

# Error Performance of BPSK and QPSK Signals in Mobile-Satellite Communication Channel

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## 이동 위성 통신 채널에서의 BPSK 및 QPSK의 오류 특성

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### ABSTRACT

The error performance of BPSK and QPSK signals in mobile-satellite channel is investigated considering nonlinearity of TWTA (Traveling Wave Tube Amplifier) in the presence of AWGN (Additive White Gaussian Noise) on the uplink and downlink paths. It is assumed that the fading on the downlink path forms a Rician distribution.

The Rician distribution is approximated by discrete probability values. The values are firstly found by Classical Moment Technique. Finally, the error probability is evaluated using approximate discrete values of Rician distribution and the Gaussian Quadrature Formula.

### 要 約

업링크와 다운링크 경로상에 각각 가산성 백색 가우스잡음이 존재하는 이동 위성 통신로에서 TWTA의 비선형성에 의한 BPSK 신호와 QPSK 신호의 오류성능을 조사하였다. 다운링크 경로상의 페이딩은 Rice 분포를 한다고 가정하였다. Rice 분포에 대한 근사 이산 확률분포를 고전적인 모멘트 기법(CMT)을 이용하여 처음으로 유도하였다. 최종적으로 오류는 근사 이산 확률 분포와 Gauss Quadrature Formula를 이용하여 계산하였다.

### I. INTRODUCTION

The major difference between mobile-satellite communication and terrestrial mobile communication is in path loss, noise environment, and fading characteristics<sup>(1)</sup>. That is, in the analysis of mobile-satellite communication channel, not only an analysis for the environment of the fixed satel-

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lite communication channel but also analysis for the mobile environment should be considered.

The objective of this paper is to investigate the error performances of BPSK (Binary Phase Shift Keying) & QPSK (Quaternary Phase Shift Keying) signals transmitted over the mobile-satellite channel considering nonlinearity of TWT amplifier in the presence of AWGN (Additive White Gaussian Noise) on the uplink and downlink paths.

For mobile-satellite communication, the channels can be modeled in most cases, as non-frequency selective Rician channels for which the fading amplitude obeys a Rician distribution. It is known that fading degrades the performance of communication systems. The fade margin for BPSK and QPSK systems on channel impaired by Rician fading has been examined<sup>(2)</sup>, and some error performances considering just the path loss on the downlink and Rician channel had been driven<sup>(3), (4)</sup>.

For the fixed satellite communication, however, the error performances considering the path losses on the uplink and downlink had been analyzed, and it had been proven through such analyses that the uplink noise parameter has a serious effect on the system performance<sup>(5), (6), (7), (8)</sup>.

Therefore, in mobile-satellite communication, the analysis on the error performance including the uplink noise parameter as well as nonlinear TWT amplifier in addition to the conventional analyses considering the fading effect on the downlink channel is required in order that it may be used as a method for design of the system to reduce the degradation of such system.

Accordingly, this paper provides the overall performance analysis considering the physical environments, that is, the path losses on the uplink and downlink, nonlinear TWT (Travelling Wave Tube) amplifier, and Rician channel.

The overall performance is obtained by using the discrete values for the Rician pdf. The values are firstly found in this paper by applying the Classical Moment Technique that is more accurate than Gauss Quadrature Formula being generally used to solve complex performance function.

## II. SYSTEM MODEL

A bandlimited nonlinear mobile-satellite communication system is modelled as shown in Fig. 1 in which the effect of fading is considered in the downlink path with AWGN in the uplink and the downlink paths. We also assume that the bandwidth of the downlink filter is wide enough to pass the transponder output without significant distortion.

Let us consider the PSK signal sequence to be as

$$m(t) = \sqrt{2P} \sum_{\mathbf{k}} p(t-kT) \cos(\omega_c t + \theta_{\mathbf{k}}) \quad (1)$$

where  $P$  is the transmitted signal power,  $T$  is the symbol duration,  $\omega_c$  is the angular frequency of carrier,  $\theta_{\mathbf{k}}$  is the transmitted phase taken from one of  $M$ -phases  $\left\{ \frac{2\pi}{M} i, i=0, 1, \dots, M-1 \right\}$ , and  $p(t)$  represents pulse shaping and unity over each symbol duration. We assume  $\theta_{\mathbf{k}}$ , the phases of transmitted signal, to be equally likely. Supposing that the front end filter at the satellite repeater is transparent, we can express the uplink signal as

