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일반적 큐의 큐길이 분포에 대한 근사방법 $M+\sum N_j D_j/M/1$ 큐에의 응용

正會員 李 奎 錫* 正會員 金 永 燮* 正會員 朴 弘 植*

Approximate Queue Length Distribution of General Queues : Application to The $M+\sum N_j D_j/M/1$ Queue

Kyu-Seok Lee*, Young Sup Kim*, Hong Shik Park* *Regular Members*

要 約

본 논문에서는 일반적인 큐잉 시스템의 큐길이 분포에 대한 근사 이론을 제안하였다. 제안된 근사 이론은 하한치와 상한치를 단계적으로 찾는 2단계 해석적인 근사 방법을 기초로 한다. 이를 이용하면 다양한 트랙픽원이 다중화된 모델의 큐길이 분포를 신속히 계산할 수 있다. 본 논문에서는 $M+\sum N_j D_j/M/1$ 큐잉 시스템에서 도착하는 고객이나 떠나는 고객이 관측한 큐길이 분포를 계산하고 시뮬레이션을 통해 얻은 결과와 비교하여 제안된 근사이론이 시뮬레이션 결과에 근접함을 확인하였다. 특히 M/M/1큐에 대해 근사이론으로 유도된 공식은 정해와 같았으며, D/M/1 큐에 대해서는 간단한 해석적인 공식을 얻을 수 있었다.

ABSTRACT

In this paper we develop an approximation formalism for the queue length distribution of general queueing models. Our formalism is based on two steps of analytic approximation employing both the lower and upper bound techniques. It is favorable to a fast numerical calcuation for the queue length distribution of a superposition of a superpostition of arbitary type traffic sources. In the application, $M+\sum N_j D_j/M/1$ is considered. The calculated result for queue length distribution measured by arriving or leaving customers shows a good agreement with the direct simulation of the system. Especially, we demonstrate that our formula for $M/M/1$ is equivalent to the exact solution, while that $D/M/1$ is simplified in an analytic form.

*韓國電子通信硏究所
Electronics And Telecommunications Research Institute
論文番號 : 93224
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I. INTRODUCTION

Statistical analysis of queueing systems is extremely important in the broad area of our society. For instance, in the future Broadband Integrated Service Networks(BISDN), a link of Asynchronous Transfer Mode (ATM) will be required to carry a wide variety of traffic types as well as large numbers of sources. Fast analysis of the statistical distribution of various parameters must be performed carefully to provide high quality services and to ensure efficient operation of switching networks. One of the most important information one needs to acquire in traffic analysis is the queue length distribution (QLD), which is defined as probability $P[Q>r]$ that the queue length Q is larger than r. In this paper, we consider the queue length as the number of customers in the system consisting of a queue and a server. In general, $P[Q>r]$ can be measured by either a random observer outside the system or arriving costumers at the system, though the probability measured by the latter observers is more appropriate for an estimation of the loss probability at a switching node with the finite size buffer.

In the fast few years, numerous approaches such as the analytic approximation and the direct numerical simulation of the system have been studied on the QLD of various complicated traffic models that have no exact analytic formulas.[1-8] The simulation method is one of the easiest way to acquire more of less an accurate QLD for most queueing systems, but it requires an excessive time to get a good sccuracy for large r values. For instance, a direct numerical simulation of a typical queueing ststem needs to generate more than 10^{12} customers to find the condition that r satisfies $P[Q>r]\approx 10^{-10}$. Hence, it is not feasible to implement this scheme in the future ATM switching systems where the loss rate is required to be of below the order of 10^{-10}.

On the oteher hand, most analytic approaches[5][6] have concentrated to find either the upper or lower bounds on the QLD. In such an approach, it is desirable to find a formalism which has following advantages ; (i) the formalism is favorable to a fast numerical computation, (ii) it can be applicable to the superposition of arbitrary type traffic sources, sources, and (iii) the deviation from the exact solution is small. It is noted that Nakagawa[6] made an approach based on the fundamental recursion formula and the Chernoff bound technique for general queueing problems. However, his formalism was not rigorous in a modification that the calculated result is further divided by a constant factor to match the exact solution for for $P[Q>0]$.

