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Detection of Local Structural Changes in Time Series[1)]

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Abstract

In time series data, atypical observations are not rare. Several approaches have been proposed to detect a single outlier, but the effectiveness of those procedures is in doubt when patchy outliers are present. In this paper, the atypicality in patchy outliers is interpreted as a local structural change, and a model is introduced to entertain its effect on the series. Based on this model, a statistic and a procedure are proposed for identifying those local structural changes. The performance of the proposed procedure is evaluated through simulation study and the analysis of real data sets.

1. Introduction

Atypical observations are commonly encountered in statistical data analysis. The presence of those extraordinary events could easily mislead the conventional data analysis procedure into erroneous conclusions. Therefore, identifying and handling those observations are essential to enhance the accuracy of the statistical analysis. In the case of independent observations, one usually deletes a single observation at a time and computes various diagnostic statistics. However, the time series situation differs from the case of independent observations, especially because a certain structure between observations is imposed by time ordering, and atypical observations often come in the form of a patch or local structural changes extending over observations in sequence. In this article, a model is introduced to characterize atypical patch of observations in a time series and a procedure is proposed to detect them.

Suppose that a time series X_t follows an autoregressive moving average [ARMA(p,q)] model,

$$\Phi(B) X_t = \delta + \theta(B) \varepsilon_t, \qquad (1.1)$$

where $\Phi(B) = 1 - \Phi_1 B - \cdots - \Phi_p B^p$ and $\theta(B) = 1 - \theta_1 B - \cdots - \theta_q B^q$ are polynomials in B of degrees of p and q, respectively, B is the backshift operator such that $B X_t = X_{t-1}$, and $\{\varepsilon_t\}$ is a sequence of independent Gaussian variates, called innovations, with mean zero and

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variance σ^2. In model (1.1), it is assumed that all of the zeros of $\Phi(B)$ and $\theta(B)$ are outside the unit circle and that $\Phi(B)$ and $\theta(B)$ have no common factors.

Atypical observations are usually referred to as outliers in time series. A realization of (1.1) with outliers, Y_t, can be written in parametric form:

$$Y_t = f(t) + X_t, \tag{1.2}$$

where $f(t)$, deterministic or stochastic, is a generating mechanism of outliers. In this paper, $f(t)$ is called outlier effect. In model (1.2), Y_t is the observed series of X_t, possibly contaminated by outlier effects.

Several approaches have been proposed in the literature for handling outliers in time series. Fox(1972), Chang(1982), Chang, Tiao, and Chen(1988), and Tsay(1986, 1988) employed various versions of model (1.2) to describe the generating mechanism of an outlier. The intervention model of Box and Tiao(1975) has been used to characterize $f(t)$. Additive outlier (AO) and innovational outlier (IO) are intensively studied in their works. Martin(1981) and Kunsch(1984) proposed a robust approach for parameter estimation. In addition, Martin and Yohai(1986) introduced influence functionals in the context of time series. Bruce and Martin(1989), Pena(1990), and Ledolter(1990) proposed deletion diagnostics in time series context. In all of these approaches, the correct model form of outlier-free series X_t in (1.1) is assumed known.

The procedures by Chang(1982) and Tsay(1986) seem to be most prominent in detecting a single AO or IO. Meanwhile, their methods often fail to work correctly when multiple outliers or patchy outliers are present (Chang et al., 1988 and Lee, 1990). If some structure of outlier effects wasn't included in the set of outlier types considered, their iterative procedures would either disregard the outlier effects or identify one of the pre-setted types which is the most closest. Besides AO and IO various forms of outlier types could be introduced, but it is impractical because of the work load and computational burden in the stages of the identification of outlier type and estimation of parameters.

The paper is organized as follows. The motivation and procedure, including the model, estimation, and a detection criterion, are given in Section 2. In Section 3, the performance of the proposed procedure is investigated by simulation study and the analysis of the SERIES A data given in Box and Jenkins (1976). In Section 4, the application of the procedure to the data set with multiple outliers is discussed and the annual spirits consumption data of the United Kingdom is analyzed.

2. A Procedure for Detecting Local Structural Changes

A new interpretation of modelling outlier effects is introduced and a model with minimum structure is proposed to entertain various types of outliers, namely local structural changes.