$$x(t) = m(t) + n_u(t) \quad (2)$$

where

$$n_u(t) = n_{uc}(t) \cos \omega_c t - n_{us}(t) \sin \omega_c t. \quad (3)$$

In Eq. (3),  $n_u(t)$  is a narrowband Gaussian noise and  $n_{uc}(t)$ ,  $n_{us}(t)$  are baseband Gaussian process. Since the uplink noise is a narrowband Gaussian process with zero-mean and power equal to  $\sigma_u^2$ , the envelope  $R$  and the phase  $\eta$  of signal in uplink path is identical with the case of a sine wave plus a narrowband Gaussian process as given in Ref. (5)

$$x(t) = R(t) \cos(\omega_c t + \theta_0 + \eta(t)). \quad (4)$$

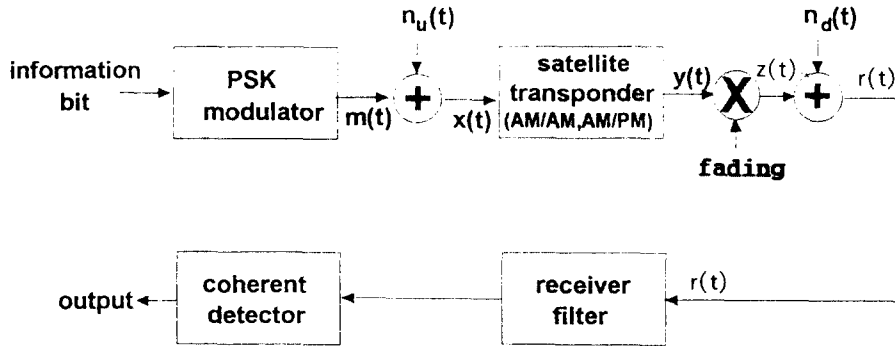


Fig. 1. System model for PSK mobile satellite communication system.

The probability density function of  $R$  and  $\eta$  is given in Ref. (9)

$$p_u(R, \eta) = \frac{R}{2\pi \sigma_u^2} \exp\left[-\frac{R^2 + 2P - 2\sqrt{2P} R \cos \eta}{2 \sigma_u^2}\right]. \quad (5)$$

The output signal of the transponder is expressed as<sup>(5)</sup>

$$y(t) = f(R) \cos(\omega_c t + \theta_0 + \eta(t) + g(R)) \quad (6)$$

where  $f(R)$  represents the AM/AM conversion and  $g(R)$  AM/PM conversion at the satellite transponder. These two functions characterize the memoryless bandpass nonlinearity. The transponder output  $y(t)$  is attenuated and faded in downlink path. The envelope of attenuated and faded signal becomes  $F(R)$ . We assume the slow fading with a normalized amplitude  $\rho = \left(\sqrt{\frac{F(R)^2}{F(R)^2}}\right)$  which is Rician distributed<sup>(10)</sup>,

$$p(\rho) = 2\rho(1+K) \exp[-K - \rho^2(1+K)] \times I_0(2\rho\sqrt{K(1+K)}) \quad ; \rho \geq 0 \quad (7)$$

where the parameter  $K$  is the power ratio of coherent (line-of-sight) component to noncoherent (diffuse) component. Rayleigh fading is a special case of the Rician fading model and is corre-

sponding to  $K=0$  which characterizes terrestrial mobile radio channel.

And then the attenuated and faded signal is mixed with downlink Gaussian noise  $n_d(t)$  with zero-mean and power equal to  $\sigma_d^2$ .

We assume a conventional demodulator which includes a sampler taking samples at an optimum instant on the quadrature and inphase channels. It is assumed that the carrier tracking loop will provide the mean phase  $g$  of the phase randomness due to AM/PM distortion. The decision device shall determine the transmitted phase based on the observation of the sample which can be expressed as

$$r = z + n_{di}, \quad z = F(R) \cos(\theta_0 + \eta + g(R) - \bar{g}) \quad (8)$$

$$r = z + n_{ds}, \quad z = F(R) \sin(\theta_0 + \eta + g(R) - \bar{g}) \quad (9)$$

The pair  $(r, r)$  is processed by the signal detector to determine which one of  $M$  phases,  $\theta_0$ , was transmitted according to the phase decision zone in which  $(r, r)$  lies (see Fig. 2).

