tau)$ increases monotonically as $\tau$ increases, because the accumulation of th greater number of cells in queue usually needs thelonger time interval. As $\rho$ increases, the right termof (13), as a function of t, is enhanced at larger t values, and consequently $\tau(\tau)$ increases. However, for a given r, it remains finite unless the utilizationfactor exceeds one. The characteristics of $\tau(\tau)$ sensitively depend on multiplexing conditions of arriving customers as well as on the type of the traffic model.

## Ⅳ. APPLICATION TO MMPP/D/1

In this section, we apply our approximation formalismto MMPP/D/1 in an ATM multiplexer. We also consider our approximation for a superposed MMPP/D/1 and M/D/1. An MMPP is a doubly stochasticPoisson process where the rate process is determined by the state of a continuous time Markow chain. In this paper, we consider a two state Markov chain, where the mean sojourn times in states 1 and 2 are $r_1^{-1}$ and $r_2^{-1}$, respectively. When thechain is in state $i$, the arrival process is assumed to be Poisson with the mean arrival rate $\lambda_i$. Theservice process is deterministic with the transmission capacity of one cell per time slot.

We will calculate $Q^{\sim}(r;II)$ using(14b). To obtain the QLD in Scheme $I$, the calculated result of $Q^{\sim}(r;II)$ is substituted into (15). In Scheme $II$, the time domain is restricted to positive integers representing the number of time slots. The PGF of the numbr of arrivals in $s$ time slots is given by[8]

$$\Psi_A(z,s) = \pi \exp\{[R + (z-1)\Lambda] s\} e, \tag{16}$$

where, forthe two-state MMPP,

$$R = \begin{pmatrix} -r_1 & r_1 \\ r_2 & -r_2 \end{pmatrix}, \Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, e = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

and the equilibrium probability vector is given by

$$\pi = \frac{1}{r_1 + r_2}(r_2, r_1).$$

The exponential function of a square matrix cna be easily calculated[9]. On the other hand, the PGF of the service capacity in the same interval is

$$\Psi_S(z^{-1}, s) = \left(\frac{1}{z}\right)^{s-1}. \tag{17}$$

Substituting (16) and (17) into (14b), we have

$$Q^{-}(r, \Pi) = \max_{s>1}\left\{\min_{z>1}\left\{\frac{z^{-(r+s+r0)}}{x_+ - x_-}\left[e^{sx_+}(y\lambda - x_-) + e^{sx_-}(y\lambda - x_+)\right]\right\}\right\}, \tag{18}$$

where

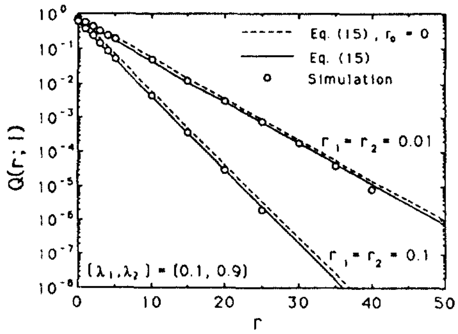
$$\lambda = \frac{\lambda_1 r_2 + \lambda_2 r_1}{r_1 + r_2}$$

represents the mean arrival rate of cells in the stationary two-state MMPP. $\lambda$ is equivalent to the utilization factor $\rho$ in this case. $\chi_\pm$ denote two eigenvalues of the matrix $R+(z-1)\Lambda$,

$$x_\pm = \frac{1}{2}\{-(r_1+r_2) + y(\lambda_1 + \lambda_2) \pm\sqrt{[r_1 - r_2 - y(\lambda_1 - \lambda_2)]^2 + 4r_1 r_2}\},$$

and $y \equiv z-1$.

Fig. 1 shows the calculated QLD for arriving customers in a MMPP/D/1 queue as a function of the queue length. $\lambda_1$ and $\lambda_2$ of the two-state MMPP were taken to be 0.1 and 0.9, respectively. We considered two sets of $\{r_1, r_2\}$ = {0.01, 0.01} and {0.1, 0.1} at a fixed utilization factor $\rho$ = 0.5. The dashed and solid lines are the calculated results obtained using (18) and (15) with $r_0$ = 0 and $r_0$ satisfying (6b), respectively. It should be noted that the calculated results are defined only at integer values of r and the drawn lines are to guide the eye. The empty circles are a few data obtained from a direct simulation of the system. Both of the calculated results show a good agreement with those obtained by a simulation of the system, though the solid lines are slightly closer to the empty circles. For $r_1$ = $r_2$ = 0.1, and 0.01, the calculated $k(0)$ is given by 1.15 and 1.35, respectively. For the same utilization factor, both $k(0)$ and QLD increase as $r_i$ decreases, in other words, the duration of two phases of the two-state MMPP increases. We consider that $k(0)$ is useful to characterize various traffic models in an ATM multiplexer and needs a further study.