Lee(1990) introduced this idea and proposed an outlier detection procedure which can be applied to AR model. In this section, the procedure is modified in parameter estimation and cutoff criterion. Thereupon, the proposed procedure can be applied to ARMA model and clear-cut outlier detection can be achieved.

2.1 The Model

Suppose that a time series Y_t follows MA(1) model with no constant term, but contaminated by an IO at time T_0 with magnitude ω. Then, Y_t can be expressed as

$$Y_t = X_t + \omega \cdot P_t(T_0) - \omega \cdot \theta \cdot P_t(T_0+1),$$

where $P_t(T_0) = 1$ if $t = T_0$, and $= 0$ otherwise. Here, we note AO type outliers at time T_0 and T_0+1, with magnitudes ω and $-\omega\theta$, respectively. For a time series Y_t of AR(1) model with IO effect at T_0, it can be shown that $Y_t = X_t + \sum_{j=0}^{\infty} \omega \Phi^j P_t(T_0+j)$ where ∞ is replaced by $n - T_0$ for finite series. Since Φ^j die out exponentially as j increase, Y_t can be approximated by $Y_t = X_t + \sum_{j=0}^{k-1} \omega \Phi^j P_t(T_0+j)$ for some k. Based on these examples, it can be noted that any local structural change can be approximated by modeling AO effects at consecutive time points.

The model we propose for a time series with any local structural changes is:

$$Y_t = X_t + \sum_{j=0}^{k-1} \omega_{j,T_0}(k)\, P_t(T_0+j), \tag{2.1}$$

where $\omega_{j,T_0}(k)$, called impact parameters, is the magnitude of outlier effect in Y_{T_0+j}. The T_0, j, and k indicate the starting position of the local structural change, the position in the patch, and the length of the patch, respectively. From now on, we shall use $\omega_j(k)$ for $\omega_{j,T_0}(k)$ unless it causes confusion.

2.2 Estimation

For the estimation of $\omega = (\omega_0(k),\dots,\omega_{k-1}(k))^t$, consider the residuals e_t computed by fitting Y_t into the model of X_t. When parameters $\beta = (\Phi,\theta)^t$, T_0, and k are known, e_t can be expressed as a multiple linear regression equation

$$\begin{aligned} e_t &= \pi(B) Y_t \\ &= \omega_0(k) x_{0,t} + \omega_1(k) x_{1,t} + \cdots + \omega_{k-1}(k) x_{k-1,t} + \varepsilon_t + \delta/(1-\theta_1-\cdots-\theta_q), \end{aligned} \tag{2.2}$$

where $\pi(B) = \Phi(B)/\theta(B)$ and $x_{j,t} = \pi(B) P_t(T_0+j) = -\pi_{t-(T_0+j)+1}$, $j = 0,1,\cdots,k-1$. Here

$x_{j,t}$ is not a realization of X_t, but is used to denote the independent variable of regression equation. The convention $\pi_i = 0$ for $i < 0$ and $\pi_0 = -1$ are used. For the finite series of size n, an explicit expression can be written by

$$Z = X(k)\,\omega(k) + \varepsilon, \tag{2.3}$$

where

$$X(k) = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 1 & 0 & \cdots & 0 \\ -\pi_1 & 1 & \cdots & 0 \\ -\pi_2 & -\pi_1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ -\pi_{n-T_0} & -\pi_{n-T_0-1} & \cdots & -\pi_{n-T_0-k+1} \end{bmatrix}, \quad \omega(k) = \begin{bmatrix} \omega_0(k) \\ \omega_1(k) \\ \vdots \\ \omega_{k-1}(k) \end{bmatrix}$$

and the j-th element of $(n \times 1)$ vector Z is $e_j - \delta/(1-\theta_1 - \cdots - \theta_q)$. Note that the T_0-th row has one in the first column of $X(k)$. By least squares estimation, $\hat{\omega}(k) = [X(k)^t X(k)]^{-1} X(k)^t Z$ and $Cov(\hat{\omega}(k)) = [X(k)^t X(k)]^{-1} \sigma^2$. It is well known that the $\hat{\omega}(k)$ follows a multivariate normal distribution with mean vector $E[\hat{\omega}(k)]$ and covariance matrix $Cov[\hat{\omega}(k)]$, under the Gaussian assumption on $\{\varepsilon_t\}$.