In this mobile satellite system, the TWT amplifier is typically characterized for AM/AM and AM/PM as follows<sup>(8)</sup>

$$f(R) = \alpha_a R / (1 + \beta_a R^2), \quad (10)$$

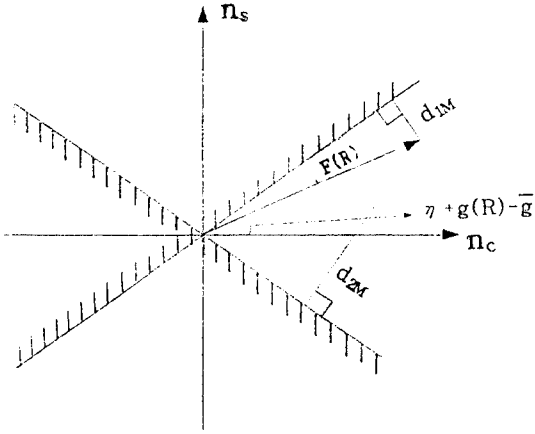


Fig. 2. Decision zone for detecting transmitted phase.

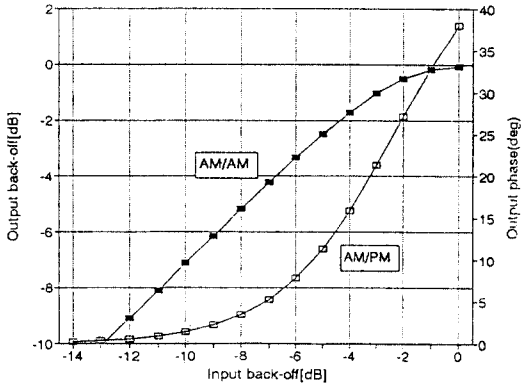


Fig. 3. Transfer characteristic of TWTA.

$$g(R) = \alpha_b R^2 / (1 + \beta_b R^2) \quad (11)$$

where

$$\begin{aligned} \alpha_a &= 1.9638, & \beta_a &= 0.9945, \\ \alpha_b &= 2.5293, & \beta_b &= 2.8168. \end{aligned}$$

The transfer function of the above typical TWT amplifier is shown in Fig. 3.

### III. CONDITIONAL ERROR RATE

Without loss of generality, we assume that the transmitted phase is zero, i.e.,  $\theta_0 = 0^\circ$ . The receiver bases its decision on the pair  $(r, \bar{r})$  where an error occurs if and only if  $(r, \bar{r})$  falls outside of the decision zone of  $\frac{2\pi}{M}$  radian centered at  $\theta_0 = 0^\circ$ . Then, the conditional average error rate may be expressed as<sup>(11), (12)</sup>

$$P_e(n_{dc}, n_{ds}, |R, \eta, \gamma) = \frac{1}{2} \operatorname{erfc}\left[\frac{z}{\sqrt{2} \sigma_d}\right] \text{ for BPSK,} \quad (12)$$

$$\begin{aligned} P_e(n_{dc}, n_{ds}, |R, \eta, \gamma) &= \frac{1}{2} \operatorname{erfc}\left[\frac{d_{1M}}{\sqrt{2} \sigma_d}\right] \\ &+ \frac{1}{2} \operatorname{erfc}\left[\frac{d_{2M}}{\sqrt{2} \sigma_d}\right] \\ &- \frac{1}{4} \operatorname{erfc}\left[\frac{d_{1M}}{\sqrt{2} \sigma_d}\right] \\ &\cdot \operatorname{erfc}\left[\frac{d_{2M}}{\sqrt{2} \sigma_d}\right] \text{ for QPSK} \end{aligned} \quad (13)$$

where

$$z = F(R) \cos(\eta + (g(R) - \bar{g})),$$

$$d_{1M} = F(R) \sin\left(\frac{\pi}{4} - \eta - (g(R) - \bar{g})\right),$$

$$d_{2M} = F(R) \sin\left(\frac{\pi}{4} + \eta + (g(R) - \bar{g})\right).$$

$$\gamma = F(R)^2 / 2 \sigma_d^2$$

### IV. AVERAGE ERROR RATE

The conditional error rate  $P_e(n_{dc}, n_{ds} | R, \eta, \gamma)$  should be averaged over the statistics of  $R, \eta$  which are function of the uplink noise and  $\gamma$  of Rician fading.

