◉ 論 文

Non-Linear Response of a Semi-Submersible with Non-Linear Restoring Forces

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비선형 복원력을 가지는 반잠수식 해양구조물의 비선형 응답

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Key Words : Non-Linear Responce(비선형 응답), Restoring Force(복원력), Time Domain Analysis(시간영역해석), Spectral Analysis(스펙트럼해석)

Abstract

일반적으로 규칙파 또는 불규칙파중에서의 반잠수식 해양구조물의 응답을 추정할때, 선형계에 적합한 주파수 영역해석법을 사용하고 있다. 대다수의 해양구조물은 Lower Hull과 단면적이 일정한 Column으로 구성되어 있지만, 만약 Column의 단면적이 흘수에 따라 변화한다면 복원력항에 비선형계를 적용해야만 한다.

따라서 본 논문에서는 비선형 복원력을 고려한 반잠수식 해양구조물의 응답을 추정할 수 있는 시간 영역 해석법을 개발하였다. 그리고, Column형상이 다른 5개의 모델을 선정하여, 이들의 시간 영역 해석 결과와 주파수 영역 해석 결과를 서로 비교하였다. 또한 파랑외력으로서 불규칙파를 적용할 때, 비선형 복원력이 해양구조물에 응답에 미치는 영향을 조사하였다.

1. INTRODUCTION

As modern civilization have been developed, natural resources in lands have rapidly exhausted. Therefore we began to be interested in the ocean having abundance of resources. In order to develope these resources, several types of offshore structures have been constructed. There were many problems in the safety of the structures. Among the several types, a typical type of the offshore structures operated in deep sea is a semi-submersible rigs with columns and lower hulls. Many researches have been performed to

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estimate the dynamic response of the semi-submersible operated under the external environments. Most of these studies were performed on the assumption that restoring forces in the equations of motion are linear.

However, if the responses are large or the structure has varying water plane area, we must consider non-linear restoring forces coefficients.

In this study, the dynamic response of semi-submersibles composed of two lower hulls and eight columns with nonuniform cross section are calculated and are investigated the effects of non-linear restoring force. We developed the time domain simulation program for these purpose. We chose five models which had differrent characteristic of restoring force coefficients.

At first, we carried out calculations of the response in frequency domain using 3D-Singularity Distribution Method(SDM) for each model. For the estimation of non-linear response, we introduced integro-differential equations as the equation of motion in time domain. In the equation, hydrodynamic forces are represented by the convolution integral by the multiples of memory effect functions and velocity of the motions. Restoring force coefficients are introduced by the functions of displacement using quadratic equations. However, right hand side of the equation is composed of the first order wave exciting force.

We compared the heave responses of the semi-submersibles with different column shapes and discussed the characteristics of non-linear restoring force coefficients in irregular waves as well as regular waves. The time history of irregular waves was generated using ISSC spectrum.

Finally we got the significant values of the responses from the response spectra. The nonlinearty in restoring force function reveals some interesting features which can't be obtained when we consider the linear restoring force functions.

2. THEORETICAL ANALYSIS

2. 1 GOVERNING EQUATION

The coordinate system is introduced by othorgonal coordinate axes with origin at the calm water plane, X pointing forwards, Y to port and Z vertically upwards. Employed models are a semi-submersibles with two planes of symmetry about the OXY and OXZ plane. It is usual to formulate these matrix equations in the six rigid body degrees of surge, sway, heave, roll, pitch and yaw described by a column vector X. Therefore, the equation of motion in time domain can be written as the following form.

$$\sum_{j=1}^{6}[(M_{kj}+m_{kj}(\infty))\ddot{x}(t)+\int_{0}^{t}k_{kj}(t-\tau)\dot{x}_j(\tau)d\tau + b_{0kj}\dot{x}_j(t)\,|\dot{x}_j(t)|+c_{kj}(x_j)x_j(t)]=F_k(t)$$
$$k=1,2,\cdots,6 \quad \cdots\cdots (1)$$

where, M_{kj} : the structure physical mass
$m_{kj}(\infty)$: the added mass of models at infinite frequency
$k_{kj}(t)$: the memory effect function
b_{0kj} : the viscous damping coeficient
$c_{kj}(x_j)$: the non-linear restoring force coefficient
$F_k(t)$: the wave exciting force

These equations express the hydrodynamic force in terms of a convolution integral represented by the memory effect functions and the velocities of each motion modes, and the drag forces proportional to square of the velocity of motions. The restoring force terms are expressed by the multiples of the non-linear restoring force coefficient and the displacement of body. The non-linear restoring functions are obtained by fitting the appropriate non-linear algebraic equations. A numerical solutions of the equations were obtained by using the Newmark-β Method and we estimated the displacement, velocity and acceleration of each modes of motions at each time steps.