In practice, the true parameters β are usually unknown, but they can be replaced by some consistent estimates. When outliers are present in a time series, consistency of parameter estimates wouldn't be guaranteed. Pretending those suspicious observations as missing, any missing data algorithm which can provide the consistency of model parameters may be used. In this approach, the model parameters and impact parameters could be estimated recursively. The EM algorithm may be used, but almost impractical because of computational burden. To reduce computation time, the Yule-Walker (Y-W) type estimator can be used. For a time series with missing data, Dunsmuir and Robinson(1981) showed the consistency and asymptotic normality of the Y-W estimates for AR models.

2.3 A Criterion and A Procedure

For detecting the existence of outlier effects at the time points between T_0 and T_0+k-1, consider a statistic $\lambda(k, T_0)$ defined by

$$\lambda(k, T_0) = \hat{\omega}(k)^t [X(k)^t X(k)]\, \hat{\omega}(k)/\sigma^2 \tag{2.4}$$

where T_0 and k signify the starting time point and the length of patch, respectively. From the distributional property of $\hat{\omega}(k)$, $\lambda(k, T_0)$ under H_0 (no outlier effects at the times between T_0 and T_0+k-1) follows χ^2 distribution with k degrees of freedom(df).

Meanwhile, the $\lambda(k,T_0)$, under H_1, has a non-central χ^2 distribution with k df and non-centrality parameter defined by $\alpha = E[\hat{\omega}(k)]^t Cov[\hat{\omega}(k)]^{-1} E[\hat{\omega}(k)]/2$. We note that the outlier effects estimated by $\hat{\omega}$ are summarized into α.

Since we test the existence of outlying patch, we should check every time point t for $t = 1,2,\cdots,n$. Therefore, the statistic $Max\lambda(k) = Max_t \lambda(k,t)$ may be employed as a diagnostic for the detection of local structural changes in each iteration k. For a fixed k, the value of statistic $Max\lambda(k)$, say $\lambda(k,T_0)$ can be compared with a certain percentile of $\chi^2{}_k$. Since the statistic of Chang to test a single AO effect is $\sqrt{\lambda(k,T_0)}$, the cutoff values which Chang suggested may lead to a reasonable criterion for $\lambda(k,T_0)$. The p-values corresponding to C = 3.0, 3.5, and 4.0 of Chang's Criteria are 0.0027(C_1), 0.0005 (C_2), and 0.0001(C_3), respectively.

Meanwhile, direct use of the criteria can cause some problem in the iteration process for $k > 1$. For example, consider a time series with AO at $t = T_0$. For $k = 1$, only $\lambda(1,T_0)$ would be significant. When the effect of AO is highly significant, both $\lambda(2,T_0-1)$ and $\lambda(2,T_0)$ could be significant and have nearly the same values. Thus, correct determination of k and T_0 wouldn't be obtained. Therefore, for $k > 1$,

$$DMax\lambda(k) = Max_t(\lambda(k,t) - \lambda(k-1,t)) \tag{2.5}$$

is compared with the quantiles of $\chi^2{}_1$ distribution, and $\chi^2{}_1(0.0015) = 10.0$ is recommended, based on simulation study. As a rough cutoff criterion, $(Y_{t_0},\cdots,Y_{T_0+k-1})$ could be decided as patch outliers at k-th iteration if

i) $\lambda(k,T_0) = Max_t\lambda(k,t)$,

ii) $(Y_{t_0},\cdots,Y_{T_0+k-2})$ were patch outliers at $(k-1)$-th iteration, (2.6)

iii) $DMax\lambda(k) = \lambda(k,T_0) - \lambda(k-1,T_0) > 10$.

A procedure for the detection of outlying patch is summarized as follows:

step 1. Compute $\lambda(1,t)$, $t = 1,2,\cdots,n$. And choose the time point T_0 where $\lambda(1,t)$ is maximized. Compare $\lambda(1,T_0)$ with the quantiles of $\chi^2{}_1$.

step 2. Compute $\lambda(k,t)$, $t = 1,2,\cdots,n$ for increasing $k(>1)$ until no data points satisfies the conditions(2.6).

step 3. The T_0 and k are determined by the last iteration where the conditions were satisfied in step 2.

3. Numerical Examples

3.1 A Simulation Study

The proposed procedure is designed to detect the existence and the positions of outliers in sequence. To assess the performance of the procedure, the probability of correctly detecting the locations and the number of outliers is concerned. In this study, we focus on the issues of (a) no outlier, single, 2 consecutive AOs, (b) time series structure of Y_t, and (c) three cutoff criteria.