Thus, the average error rate is

$$\begin{aligned} P_E &= \int_0^\infty \int_0^{2\pi} \int_0^\infty P_e(n_{dc}, n_{ds} | R, \eta, \gamma) \\ &\cdot p_u(R, \eta) p(\gamma) dR d\eta d\gamma. \end{aligned} \quad (14)$$

In order to find the more exact average error rate and simplify the calculation, the standard Gaussian Quadrature Rule to carry out the expectation over R is used as follows :

$$\begin{aligned}
 p_u(R, \eta) &= \frac{R}{2\pi \sigma_u^2} \exp\left[-\frac{R^2 + 2P - 2\sqrt{2P} R \cos \eta}{2\sigma_u^2}\right] \\
 &= \frac{R}{2\pi \sigma_u^2} \exp\left[-\frac{P}{\sigma_u^2} \sin^2 \eta\right] \\
 &\quad \cdot \exp\left[-\frac{(R - \sqrt{2P} \cos \eta)^2}{2\sigma_u^2}\right]. \quad (15)
 \end{aligned}$$

In the Eq. (15), for convenience, we define a variable

$$x = \frac{R - \sqrt{2P} \cos \eta}{\sqrt{2} \sigma_u} \quad (16)$$

Thus the Eq. (14) can be expressed as follows<sup>(6)</sup> :

$$\begin{aligned}
 P_E &= \int_0^\pi \int_0^{2\pi} \int_a^\infty h(R', \eta) p_1(R', \eta) \\
 &\quad \cdot p(\gamma) e^{-x^2} dx d\eta d\gamma, \quad (17)
 \end{aligned}$$

$$p_1(R', \eta) = \frac{R'}{\sqrt{2} \pi \sigma_u} \exp\left[-\frac{P}{\sigma_u^2} \sin^2 \eta\right],$$

$$h(R', \eta) = P_e(n_{dc}, n_{ds} | R', \eta, \gamma),$$

$$R' = \sqrt{2} (\sigma_u x + \sqrt{P} \cos \eta),$$

$$a = -\frac{\sqrt{P}}{\sigma_u} \cos \eta.$$

Expressing again the Eq. (7) with  $\rho = \sqrt{\frac{\gamma}{\gamma_0}}$  ( $\gamma$  : instantaneous downlink CNR<sub>d</sub>  $\left( = \frac{F(R)^2}{2\sigma_d^2} \right)$ ,  $\gamma_0$  : average CNR<sub>d</sub>  $\left( = \frac{F(R)^2}{2\sigma_d^2} \right)$ ),

$$\begin{aligned}
 p(\gamma) &= \frac{(1+K)}{\gamma_0} \exp\left[-K - \frac{\gamma}{\gamma_0} (1+K)\right] \\
 &\quad \cdot I_0\left[2\sqrt{\frac{\gamma(K+K^2)}{\gamma_0}}\right]. \quad (18)
 \end{aligned}$$

Then, the Nth moment of  $\gamma$  is

$$\mu_n = \int_0^\infty \gamma^n p(\gamma) d\gamma. \quad (19)$$

From Eqs. (21) and (22), we can show that (see APPENDIX A)

$$\mu_n = \exp(-K) \sum_{m=0}^{\infty} \frac{K^m \Gamma(m+n+2)}{m! \Gamma(m+1) (1+m+n) \left(\frac{K+1}{\gamma_0}\right)^n} \quad (20)$$

The approximate discrete probability distribution  $\gamma_\ell$ ,  $\omega_\ell$  ( $\ell = 1, 2, \dots, \nu$ ) for the random variable  $\gamma$  is required to satisfy the moment constraints

$$\mu_K = \sum_{\ell=1}^{\nu} \omega_\ell \gamma_\ell^K. \quad (21)$$

The above discrete probability distribution  $\gamma_\ell$ ,  $\omega_\ell$  is obtained by the Classical Moment Technique (see Appendix B).

In general, as the order of approximation is increased, we can obtain the more accurate performance. In this study, we have found that only a relatively low order of approximation ( $\nu \leq 6$ ) is needed for the Rician distribution since the numerical results approach the same quantity as the order of approximation is increased (see Fig. 4).