2.2 HYDRODYNAMIC FORCE

In the time domain analysis, when we represent the hydrodynamic forces due to radiation potential, we use the convolution integral with the multiples of the memory effect function and body motion velocity. The memory effect function is expressed as

$$K_{kj}(t) = \frac{2}{\pi} \int_0^{\infty} b_{kj}(\omega) \cos \omega t d\omega \quad \cdots\cdots\cdots\cdots\cdots (2)$$

where, $b_{kj}(\omega)$ is the wave making damping coefficient calculated by the SDM in the frequency domain.

To verify the accuracy of the memory effect function shown above, we compared the original added mass and wave making damping coefficient estimated by SDM with those calculated by Fourier Transform of the memory effect function.

Viscous damping coeficient b_{0kj} is estimated by the summation of drag forces on each components of the structure as follows,

$$b_{0kj} = \sum_{i=1}^{M} \frac{1}{2} \rho A_i \; C_D \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3)$$

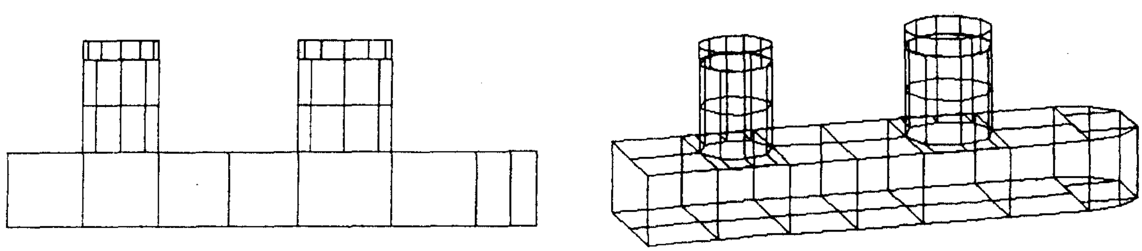
where, A_i : area of the cross section of each component

C_D : drag coefficient

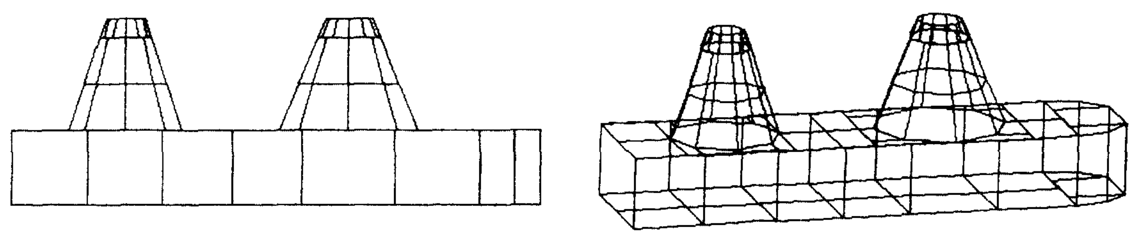
2.3 RESTORING FORCE

In the equations of motion, restoring force terms are expressed by the multiple of the restoring force coefficient and displacement of the body. The restoring force coefficient is a function of displacement. We determined the coefficients by using the quadratic equations. We selected five types of columns as shown in Fig. 1. The restoring force coefficients of the models were formulated and represented in Fig. 2. For example, when it comes to Model-D, the restoring force can be represented by two regions as follows,

$$c(z) = 4543.33z^2 + 447.553z + 13.225 \quad 0 \leq z \leq 0.06$$
$$c(z) = 9299.68z^2 - 1639.67z + 119.089 \quad 0.06 \leq z \leq 0.12$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (4)$$



Model－A(Mesh : 460)



Model－B(Mesh : 460)