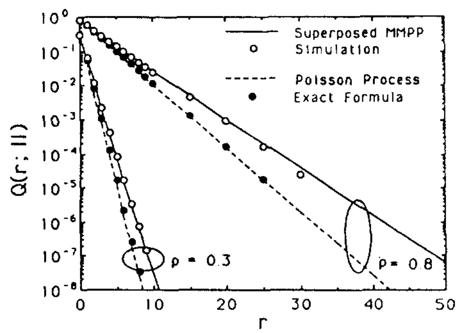

1) The QLD of MMPP/D/1 in Scheme I at $\rho$ = 0.5. The solid and dashed lines are calculated by substituting (18) into (15) with $r_0$ satisfying (6b) and $r_0$ = 0, respectively, and the empty circles are associated with a direct simulation of the system.

If m types of heterogeneous and independent Markov Modulated Poisson sources are superposed at the input of an ATM multiplexer, the PGF of the number of arrivals can be obtained by the product of the PGF of each source,

$$\Psi_A(z, s) = \prod_{j=1}^{m} \Psi_{A_j}(z, s)^{N_j},$$

where $\Psi_{A_j}$ and $N_j$ are the PGF and number of sources of type j MMPP, respectively.

Fig. 2 displays the calculated results for QLD of a superposed MMPP/d/1 queue in Scheme *II* with the solid lines. For this calculation we used (6b) and (14b). Using three types of MMPP, j = 1, 2, and 3, we took $\{\lambda_{j1}, \lambda_{j2}; 1/r_{j1}, 1/r_{j2}\}$={0.3, 0.05; 2, 8}, {0.03, 0.005; 20, 80} and {0.003, 0.0005; 200, 800}. We calculated the $Q(r;II)$ for two utilization factors $\rho$ = 0.3 and 0.8 with the number of sources $\{N_j\}$ = {1, 10, 100} and {3, 30, 200}, respectively. The empty circles represent the results by a simulation of the system. The calculated results also show a good agreement with a simulation of the system.




2) The QLD of a superposed MMPP/D/1 and M/D/1 in Scheme II at $\rho$ = 0.3 and 0.8. The solid lines (the empty circles) represent the calculated results (a direct simulation) of a superposed MMPP/D/1, respectively. The number of sources in three types of MMPP are given in the text. The dashed lines (the filled circles) refer to the calculated results (the exact formula in (21)) of M/D/1.

As a special case of MMPP/D/1, let us also consider the QLD of M/D/1 in Scheme *II*. Since the formula obtained in our approach can be represented in a simple form for this queueing system, it would be instructive to consider it in more details below. For M/D/1, one may simply set $\lambda_1=\lambda_2=\lambda$ in all of the equations driven for the QLD of MMPP/D/1. The PGF of the number of arrivals from a Poisson source in s time slots is given by

$$\Psi_A(z,s)=e^{\lambda s(z-1)}. \tag{19}$$

Hence, substituting (17) and (19) into (13b), we have

$$Q'(r;II)=\max_{s\geq}\left\{z_s^{-(r+s+r_0)}e^{\lambda s(z_s-1)}\right\}, \tag{20}$$

where

$$z_s=\max\left\{1,\frac{r+s+r_0}{\lambda s}\right\},$$

and $r_0$ is chosen to satisfy $Q'(0;II)=\rho$. In Fig. 2, we also plotted the calculated results of QLD of M/D/1 for $\rho$ = 0.3 and 0.8. The dashed lines represent $Q'(r;II)$ obtained using (20), while the filled circles denote the exact formula[10] of M/D/1 given by

$$Q(r;II)=1-(1-\rho)\sum_{i=0}^{r}\frac{(-\rho i)^{r-i}}{(r-i)}e^{\rho i}, \tag{21}$$