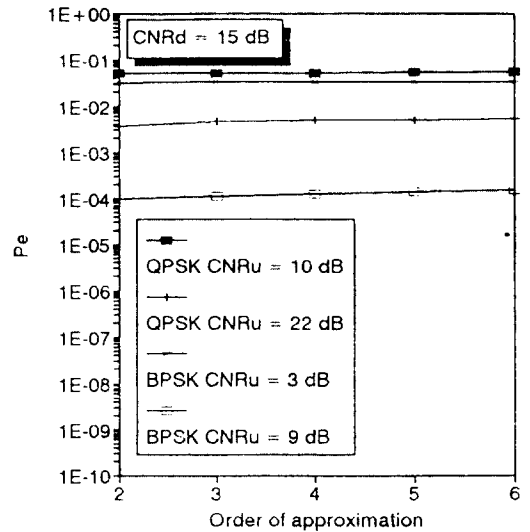


Fig. 4. Accuracy of the classical moment applied to the performance evaluation of BPSK and QPSK.

Then by the Gaussian Quadrature Rule and the Classical Moment Technique for obtaining the approximate discrete probability density  $\gamma_\ell$ ,  $\omega_\ell$  for the above  $p(\gamma)$ ,  $P_E$  can be expressed as

$$P_E = \sum_{\ell} \sum_n \sum_m \omega_\ell C_n C_m h(x_m, \eta_n, \gamma_\ell) p_1(x_m, \eta_n) \cdot u(x_m + \frac{\sqrt{P}}{\sigma_u} \cos \eta_n) \quad (22)$$

where

$$u(\cdot) = \begin{cases} 1, & \geq 0 \\ 0, & < 0 \end{cases}$$

$(C_m, x_m)$ : Gauss-Hermite quadrature parameters given in Ref. (13)

$(C_n, \eta_n)$ : Equal weight parameters from the trapezoidal integration rule

$(\omega_\ell, \gamma_\ell)$ : Weight parameters obtained by Classical Moment Technique.

### V. NUMERICAL RESULTS

The numerical results given here are for BPSK and QPSK signals with fading factor  $K$ , using the numerical technique presented in this previous section. The characteristics of the TWTA for this paper are shown by the set of curves given in Fig. 3, and the input back-off for evaluating the numerical results is 2.5 dB (see Fig. 3). In order to demonstrate the convergence of the numerical results with respect to the order of approximation used for the Rician distribution, Fig. 4 shows the error performance of BPSK and QPSK in the same environment. With investigation from Fig. 5 to Fig. 9, it is obvious that uplink noise parameter has a serious effect on the system performance, and particularly the case accompanying the fading on the downlink has more serious effect. And also we can see that for a fixed fading factor  $K$  the curves of error performance converge to constant values at low uplink and downlink CNR's. Therefore, this presents a good

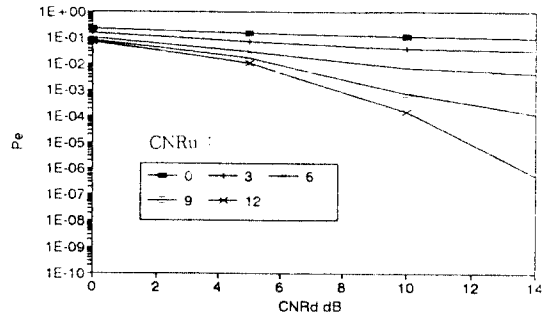


Fig. 5. Performance of coherent BPSK system with non fading.

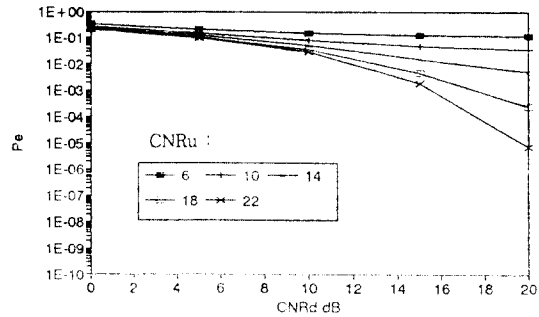


Fig. 6. Performance of coherent QPSK system with non fading.