For the simulation we geneated data from AR(1) and MA(1), with sample size n=100. All the procedures are repeated 500 times. We used GGNML in IMSL to generate the normal random numbers with mean 0 and variance 1 using SUN SPARC II. In our study we considered three cases: no-outlier, a single AO at t=50, and two AOs at t=50 and 51 for each model. The magnitudes of outlier effects considered are 5σ. In the stage of parameter estimation, the backcasting method has been used. The percentage of right decision and the mean and $\sqrt{MSE}$ of Max $\lambda(k)$ of simulation results for three cases are reported in Table 3.1-3.3, respectively. From Tables 3.1-3.3, the followings can be observed:

(1) The three cutoff criteria were proposed in Section 2.3. As shown in Table 3.1, the percentages of right decisions(no outliers) are relatively low for the criterion C_1. For C_2 criterion, approximately 90% for AR(1) model and relatively higher percentages for MA(1) model can be observed. For C_3 criterion, the percentages of right decisions are even higher(greater than 95%) then the results of C_2. It can be noted that the percentages are nearly consistent for AR(1) model, but the percentage increases as θ increases for MA(1) model. Since no outlier effect is imposed in the series, each value is the probability of type I error. Based on the result of Table 3.1, from now on the criterion C_1 is excluded in this study.

(2) The result of simulation for a single AO case is given in Table 3.2. For AR(1) model, the percentage of right decision is approximately 95% when the C_2 criterion is applied. The results are almost the same when C_3 is used, except ϕ=0.2. Based on these results, the power of correct decision for AR(1) model would be about 95%. For MA(1) model, the powers are lower than those of AR(1) model, especially for higher θ values. In many cases, detection of t=49 or 50 at k=2, or no detection but maximum $\lambda(1,50)$ are observed, which haven't been counted in the percentage.

(3) The result of simulation for two AOs at t=50 and 51 is shown in Table 3.3. For AR(1) model, the percentage is approximately 95% for criterion C_2 and C_3, respectively.

For MA(1) model, the powers are about 95% for θ=0.2, 93% for θ=0.5, and 74% for θ=0.8. Further looking into the case of θ=0.8 showed that t=50 or 51 at k=1 was detected in many cases.

Table 3.1 Simulation Results for NO Outlier Series

(a): AR(1) with No Outlier

ϕ	C1			C2			C3		
	%	Mean	$\sqrt{MSE}$	%	Mean	$\sqrt{MSE}$	%	Mean	$\sqrt{MSE}$
0.2	62.5	7.25	1.05	89.8	8.14	1.68	96.6	8.49	2.09
0.5	63.7	7.14	1.11	91.0	8.04	1.73	98.2	8.42	2.16
0.8	65.0	6.98	1.18	90.7	7.87	1.78	98.5	8.30	2.16

(b): MA(1) with No Outlier

ϕ	C1			C2			C3		
	%	Mean	$\sqrt{MSE}$	%	Mean	$\sqrt{MSE}$	%	Mean	$\sqrt{MSE}$
0.2	67.3	6.99	1.11	90.7	7.81	1.74	96.9	8.15	2.14
0.5	87.2	5.93	1.47	97.6	6.37	1.90	99.6	6.50	2.11
0.8	96.2	4.60	1.72	99.7	4.78	1.94	100.	4.81	1.99

Table 3.2 Simulation Results for Single AO Series

(a): AR(1) with AO at t=50

ϕ	C2			C3		
	%	Mean	$\sqrt{MSE}$	%	Mean	$\sqrt{MSE}$
0.2	95.4	29.91	12.03	89.8	30.92	11.69
0.5	96.2	34.58	13.28	94.0	35.07	13.03
0.8	94.2	43.80	15.34	94.2	43.80	15.34

(b): MA(1) with AO at t=50

ϕ	C2			C3		
	%	Mean	$\sqrt{MSE}$	%	Mean	$\sqrt{MSE}$
0.2	93.6	31.12	12.65	89.2	31.99	12.32
0.5	83.2	37.36	17.28	81.2	37.93	17.09
0.8	68.6	37.70	18.66	67.2	38.20	18.53