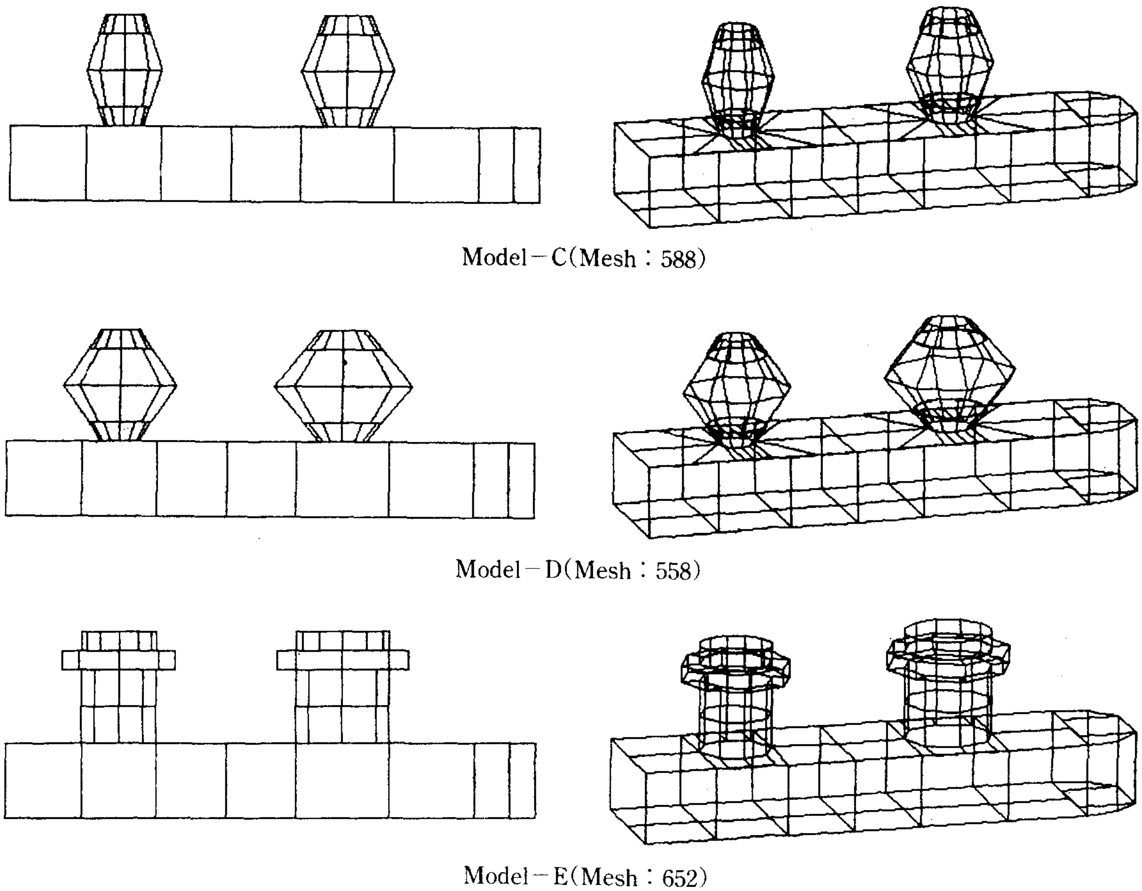


Fig. 1 Configuration of the semisubmersibles used in calculation.(1/4 body)

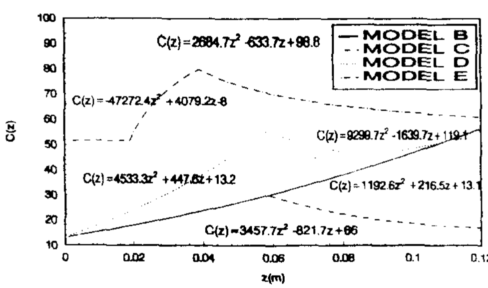


Fig. 2 Non-linear restoring force coefficient for each models.

2.4 WAVE EXCITING FORCE

When we estimated the wave exciting forces in time-domain, we introduced the convolution integral of the impulse response function of the wave exciting forces and the water surface elevations as follows,

$$F(t)=\int_0^t h(\tau)\zeta(t-\tau)d\tau \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (5)$$

where h is impulse response function of the wave exciting force ζ is the water surface elevation at the center of gravity of the body. These surface elevations were simulated by using the superposition of the regular waves with random phase lags.

The impulse response function is given by the Fourier Transform of the transfer function of the wave exciting force as follows,

$$h(\tau)=\frac{1}{2\pi}\int H(\omega)e^{i\omega\tau}d\omega \quad \cdots\cdots\cdots\cdots (6)$$

where, H(ω) is the transfer function of the wave exciting force calculated by SDM.

3. NUMERICAL RESULTS AND DISCUSSION

In this paper, we calculated the heave response of the semi-submersibles with five different types of columns as shown in Fig. 1 using frequency domain and time domain analysis.