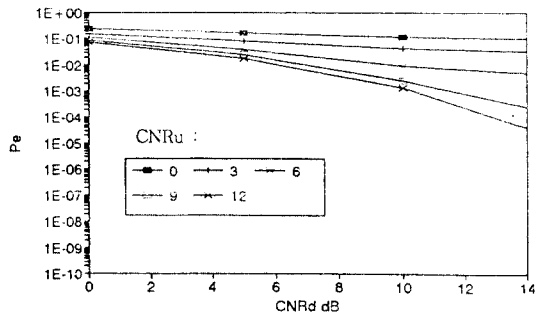


Fig. 7. Performance of coherent BPSK system with fading ( $K = 10$ )

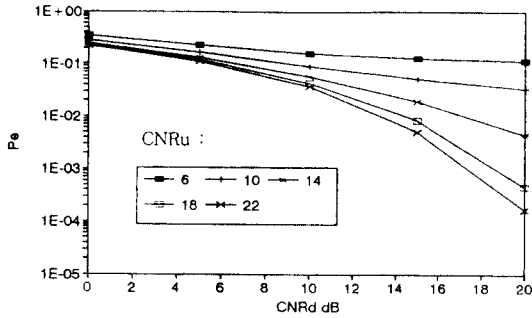


Fig. 8. Performance of coherent QPSK system with fading ( $K = 10$ )

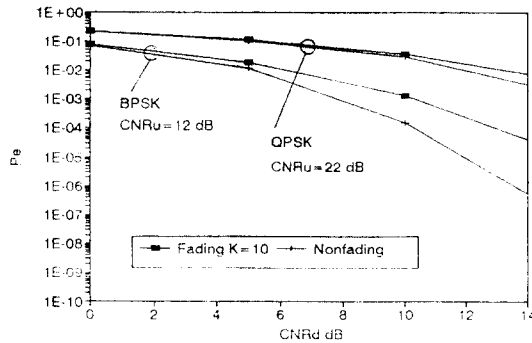


Fig. 9. The effect of fading on BPSK and QPSK at given  $CNR_u$ .

reason why an equalizer or diversity scheme should be necessarily adopted in mobile-satellite communication system.

### VI. CONCLUSIONS

This paper provides an overall performance analysis to optimize the design of the BPSK and QPSK modulation schemes including the uplink noise parameter as well as nonlinear TWT amplifier in addition to the downlink fading effect presented in the conventional analyses on mobile satellite communication. To make possible the performance evaluation having numerical complexity, the discrete probability values of the Rician pdf

are firstly obtained by using the Classical Moment Technique. The accuracy of introducing the discrete values in evaluating the performance is confirmed graphically in this paper. Therefore, it is shown that it is sufficiently accurate to be used as a method for design of the system to reduce the degradation in the performance of mobile-satellite channel considering nonlinearity in TWT amplifier.

The results of numerical evaluation are given for a fixed fading factor. It is shown that (1) the uplink noise parameter has a serious effect on the system performance, (2) particularly the case accompanying the fading on the downlink has more serious effect, (3) for a fixed fading the curves of error performance converge to constant values at low uplink and downlink CNR's, (4) we should be careful not to neglect the effects of uplink noise in analyzing the performance of a mobile-satellite transmission system, and (5) it is necessary that a processing function (equalizer) be added before the high power amplifier of the transponder and also the diversity scheme be added at the receiving end for compensation of fading.

### APPENDIX A DERIVATION OF Eq. (20)

$$\begin{aligned}
 \mu_w &= \int_0^\infty \gamma^n \frac{(K+1)}{\gamma_0} \exp\left[-K - \frac{\gamma}{\gamma_0} (K+1)\right] \\
 &\quad \cdot I_0 \left[ 2\sqrt{\gamma \frac{(K^2+K)}{\gamma_0}} \right] d\gamma \\
 &= \frac{(K+1)}{\gamma_0} \int_0^\infty \gamma^n \exp(-K) \exp\left[-\frac{\gamma}{\gamma_0} (K+1)\right] \\
 &\quad \cdot I_0 \left[ 2\sqrt{\gamma \frac{(K^2+K)}{\gamma_0}} \right] d\gamma \\
 &= \frac{(K+1)}{\gamma_0} \exp(-K) \int_0^\infty \gamma^n \exp\left[-\frac{\gamma}{\gamma_0} (K+1)\right] \\
 &\quad \cdot I_0 \left[ 2\sqrt{\frac{\gamma(K^2+K)}{\gamma_0}} \right] d\gamma \quad (A.1)
 \end{aligned}$$