where $\rho$ is the utilization factor which is equal to $\lambda$ in the present case. For a stable queue and r > 0, our approximation shows an extremely tight lower bound below the exact solution. Furthermore, both the exact and approximate solutions are almost logarithmically linear in r. This results may be uderstood if we consider a further approximation of (20) for real numbers s. In this approximation, $Q'(r;II)$ can be simplified in the form

$$Q'(r;II)=\left(\frac{1}{\sigma}\right)^{r+r_0}, \tag{22}$$

where $\sigma$ is the solution to the following equation,

$$\lambda(\sigma-1)=\ell n(\sigma),\ \sigma>1, \tag{23}$$

$r_0$ is obtained from the conditon $\sigma^{-r_0}=\rho$. $\gamma(r)$ is

$$\gamma(r)=\frac{r+r_0}{\lambda\sigma-1},$$

and the corresponding $z_r$ is

$$z\tau = \sigma .$$

It is noted that $\sigma$ is analogous to $1/\rho$ of M/M/1. Though we did not show in the figure, we observed that the difference between (20) and (22) is negligible for most $\rho < 1$.

## V. CONCLUSION

In summary, we proposed an approximation approach to the QLD of single server queueing system. Our formalism is based on two steps of bound techniques, one lower bound below the exact solution and a subsequent upper bound on the former approximation. The calculation of our formula for the QLD is to find a global saddle point in two dimensions. Utilizing the PGF of arrival and service processes independdently, the QLD of a superposition of general input sources can be obtained within a good approximation. In this paper we applied our approximation method to MMPP/D/1 in an ATM multiplexer. The calculated result showed an excellent with either the exact solution or a simulation result of the system. Furthermore, numerical algorithms based on our approach provide an extremely fast result. For instance, on a 486 IBM PC, the algorithm for MMPP/D/1 requires a few milliseconds to calculate the QLD in case of 2 state MMPP. Hence, our formalism may be useful for the real-time analysis on the QLD of general traffic in an ATM multiplexer.

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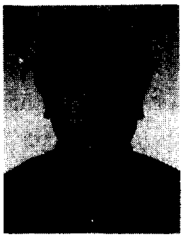
## REFERENCES

1. R. Nagarajan, J. F. Kurose, and D. Towsley, "Approximation techniques for computing packet loss in finite-buffered voice multiplexers", IEEE J. Selected Areas Commun., vol.9, no.3, pp.368-377, 1991.
2. P. Humblet, A. Bhargava, and M. G. Hluchyi, "Ballot theorems applied to the transient analysis of nD/D/1 queues", IEEE/ACM Trans. Network., vol.1, no.1, pp.81-95, 1993.
3. For instance, R. O. Onvural, Asynchronous transfer mode netwoeks, performance issues, Artech House, Inc. Boston, Massachusetts, 1993.
4. K-S. Lee and H. S. Park, "Approximation of the queue length distribution of general queues", ETRI J., vol.15, no3/4, pp.35-45, 1994.
5. K-S. Lee, Y. S. Kim, and H. S. Park, "Approximate queue length distribution of general queues: Application to the M+$\Sigma N_j D_j$/M/1 queue", Submitted to J. Kor. Inst. Commun. Sci.
6. K. Nakagawa, "Loss and waiting time probability approximation for general queueing", IEICE Trans. Commun., vo.E76-B, no.11, pp.1381-1388, 1993.
7. F. Hubner, "Analysis of a finite-capacity asynchronous multiplexer with deterministic traffic sources", 7th ITC Seminar, Morriston, New Jersey (Oct 1990) paper 11. 2.
8. M. F. Neuts, "A versatile Markovian point process", J. Appl. Prob. vol.16, pp.764-779, 1979.
9. P. L. Lancaster and M. Tismenetsky, The theory of matrices with applications, 2nd ed. Academic Press, Inc. Orlando, Florida, 1985.
10. J. W. Roberts and J. T. Virtamo, "The superposition of periodic cell arrival stream in an ATM multiplexer", IEEE Trans. Commun., vol.39, no.2, pp.298-303, 1991.



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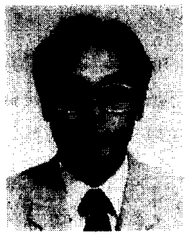

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