In this paper, we develop a new approximation formalism on the QLD for general queueing problems. Our formalism is based on two steps of analytic approximation. The first step is to find a lower bound below the exact formula. After this step the service and arrival processes can be considered separately. However, as the resulting form is still difficult to solve, we apply the Chernoff bound techniqe to find an upper bound on the distribution function obtained in the previous step. In this latter process, the probability generating function (FGF) of both the arrival and service processes are introduced in a natural manner. Consequently, we can handle the superposition of arbitrary type traffics easily, because the PGF of a superposed traffic sources can be represented by the product of PGFs of all individual sources. The bound characteristics is lost in our approach using mixed bound techniques. However, our philosophy on the problem is that this approach gains the better approximation than other formalisms using multiple bounds in the same direction[5][6]. Furthermore, our approach has other advantages mentioned above.

In the application of the proposed formalism, we will consider $M+\sum N_j D_j/M/1$ where a Poisson and a group of heterogeneous constant bit rate

(CBR) sources are superposed at the server whose service follows a Poisson process. This model is the generalization of $M/M/1$ but a special case of G/M/1 while maintaining the non-burstiness properties. The $G/M/1$ queue can find its application in packet switch networks. To the best of our knowledge, except for $M/M/1$, no analytic solution for the QLD of $M+\sum N_j D_j/M/1$ is known. We therefore compare our calculated results with the direct numerical simulation of the system. Especially for $M/M/1$, we will demonstrate that our formalism gives the well known analytic solutions.

In the application to other queueing problems such as $M+\sum N_j D_j/D/1$, our formalism showed a good agreement with other formalisms.[9] We also developed an approximate formalism for delay and waiting time distributions for single server queues utilizing the moment generation functions of arrival and service processes in the similar mixed bound techniques. This will be subject of another paper.[10]

II. FORMALISM

We consider a queue with the customer arrival producing a stable queue. The stable queue may be defined as that for an arbitrary $0 < \in < 1$, there exists a finite time interval T such that $1-P[N_0(t) > 1] < \in$ for $t > T$, where P denotes probability and $N_n(t)$ refers to the number of the event of $Q=n$ in an interval t.

Let A_t be the number of arriving customers in t and S_t be the number of customers that a server can finish its service without any idle period in the same interval. S_t is independent of A_t. The utilization factor of a queueing system is defined as

$$\rho=\lim_{t\to\infty}\frac{E[A_t]}{E[S_t]},$$

where $E[*]$ refers to the mean or expectation value. A stable queue is guaranteed when $\rho < 1$. For a stable queue, T is the sufficient time interval of an ensemble process for statistical analysis of the QLD. This means that the queue length at the present time is only affected by the arrival and transmission events occurred within T in the past. So we assume that the present time t_1 lies beyond T after the switching system is turned on, or in other words that the switching system is stationary.

For the analysis of a queueing process, we divide the time interval $(t_1 - T, t_1)$ into sufficiently large numbers of subinterval. These intervals are labeled as 1, 2, 3⋯ starting from t_1 to the past. Without loss of generality, the queue length is measured at the end of each section. In other words, we suppose that observers who can be either the incoming customers, the server or anyone, and they measure the queue length at the end of each section. It is noted that subintervals are not necessarily uniform. We have the following property of queue length q_i at the end of the i-th section.

$$q_i=q_{i+1}+a_i-b_i \geqq q_{i+1}+a_i-s_i, \tag{1}$$

where a_i and b_i the number of arriving and leaving customers in the i-th section, while s_i is associated with the number of customers that can be transmitted by a busy server in the same period. In other words, s_i is the sum of b_i and the number of customers that the server could have further served in its idls period. Using the recursion relation in Eq. (1), we find that the queue length at t_1 has the following relation for all $i \geqq 1$,

$$Q=q_1 \geqq q_{i+1}+A_i-S_i - S_i, \tag{2}$$

where $A_i=\sum_{j=1}^{i} a_j$ and $S_i=\sum_{j=1}^{i} s_j$. As Eq. (2) is true for all ensembles, it implies