Using the formula

$$I_0(2\sqrt{b\gamma}) = \sum_{m=0}^{\infty} \frac{(2\sqrt{b\gamma}/2)^{2m}}{m! \Gamma(m+1)} = \sum_{m=0}^{\infty} \frac{(b\gamma)^m}{m! \Gamma(m+1)} \quad (\text{A.2})$$

and

$$\int_0^{\infty} \exp(-at^x) t^y dt = (1+y)^{-1} \Gamma\left(\frac{x+y+1}{x}\right) a^{-\frac{x+1}{x}} \quad (\text{A.3})$$

$Re\ x > 0, \quad Re\ y > -1, \quad Re\ a > 0$

we can obtain  $\mu_n$  as follows :

$$\begin{aligned} \mu_n &= \frac{(K+1)}{\gamma_0} \exp(-K) \sum_{m=0}^{\infty} \frac{(\frac{K^2+K}{\gamma_0})^m \Gamma(m+n+2)}{m! \Gamma(m+1) (1+m+n) (\frac{K+1}{\gamma_0})^{(m+n+1)}} \\ &= \exp(-K) \sum_{m=0}^{\infty} \frac{(\frac{K^2+K}{\gamma_0})^m \Gamma(m+n+2)}{m! \Gamma(m+1) (1+m+n) (\frac{K+1}{\gamma_0})^{(m+n)}} \\ &= \exp(-K) \sum_{m=0}^{\infty} \frac{K^m \Gamma(m+n+2)}{m! \Gamma(m+1) (1+m+n) (\frac{K+1}{\gamma_0})^n} \end{aligned} \quad (20)$$

#### APPENDIX B

##### A CONSTRUCTION METHOD FOR CMT

In this section, we shall present an algorithm to construct the approximate discrete probability distribution  $\gamma_\ell, \omega_\ell$  ( $\ell = 1, 2, \dots, \nu$ ) for a random variable  $\gamma$  where only moments  $\mu_K, K = 0, 1, 2, \dots, N$  are known. This approximate probability distribution is required to satisfy the moment constraints

$$\mu_K = \sum_{\ell=1}^{\nu} \omega_\ell \cdot \gamma_\ell^K, \quad \text{for } K \leq N \quad (\text{B.1})$$

We set  $N = 2\nu - 1$  and define two polynomials

$$g(Z) = \sum_{k=0}^N \mu_k Z^k \quad (\text{B.2})$$

$$h(Z) = \sum_{k=0}^{\nu} \mu_{N+1+k} Z^k \quad (\text{B.3})$$

where  $\mu_{N+1+k}, K = 0, 1, 2, \dots$ , are extended moments defined by

$$\mu_K = \sum_{\ell=1}^{\nu} \omega_\ell \cdot \gamma_\ell^K, \quad \text{for } K \leq N+1, N+2, \dots \quad (\text{B.4})$$

One can show that the polynomial

$$\begin{aligned} \mu(Z) &= \sum_{k=0}^{\nu} \mu_k Z^k \\ &= g(Z) + Z^{N+1} h(Z) \end{aligned} \quad (\text{B.5})$$

or

$$= \sum_{\ell=1}^{\nu} \frac{\omega_\ell}{1 - \gamma_\ell Z} \quad (\text{B.6})$$

Let  $\sigma(Z) \equiv \prod_{\ell=1}^{\nu} (1 - \gamma_\ell Z)$  where the degree of the  $\sigma(Z)$  is less than or equal to  $\nu$ . Then, one has

$$g(Z) \sigma(Z) + Z^{N+1} h(Z) \sigma(Z) = \eta(Z) \quad (\text{B.7})$$

where

$$\eta(Z) \equiv \sum_{\ell=1}^{\nu} \omega_\ell \prod_{e \neq \ell} (1 - \gamma_e Z). \quad (\text{B.8})$$

Since the second term is of no concern, as will become clear later, we shall utilize the approach of the Euclid decoding algorithm to obtain  $\sigma(Z)$  and  $\eta(Z)$ .