Table 3.3 Simulation Results for Two AOs Series

(a): AR(1) with AOs at t=50, 51

ϕ	C2			C3		
	%	Mean	$\sqrt{MSE}$	%	Mean	$\sqrt{MSE}$
0.2	93.6	48.90	16.36	93.6	48.90	16.36
0.5	93.8	44.11	14.84	93.8	44.11	14.84
0.8	94.0	47.63	16.04	94.0	47.63	16.04

(b): MA(1) with AOs at t=50, 51

ϕ	C2			C3		
	%	Mean	$\sqrt{MSE}$	%	Mean	$\sqrt{MSE}$
0.2	99.4	73.81	25.65	99.4	73.81	25.65
0.5	93.2	142.7	345.4	93.2	142.7	345.4
0.8	74.2	157.6	142.0	74.2	157.6	142.0

(4) In geneal, the criterion C_2 works reasonable well for each cases of AR(1) model, but the power decreases significantly as θ increases for MA(1) model. We may note that the consistency of Y-W estimates when missing data are present is shown for AR model, see Dusmuir and Robinson(1981).

(5) Though not reported in this paper, simulation study for single IO at t=50 has been done for few cases. For example, when ϕ=0.5 for AR(1) model, the result of 1000 replications with C_2 criterion showed that 597 and 264 correct decisions for k=1 and 2, respectively. As expected, about 60% of outlier at t=50 and 26% at t=50 and 51 have been observed.

3.2 Series A data

The Series A data(n= 197) analyzed by Box and Jenkins(1976) is considered. The ARMA (1,1) was identified by them and used in this analysis. Time series plot is given in Figure 3.1 and the estimated values of $\lambda(k,t)$ for k=1 are given in Figure 3.2. From the analysis of Chang's procedure, two data points are detected as outliers, AO at time point 43 and IO at 64.

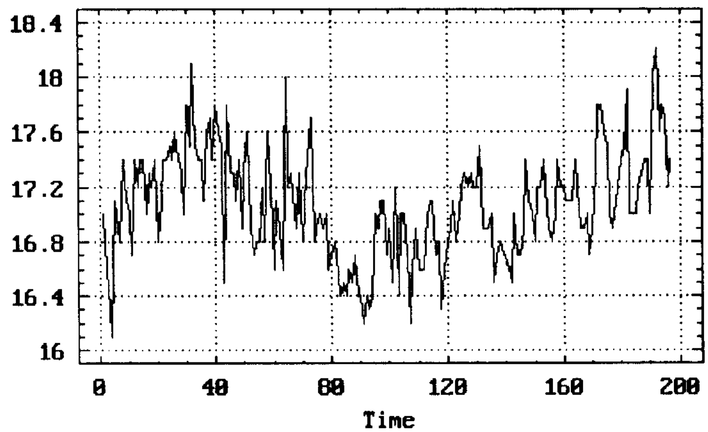

Figure 3.1 Time Series Plot of Series A Data

Two iterations have been performed. As shown in Figure 3.2, clear peaks at time points 43 and 64 have been observed. For k=2, no data point has significant diagnostic value, and thus iteration was stopped. Compared with the cutoffs for k=1, the observations at t=43 and 64 are detected as single AOs. The result of analysis is slightly different from that of Chang's.

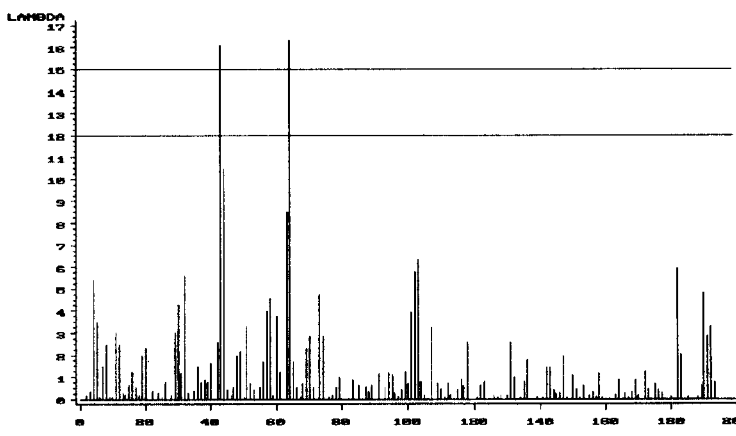

Figure 3.2 Series A Data: Lambda values at k=1

4. Discussion

4.1 Multiple Outlying Patches.

Multiple outliers are not rare in time series data analysis. The presence of multiple outliers can easily lead to masking effects, and thus the effectiveness of dectection measures is generally in doubt. By replacing suspicious observations by reasonable estimates, masking effects could be reduced. We, therefore, propose a procedure to detect multiple outlying patches as follows:

(1). using the proposed procedure, determine the most significant patch of observations

(2). replace all observations in the patch by $Y_t - \hat{\omega}(k)$.