Fig. 3 shows the added mass and the wave damping coefficients claculated by Fourier transform of memory effect function and those calculated by SDM for Model-A. The Fourier transform of memory effect function is defined as follows,

$$K^*_{kj}(\omega)=\int_0^{\infty} K_{kj}(t)exp(-i\omega t)dt \quad \cdots\cdots\cdots\cdots (7)$$

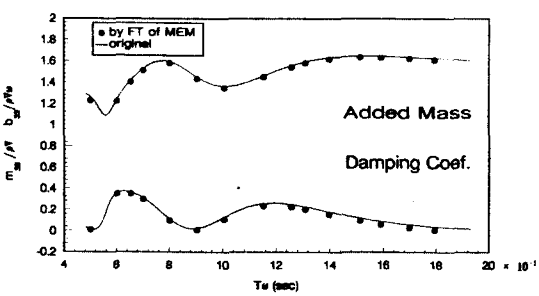


Fig. 3 Added mass and damping coefficient calculated by SDM and those by Fourier Transform of memory effect functions. (Model-A)

From this equation, the added mass $m_{kj}(\omega)$ and wave making damping coefficient $b_{kj}(\omega)$ are expressed as follows,

$$m_{kj}(\omega)=m_{kj}(\infty)+\frac{Im[K^*_{kj}(\omega)]}{\omega} \quad \cdots\cdots (8)$$

$$b_{kj}(\omega)=Re[K^*_{kj}(\omega)] \quad \cdots\cdots\cdots\cdots (9)$$

Now we can verify the accuracy of the hydrodynamic forces calculated from the time domain analysis by using these relations. When we change the shape of cross-section of the column, the characteristics of hydrodynamic forces would be changed too. Fig. 4, Fig. 5, Fig. 6 show the added masses, damping coefficients and the wave exciting forces for each models. The largest hydrodynamic forces are exerted on the Model-D and the smallest one on the Model-A which has the ordinary shapes. When we perform numerical calculation of the convolution integral to estimate the hydrodynamic force, we must truncate the memory effect function at appropriate time step. From the Fig. 7, we know that all memory effect functions of each models vanish after four seconds, so we took one hundred time steps for the calculation of convolution integral. Accordingly we picked up the time interval as 0.04 second.

We investigated the characteristics of the heave motions for each models in regular waves. Fig. 8 shows the heave motions estimated by the frequency domain analysis with linearlized restoring force.

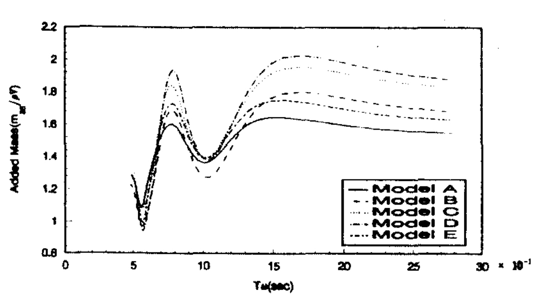


Fig. 4 Added mass for each models.

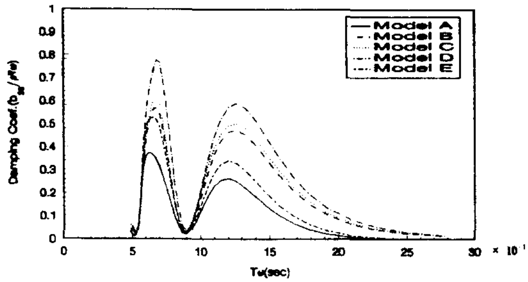


Fig. 5 Damping coefficients for each models.

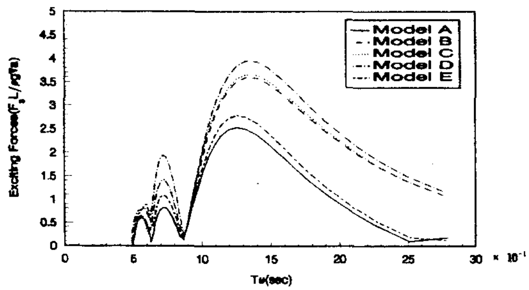


Fig. 6 Wave exciting forces for each models.