First, start with two initial polynomials  $\gamma_{-1}(Z) \equiv Z^{N+1}$  and  $\gamma_0(Z) \equiv g(Z)$ . The Euclid algorithm is performed on  $\gamma_{-1}(Z)$  and  $\gamma_0(Z)$  until the degree of the  $h_{th}$  remainder  $\gamma_h(Z)$  is less than  $\nu$ .



$$\gamma_{-1}(Z) = q_1(Z)\gamma_0(Z) + \gamma_1(Z), \quad \deg \gamma_0 > \deg \gamma_1 \quad (\text{B.9})$$

$$\gamma_0(Z) = q_2(Z)\gamma_1(Z) + \gamma_2(Z), \quad \deg \gamma_1 > \deg \gamma_2 \quad (\text{B.10})$$

⋮  
⋮

$$\gamma_{i-2}(Z) = q_i(Z)\gamma_{i-1}(Z) + \gamma_i(Z), \quad \deg \gamma_{i-1} > \deg \gamma_i \quad (\text{B.11})$$

where  $q_i(Z)$  is a quotient polynomial and  $\gamma_i(Z)$  is the  $i_{th}$  remainder polynomial. The above relation can be put in a matrix form

$$\begin{bmatrix} \gamma_{i-2}(Z) \\ \gamma_{i-1}(Z) \end{bmatrix} = Q_i(Z) \begin{bmatrix} \gamma_{i-1}(Z) \\ \gamma_i(Z) \end{bmatrix} \quad (\text{B.12})$$

where

$$Q_i(Z) = \begin{bmatrix} q_i(Z) & 1 \\ 1 & 0 \end{bmatrix}. \quad (\text{B.13})$$

From Eq. (B.12), we obtain

$$\begin{bmatrix} \gamma_{i-1}(Z) \\ \gamma_0(Z) \end{bmatrix} = Q(Z) \begin{bmatrix} \gamma_{i-1}(Z) \\ \gamma_i(Z) \end{bmatrix} \quad (\text{B.14})$$

where

$$Q(Z) = Q_1(Z)Q_2(Z) \cdots Q_i(Z) = \begin{bmatrix} U_i(Z) & U_{i-1}(Z) \\ V_i(Z) & V_{i-1}(Z) \end{bmatrix}. \quad (\text{B.15})$$

Note that  $U_i(Z)$  and  $V_i(Z)$  are functions of  $q_i(Z)$ . One can show that the matrix  $Q(Z)$  is non-singular and its determinant is equal to  $(-1)^i$ . Therefore, one has

$$\begin{bmatrix} \gamma_{i-1}(Z) \\ \gamma_i(Z) \end{bmatrix} = (-1)^i \begin{bmatrix} V_{i-1}(Z) & -U_{i-1}(Z) \\ -V_i(Z) & U_i(Z) \end{bmatrix} \begin{bmatrix} \gamma_{i-1}(Z) \\ \gamma_0(Z) \end{bmatrix}. \quad (\text{B.16})$$

From this results, we have for  $h_{th}$  division

$$U_h(Z)g(Z) - Z^{h+1}V_h(Z) = (-1)^h \gamma_h(Z). \quad (\text{B.17})$$

Comparing Eq. (B.7) and Eq. (B.17), we identify that

$$\sigma(Z) = \frac{1}{C_0} U_h(Z) \quad (\text{B.18})$$

and

$$\eta(Z) = \frac{1}{C_0} (-1)^h \gamma_h(Z) \quad (\text{B.19})$$

where  $C_0$  is the constant term of the polynomial  $U_h(Z)$ .

The roots of  $U_h(Z)$  give  $\gamma_1^{-1}, \gamma_2^{-1}, \dots, \gamma_h^{-1}$  whose weights can be obtained from Eq. (B.19)

$$\omega_\epsilon = \frac{1}{C_0} \left( \frac{(-1)^h \gamma_h(\gamma_\epsilon^{-1})}{\prod_{\ell \neq \epsilon} (1 - \gamma_\ell \gamma_\epsilon^{-1})} \right). \quad (\text{B.20})$$

Since the long division is easy to perform, the method of finding the weights  $\omega_\epsilon$  appears to be considerably simpler than other schemes which essentially require solving Eq. (B.1) for  $(\gamma_\epsilon, \omega_\epsilon)$ .

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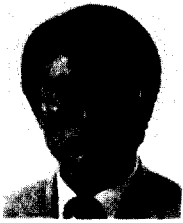
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