$$P[Q > r] \geqq P[A_i-S_i > r], \tag{3}$$

for all $i \geqq 1$ and $r \geqq 0$. Hence, we have

$$P[Q>r] \geqq \max_{i \geqq 1}\{P[A_i - S_i - r - 1 \geqq 0]\}. \quad (4)$$

In general, as the size of the subinterval can be arbitrary, we can consider Eq. (4) in the continuous time domain, and have

$$P[Q > r] \geqq \max_{t>0}\{P[A_t - S_t - r - 1 \geqq 0]\}. \quad (5)$$

In the rest of our formalism, we will restrict to the case of continuous time process. Discontinuous time process can be formulated in the similar way.

Now, we apply the Chernoff upper bound technique to Eq. (5) to obtain(See Appendix for a proof)

$$P[A_t - S_t - r - 1 \geqq 0] \leqq \min_{z \geqq 1}\{\Psi_{A_t}(z)\Psi_{S_t}(z^{-1})z^{-(r+1)}\}, \quad (6)$$

where the probability generating function (PGF) $\Psi_U(z)$ of the random variable U taking on integral values n=0, 1, 2…… is defined by

$$\Psi_U(z) = \mathrm{E}[z^U] = \sum_n P[U=n]z^n. \quad (7)$$

Finally, denoting $Q(r)$ as an approximate QLD for general queues in the continuous time process, our proposed formula is give as

$$Q(r) = \max_{t>0}\{\min_{z \geqq 1}\{\Psi_{A_t}(z)\Psi_{S_t}(z^{-1})z^{-(r+1)}\}\}. \quad (8)$$

If measurements of the queue length are performed in a deterministic rate, the time interval t is restricted to the discontinuous time space. For instance, if customers arriving at the $D/M/1$ queue measure the queue length, the interval t can be restricted to integers $i=1, 2, \cdots$ assuming that the inter-arrival time is one unit interval.

With Eq. (8), the characteristics of bound is not well determined, because both the lower and upper bound techniques are utilized in the formalism. However, we believe that for most queueing models this appoach gains a better approximation than other methods using multiple bounds of the same bound characteristics. Eq. (8) is quite simple in form and favorabel to a fast numerical calculation. Furthermore, it can be easily applicable to queueing models with a superposed arriving process and a single server, because the PGF of a superposed arrival process is given by the product of the PGF's of all sources.

Let the functions in the parentheses of Eq. (5) and Eq. (8) for given $r \geqq 0$ have the maximum values at $t = \tau(r)$ and $\tau'(r)$, respectively. As $\tau(r)$ is approximately equal to $\tau'(r)$, we will not dis tinguish them in the rest of this paper. $\tau(r)$ may be understood as the most probable time interval that an increment in queue length is greater than r with the condition of no idle state $N_0(\tau(r))=0$. $\tau(r)$ increases monotonically as r increases, because an accumulation of the longer queue length usually needs the longer time interval. As ρ increases, the parenthesis of Eq. (8) as a function of t is enhanced at larger t values, and consequently $\tau(r)$ increases. However, for a given r, it remains finite unless the service load exceeds one. The characteristics of $\tau(r)$ sensitively depend on multiplexing conditions of arriving customers as well as on the type of the traffic model.

Ⅲ. APPLICATION

Though the QLD can be dependent on both the service policy and the measurement scheme, Eq. (8) can be generally applicable to most single server queueing systems. In this paper we well focus on the conventional model that the idle server starts its service whenever a customer bring the system a load which requires a certain amount of service time. $P[Q>r]$ measured in two different ways by arriving customers (Scheme Ⅰ) and by leaving customers (Scheme Ⅱ) are equivalent if the state changes by unit step values only.[7]

However, it should be noted that our formula Eq. (8) generally approximate the QLD in the different manner in schemes I and II. This is because that PGE of a non-Markovian traffic model depends on how it is measured. But the difference between two results as well as their deviations from the exact solution are small enough to be negligible for normal traffic models with a single server.