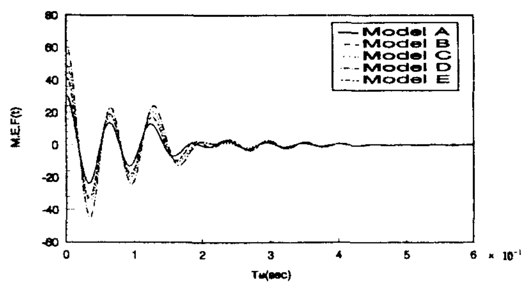


Fig. 7 Memory effect functions for each models.

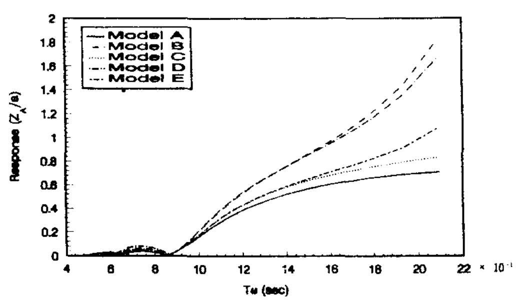


Fig. 8 Heave responses estimated by frequency domain analysis.

To grasp the effect of non-linearty, we calculated the responses by using time domain analysis in regular waves with different wave heights. In Fig. 9 and Fig. 10 non-dimensionalized heave responses in regular waves with wave amplitudes of 0.05m and 0.1m are displayed. These figures show that the heave responses of Models with non-linear restoring force tends to decrease as the wave height increases in the range of long wave periods except Model-B. In detail, Fig.11 shows the comparison of the non-dimensionalized heave amplitudes for each Models. The notation ○ represent the results by frequency domain analysis and ▲, □ by the time domain with 0.05 m, 0.1m wave amplitude respectively.

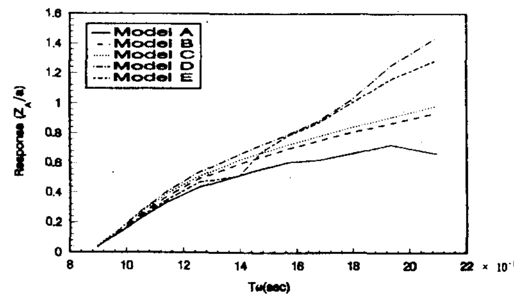


Fig. 9 Heave responses estimated by time domain analysis in regular waves.(ζ_a=0.05 m)

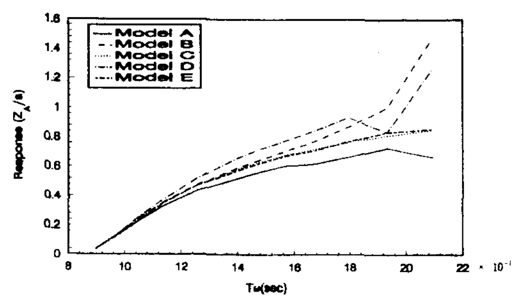


Fig. 10 Heave responses estimated by time domain analysis in regular waves.(ζ_a=0.1 m)

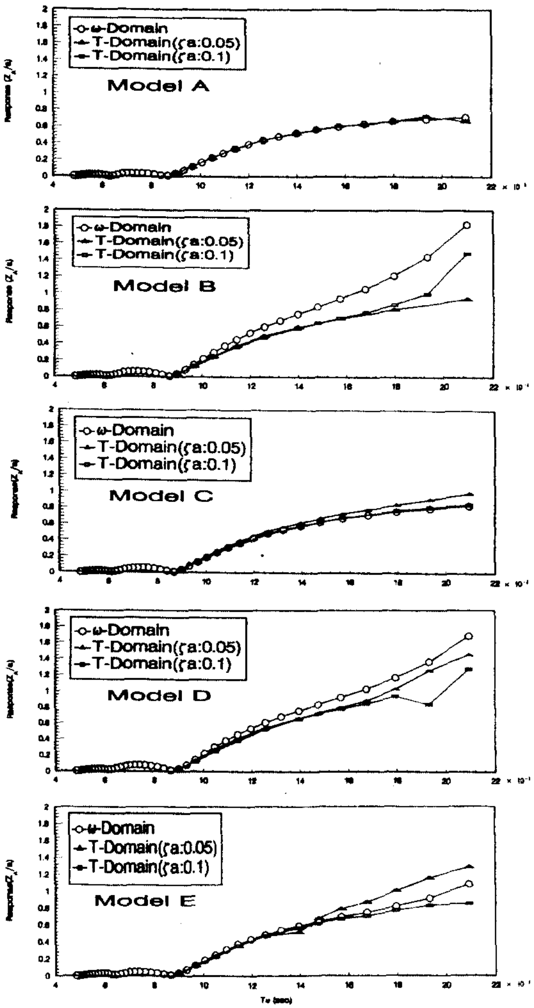


Fig. 11 Comparison of the heave responses estimated by the frequency domain and time domain for each models.