(3). repeat (1) and (2) until no significant patch is detected

Example 4.1: England Spirit Data

We analyze the data of annual consumption of spirits in the United Kingdom from 1870 to 1938. The data set was analyzed by Fuller(1976) and Tsay(1986). The observations are the residual series after fitting the time series regression model. The ARMA(1,1) model was identified by them and used in this analysis. Time series plot is given in Figure 4.1. The replacement approach has been adopted to identify multiple outlying patches. The result of iterations is summarized in Table 4.1.

At first iteration, the time point 49 was detected and the estimated ω, -0.062, was subtracted from Y_{49}. At the second iteration, Y_{46} was selected for elimination of outlier effect as single AO. The Y_{46} was modified by subtracting estimated ω= 0.0485. At third iteration, the time points 40 and 41 have been detected with estimated ω of -0.0753 and -0.0384 at k= 2. No significant outlier has been detected at iteration 4.

Table 4.1: Analysis of England Spirit Data

iteration	k	λ(k,40)	λ(k,46)	λ(k,49)
1	1	14.90	13.60	25.10
2	1	19.60	21.30	0.00
3	2	58.40	3.06	0.14

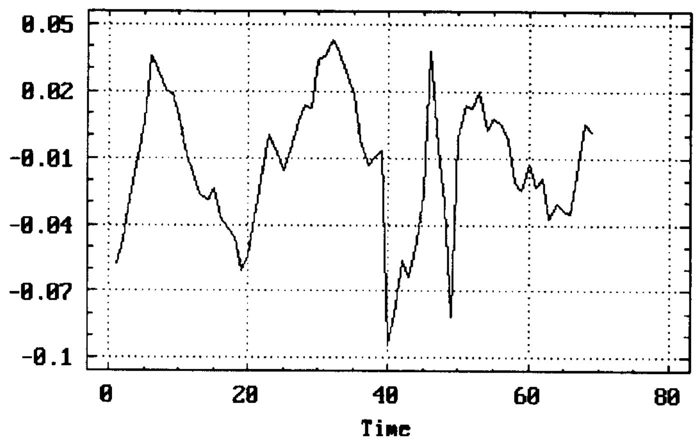

Figure 4.1 Time Series Plot of U.K. Spirit Data

4.2 Comments

It is well known that outliers can lead to model mis-identification in time series (Lee, 1990). If the positions of outliers are known and part of the data with no outlier effects is long enough, we may identify a model and estimate its parameters from it. If a time series is not long enough and the position of outliers are unknown, no known method is available at this time. Some robust model identification methods may need to be developed.

In Section 2, application of a missing data algorithm is recommended to get more accurate estimates of model parameters. Though several approaches to entertain missing data are available, e.g. the predicted estimators(Harvey and Pierce, 1984), the interpolated estimators(Pourahmadi, 1989), and the Yule-Walker type estimators(Dunsmuir and Robinson, 1981), application of each algorithm to the proposed outlier dectection procedure would result in almost same conclusion in general. Furthermore, the need of missing data algorithms seems not so critical provided a sufficiently long observed series is available, for the detection of outliers.

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시계열에서 국소구조변화의 탐지에 관한 연구[3)]

이재준[4)]

요약

시계열 자료에서 우리는 이상 관측자료들을 흔히 발견하게 된다. 한 점의 이상 관측자료를 탐지하는 방법은 여러가지가 소개되었지만 연속적인 시점에서 이상자료가 존재하는 경우에 기존의 기법은 적절하지 못한 면이 있다. 이 논문에서는 그러한 자료들을 국소구조변화의 결과로 해석하고 그 변화의 크기를 모형화하는 방법을 제시하였다. 이 모형을 이용하여 그러한 국소구조변화를 탐지할 수 있는 통계량과 탐지과정을 제안하였다. 모의실험과 실제 자료의 분석을 수행하여 제안된 기법의 유용성을 평가하였다.

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