In this section, our approximation formalism is applied to $M+\sum_j N_j D_j/M/1$. As some special cases of this queueing system, we also consider $M/M/1$, $D/M/1$, $M+D/M/1$, and $ND/M/1$. To the bast of our knowledge, except for $M/M/1$ no analytic solution is available for $M+\sum N_j D_j/M/1$. So, we will compare our formula for the QLD with a direct simulation of the system. The simulation of the system was carried out with 10^9 customers. To obtained the QLD, we considered 10^3 ensemble processes, each of which has 10^6 customers. In the simulation of all the queueing system discussed in this paper, we confirmed that the QLD in Scheme I is nearly equivalent to that in Scheme II.

1. The *M/M/1* queue

For the $M/M/1$ queue, we will prove that Eq. (8) is equivalent to the exact formula[7]

$$P[Q > r]=\left(\frac{\lambda}{\mu}\right)^{r+1}, \tag{9}$$

where λ and μ represent the mean arrival and service rates of the Poisson processes, respectively. For a stable queue, $\lambda < \mu$ is assumed. In both measuring schemes I and II, the PGF of the number of arrivals in $M/M/1$ is given as

$$\Psi_M(z) = e^{\lambda t(z-1)}, \tag{10}$$

and that of the service process is

$$\Psi_M(z^{-1}) = e^{\mu t(1/z-1)} \tag{11}$$

In this section we have dropped the subscripts A and S used with PGF's in the previous section. This must not cause any confusion, as we use different variables z and z^{-1} for the arrival and service process, respectively. Substituting Eqs. (10) and (11) into Eq. (8), we have

$$Q(r)=\max_{t>0}\{z_t^{-(r+1)}\, e^{\lambda t(z_t-1)}\, e^{\mu t(1/z_t-1)}\}, \tag{12}$$

where a real number z_t satisfies $z_t \geqq 1$ and is a unique solution that minimizes the function in the parenthesis of Eq. (12) for given t. After some lines of calculation, one can easily show that Eq. (12) is equivalent to Eq. (9). Furthermore, the time interval τ that maximizes the right-hand side of Eq. (12) is calculate as

$$\tau(r) = \frac{r+1}{\mu-\lambda}, \tag{13}$$

and the corresponding z_t is

$$z_t=\frac{\mu}{\lambda}. \tag{14}$$

2. The *D/M/1* queue

We consider a queue where the inter-arrival time is deterministic and the service follows a Poisson process. Without loss of generality, it is assumed that the inter-arrival time is a unit interval and the mean service rate satisfies $\mu > 1$ for a stable queue.

We first consider that the queue length is measured by customers arriving at the queue (Scheme I). Because there is no other sources in the system, the measurement is performed by the arriving customers at every unit time interval. Hence, the interval t is restricted to integers $i = 1, 2, 3\ldots$ In this case, we have the PGF of the number of arriving customers for i units of time interval

$$\Psi_D(z)=z^i, \quad i=1, 2, 3, \ldots, \tag{15}$$

and that of the service process following a Poisson process

$$\Psi_M(z^{-1}) = e^{\mu t(1/z-1)} \quad (16)$$

Substituting Eqs. (15) and (16) in Eq. (8), we obtain our approximate formula for the QLD of $D/M/1$,

$$Q(r;I) = \max_{i>r+1}\{e^{-\mu(r+1)},\ z_i^{-i+r+1} e^{\mu i(1/z_i-1)}\}, \quad (17)$$

where $z_i = \mu i/(i-r-1)$, $i > r+1$. When the condition $\mu < e^{\mu-1}/(r+2)$ is satisfied, Eq. (17) is simply $Q(r, I) = e^{-\mu(r+1)}$. This condition can be satisfied by high μ values and corresponding low utilization factor.