Next, we investigated the heave response in irregular waves which was simulated by the superposition of the regular waves derived by the ISSC spectrum. To investigate the effects of varying wave heights, we adopted the significant wave heights of 0.08m, 0.16m and 0.24m with same wave mean period of 1.1sec. Fig. 12 show the response spectra of heave motion for each models, which was calculated by the spectral analysis of the time history of the responses.

It is interesting to know that second peak in the response spectrum occures at the low frequency range for Model-B, Model-C and Model-D as the significant wave height increases. We expect that the phenomenon is due to non-linear effects of restoring force. This might influence the significant values of the heave response. We compared the significant response amplitudes non-dimensionalized by the significant wave height for each models in Fig. 13. It is noticed that non-dimen-

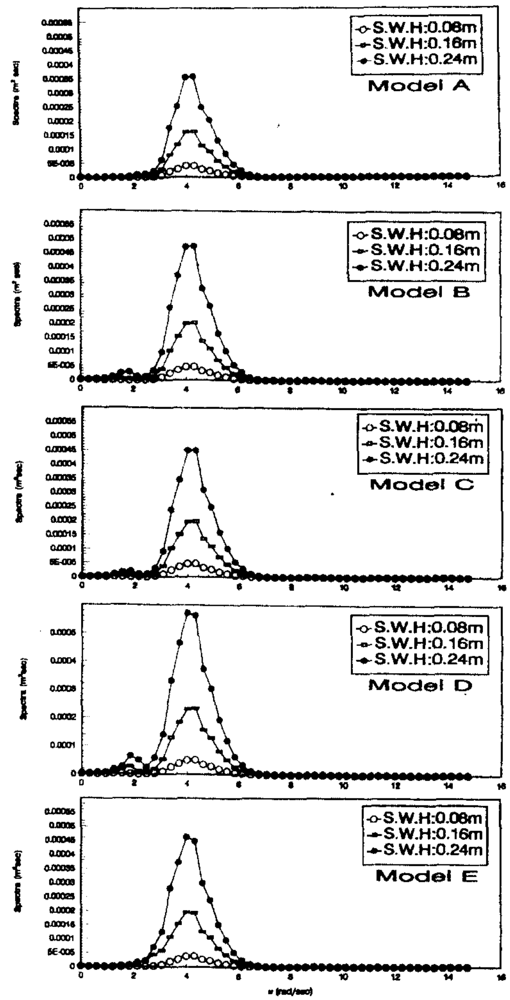


Fig. 12 Response spectra from FFT of the time history of the heave motions in irregular waves.

sionalized responses in regular waves decrease as the wave amplitude increases, but non-dimensionalized significant values of response in irregular waves tend to increase as the significant wave heights increase for the models which have non-linear restoring forces. In this paper, we calculated the hydrodynamic forces up to the mean draft line.

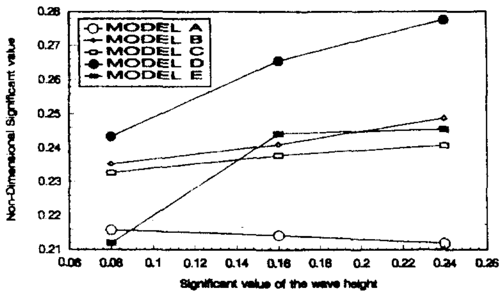


Fig. 13 Non-dimensionalized significant values of the heave responses for each models.

4. CONCLUSION

We developed the program to simulate the dynamic responses of the floating offshore structure with varying cross-sectional area.

Generally speaking, we can say that Model-A with linear restoring force experiences smaller hydrodynamic force than those of other models with non-linear restoring forces. As results a model with uniform cross sectional area has less heave motion in regular and irregular waves.

The non-linearty in restoring forces reveals some interesting features as follows. The calculation reveals that non-dimensionalized heave motion tends to decrease as wave amplitude increases in regular waves. However, the second peak in the response spectrum appears as the significant wave height increase when we consider the irregular waves. This might influence the significant values of the heave response. Those features can't be obtained when we consider the linear restoring forces.

5. REFERENCE

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