Though the interval i is restricted to integers, one may take the derivative of Eq. (17) with respect to i to get an approximate maximum value. One can easily show that Eq. (17) is simplified to the following formula.

$$Q'(r, I) = \xi^{r+1}, \quad (18)$$

where $0 < \xi < 1$ is the solution to the following equation ;

$$ln(\xi) = \mu(\xi-1). \quad (19)$$

τ that maximizes the right-hand side of Eq. (17) in this approximation and the corresponding z_τ are also calculated as

$$\tau(r) = \frac{r+1}{1-\mu\xi},\quad z_r = \frac{1}{\xi}.$$

On the other hand, let us consider our approximaion in scheme II that the queue is measured by leaving customers. For a given time interval $t > 0$, $[t]+1$ customers arrive from a CBR source with probability $\eta = t-[t]$, while $[t]$ customers with probability $1-\eta$. $[t]$ denotes the integral part of t. Hence, the PGF of the number of arriving customers for an interval $t > 0$ is given as

$$\Psi(z) = \eta z^{[t]+1} + (1-\eta) z^{[t]}. \quad (20)$$

The PGF of the service following a Poisson is the same as Eq. (11). Our approximate formula for the QLD of the $D/M/1$ queue in the Scheme II is given as

$$Q(r, II) = \max\{\min\{z^{[t]-r-1}(\eta z+1-\eta)e^{\mu t(1/z-1)}\}\}. \quad (21)$$

Considering η as a constant and $[t]$ in the real domain, we can further approximate Eq. (21) in the following analytic form ;

$$Q'(r, II) = \xi^{r+1}\left(\frac{\eta}{\xi}+1-\eta\right)e^{\mu\eta(\xi-1)} \quad (22)$$

where ξ the solution to Eq. (19) and η can be chosen to be an arbitrary number on between 0 and 1. Except the term of ξ^{r+1} in Eq. (22), the remaining term is just a constant $c(1 \leqq c \leqq 1/\xi)$ depending on the utilization factor.

Table 1. Calculated results for QLD of D/M/1 using (17), (18), (21) and a simulation (S) of the system.

		Q(r)=Value * 10^n				
ρ	r	(17)	(18)	(21)	S	n
0.3	0	3.57	4.09	7.13	4.12	-2
	2	5.87	6.83	18.16	6.69	-5
	4	1.12	1.14	3.47		-7
0.8	0	6.28	6.29	6.45	6.29	-1
	5	6.17	6.17	6.34	6.18	-2
	10	6.06	6.06	6.22	6.07	-3
	15	5.95	5.95	6.11	5.69	-4
	20	5.84	5.84	6.00	4.92	-5
	25	5.73	5.73	5.89		-6

In Table I, we show some calculated results for QLD with two values of the utilization factor $\rho = 0.3$ and 0.8 using Eqs. (17), (18), (21), and a simulation of the system. The calculated result obtained using either (17) or (18) is in an excellent agreement with the simulation for most values of ρ and r, while (21) shows an upper bound on all other formulas.

3. The $M+\sum N_jD_j/M/1$ queue

In this section, we suppose that the multiplexer handles a group of heterogeneous sources in addition to a stream following the Poisson process with the mean arrival rate λ. The mean service rate of the server is assumed to be μ. Among m types of CBR sources, there are N_j sources of type j generating customers at the rate of one per D_j time units.

In Scheme I, the PGF of the arrival process is dependent on the boundary conditions at both ends of a time interval, the resulting formula is rather complicated. To simplify our discussion, we will consider our approximation only in Scheme II. The PGF of the arrival process is given in a simple form because the phases of two boundaries of the interval seen by leaving customers are randomly distributed.

In Scheme II, we first consider the PGF of the arrival process.

$$\Psi_{M+\sum N_jD_j}(z)=\Psi_M(z)\Psi_{\sum N_jD_j}(z), \tag{23}$$

where $\Psi_M(z)$ for the Poisson arrivals was already defined in Eq. (10), and the PGF associated with $\sum N_j$ CBR sources is given as

$$\Psi_{\sum N_jD_j}(z)=\prod_{j=1}^{m}\Psi_{D_j}(z)^{N_j}, \tag{24}$$

where

$$\Psi_{D_j}(z)=\eta_j z^{\lfloor t/D_j\rfloor+1}+(1-\eta_j)z^{\lfloor t/D_j\rfloor} \tag{25}$$

and

$$\eta_j=\frac{t}{D_j}-\left[\frac{t}{D_j}\right] \tag{26}$$

Using Eqs. (11) and (23), we obtain an approximate QLD of $M+\sum N_jD_j/M/1$ in Scheme II as

$$Q(r,\ II)=\max_{t>0}\{\min_{z\geq 1}\{z^{k}exp\{t(z-1)(\lambda-\frac{\mu}{z})\}\prod_{j=1}(\eta_j z+1-\eta)^{N_j}\}\}, \tag{27}$$

where

$$k=r+1-\sum_{j=1}^{m}N_j\left[\frac{t}{D_j}\right].$$

Using Eq. (27), we considered some special cases such as $M+D/M/1$ and $ND/M/1$, as well as

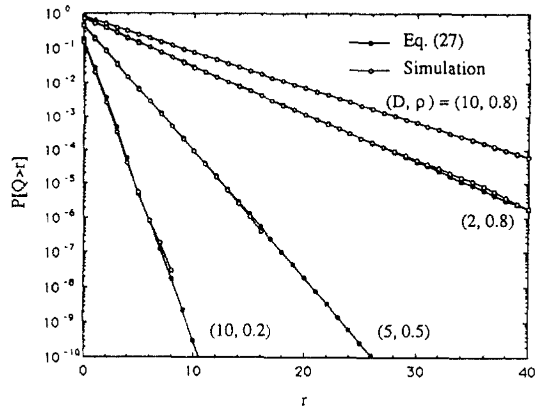

Fig. 1. The QLD of $M+D/M/1$. The mean service rate μ is set to be 1. The solid lines are guides for the eye.

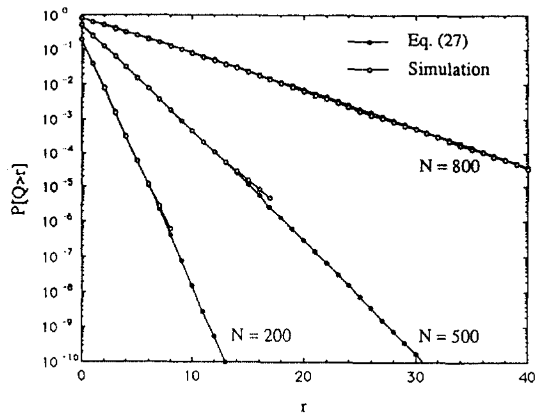

Fig. 2. The QLD of $ND/M/1$. We choose D = 1000 and μ = 1.

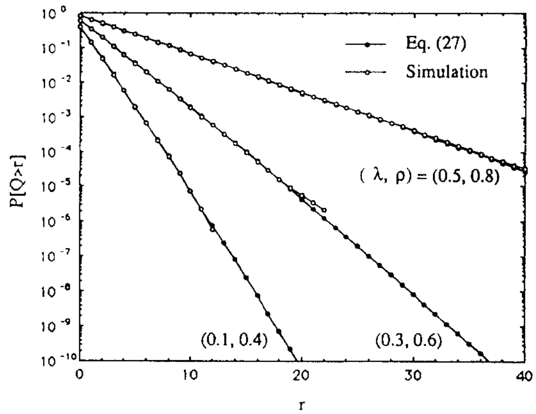

Fig. 3. The QLD of $M+\sum_{j=1}^{3} N_jD_j/M/1$. We choose $(D_1, D_2, D_3)=(10, 100, 1000)$, $(N_1, N_2, N_3)=(1, 10, 100)$, and $\mu=1$ for some values of λ the arrival rate of a Poisson source.

$M+\sum N_j D_j/M/1$. Figure 1 shows calculated QLD of $M+D/M/1$. At the input of this system customers from a Poisson and CBR sources are superposed. For this calculation, the service utilization factors are chosen to be ρ = 0.2, 0.6, and 0.8. As the mean service is chosen to be one per unit time, the mean arrival rate of customers from a Poisson source is given by $\lambda = \rho - 1/D$. The closed circles denote results using Eq. (27), while the open circles refer to results obtained by a direct simulation of the system. Each symbol will be used for the same meaning in other figures in this paper. Figure 2 displays the QLD of $ND/M/1$ for ρ = 0.2, 0.6, and 0.8 of superposed homogeneous CBR sources. The inter-arrival time of a customer stream was chosen to be 1000 for this calculation. In Figure 3, the QLD of $M+\sum_{j=1}^{3} N_jD_j/M/1$ for ρ = 0. 4, 0.6, and 0.8 are plotted. For this calculation, we considered three values of λ = 0.1, 0.3, and 5 with fixed set of CBR sources, $(D_1, D_2, D_3)=(10, 100, 1000)$ and $(N_1, N_2, N_3)=(1, 10, 100)$. The agreement between Eq. (27) and a direct simulation of $M+\sum N_jD_j/M/1$ for all situations is excellent. Though Eq. (27) is complicated, it can basically be approximated by the form of ξ^{r+1}, as is shown in these figures. We therefore have the conclusion that the QLD is exponential for the system $M+\sum N_jD_j/M/1$ in our approximation formalism. We like to remind the reader that the waiting-time distribution of $G/M/1$ is of the same form as for $M/M/1$.[7]

Ⅳ. CONCLUSION

In this paper we developed a formalism for an approximate QLD of general queueing models. Our approximation method consists of two steps of bound techniques, one lower bound and a subsequent upper bound. We showed that our formula for the $M/M/1$ queue is equivalent to the exact solution. Our calculation for the QLD of $M+\sum N_jD_j/M/1$ showed a good agreement with the direct simulation of the system. As our formalism is favorable to extremely fast numerical calculation, it can be successfully implemented in general switching systems for reliable real-time analysis of the QLD.

APPENDIX

In this appendix, we prove that

$$P[A_t - S_t - r - 1 \geqq 0] \leqq \min_{z \geqq 1}\{\Psi_{A_t}(z)\,\Psi_{S_t}(z^{-1})\, z^{-(r+1)}\}. \quad (A1)$$

Proof: At first, for random variable N taking integer values 0, ±1, ±2.... and another real variable $M > 0$, we have

$$P[N \geqq 0] = P[z^N \geqq 1], \quad z \geqq 1 \quad (A2)$$

and

$$P[M \geqq 1] \leqq E[M], \quad (A3)$$

where E[*] denotes the expectation value. From these two equations, we have

$$P[N \geqq 0] \leqq E[z^N \geqq 1], \quad z \geqq 1. \qquad (A4)$$

Hence, taking $N = A_t - S_t - r - 1$, we have the following relation

$$P[A_t - S_t - r - 1 \geqq 0] \leqq \min\{E[z^{A_t - S_t - r - 1}]\}. \qquad (A5)$$

We note that A_t and S_t are independent with each other and that $E[z^A] = \Psi_A(z)$ and $E[z^{-S}] = \Psi_S(z^{-1})$. This completes our proof.

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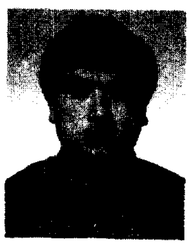
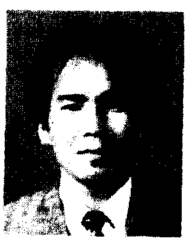


金 永 燮(Young Sup Kim) 정회원
1961년 5월 21일생
1984년 2월 : 홍익대학교 전자공학과 (학사)
1988년 8월 : 홍익대학교 대학원 전자공학과(석사)
1987년 2월~현재 : 한국전자통신연구소 ATM정합연구실 선임연구원



李 奎 錫(Kyu-Seok Lee) 正會員
1955年 6月 23日生
1979年 : 延世大學校 物理學科 學士
1981年 : 同 大學院 物理學科 碩士
1990年 : 美國 Northestern University 物理學科 博士
1991年 7月~1992年 7月 : 日本 理化學硏究所 Frontier Research Program 硏究員
1992年 9月~現在 : 韓國電子通信硏究所 先任硏究員

朴 弘 植(Hong Shik Park) 정회원
1953년 8월 16일생
1977년 2월 : 서울대학교 공과대학 (학사)
1986년 8월 : 한국과학기술원 전기 및 전자공학과(석사)
1977년 3월~현재 : 한국전자통신연구소 근무 현 ATM정합연구실장