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Simultaneous Estimation of Parameters from Power Series Distributions under Asymmetric Loss†

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ABSTRACT

Let $X_1, \ldots, X_p$ be p independent random variables, where each X_i has a distribution belonging to one parameter discrete power series distibution. The problem is to simultaneously estimate the unknown parameters under an asymmetric loss. Several new classes of dominating estimators are obtained by solving certain difference inequality.

KEYWORDS: Simultaneous estimation, Discrete power series, Unbiasedness, Asymmetric loss, Trimmed estimator.

1. INTRODUCTION

This paper is devoted to simultaneous estimation of the parameters of several independent discrete power series distribution under an asymmetric

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loss. Suppose that $X_1, \ldots, X_p$ are p independent random variables, X_i having probability mass function which is defined

$$p_{\theta_i}(x_i) = Pr(X_i = x_i) = \begin{cases} g_i(\theta_i)t_i(x_i)\theta_i^{x_i} & \text{if} x_i = 0, 1, \ldots \\ 0 & \text{elsewhere,} \end{cases} \tag{1.1}$$

and let $X = (X_1, \ldots, X_p)$.

Such distribution is called the discrete power series distribution which was first introduced by Noack(1950). Special cases include the Poisson and the negative binomial distributions. Under the normalized squared error loss, Karlin(1958) and Brown and Hwang(1982) showed the admissibility of the usual (MLE, UMVUE) estimator X of θ for $p = 1$. In the simultaneous estimation of means from p independent Poisson distributions, Peng(1975), Clevenson and Zidek(1975) and Tsui and Press(1982) all found estimators improving upon MVUE under various normalized squared error loss functions when the critical dimension for p is 2 or 3. Indeed, Hwang(1982) and Ghosh, Hwang, and Tsui(1983) considered a bigger class and obtained improved estimators under normalized squared error loss for discrete power series families whose distributions are given in (1.1). We note that all of these normalized squared error loss functions are symmetric. However, often in some practical problems, the use of symmetric loss is not appropriate. In this paper we will consider the simultaneous estimation of $\theta = (\theta_1, \ldots, \theta_p)$ under an asymmetric loss defined by

$$L(\theta, \delta) = \sum_{i=1}^{p} c_i\theta_i(\frac{\delta_i}{\theta_i} - ln\frac{\delta_i}{\theta_i} - 1), \tag{1.2}$$

where $c_i \geq 1, i = 1, \ldots, p$, $\delta = (\delta_1, \ldots, \delta_p)$ is an estimator of θ, and ln denotes the natural logarithm.

The loss(1.2) is a weighted version of the loss, so called Stein's loss, which was first introduced in James and Stein(1961) for estimation of the multinomial covariance matrix. Dey and Chung(1991) investigated the multiparameter estimation from the truncated power series distribution under Stein's loss. As indicated in Ghosh and Yang (1988), the loss(1.2) comes from the well known entropy distance (or Kullback-Leibler information number) between two distributions of p independent Poisson variables, which is defined as follows:

$$L(\theta, \delta) = \sum_{i=1}^{p} \theta_i(\frac{\delta_i}{\theta_i} - ln\frac{\delta_i}{\theta_i} - 1). \tag{1.3}$$

In general, the loss(1.3) is not applicable in the Poisson case with unequal sample sizes. Suppose that X_i is the total number of failures of the component i in n_i time periods with failure rate θ_i. Then $X_1, \ldots, X_p$ are independent Poisson with mean $n_i\theta_i$. So, we wish to estimate θ_i and not $\lambda_i = n_i\theta_i$. Then $\bar{X}_i = \frac{X_i}{n_i}$ is the unbiased estimate with variance $\frac{\theta_i}{n_i}$. Therefore the loss(1.2) is reasonable. The asymmetric loss given in (1.2) often arise in practice when the overestimation and underestimation are penalized differently. The application of the asymmetric loss for the Poisson mean estimation problem is useful in software reliability assessment. This is since the number of errors in a computer program will usually follow a Poisson distribution and underestimation of the mean number of errors will involve a large amount of penalty to the client.

By Roy and Mitra(1957), the minimum variance unbiased (also best multiple) estimator of θ is given by $\delta^0(X) = (\delta_1^0(X), \ldots, \delta_p^0(X))$, where

$$\delta_i^0(X) = \begin{cases} \frac{t_i(X_i-1)}{t_i(X_i)} & \text{if} \quad X_i = 0, 1, \ldots \\ 0 & \text{elsewhere,} \end{cases} \tag{1.4}$$

and $t_i(X_i)$ is defined as zero if $X_i < 0, i = 1, \ldots, p$.

Since $ln0$ is not defined, we consider the estimator (a corrected version) $\delta^b(X) = \delta^0(X) + b$, where $b = (b_1, \ldots, b_p), b_i > 0, i = 1, \ldots, p$. Such a correction to the unbiased estimator or the maximum likelihood estimator is quite common for the binomial and the Poisson distributions when one observes zero counts. Using asymptotic considerations, Anscombe(1956) derived such a correction. From Ghosh and Yang(1988), it follows that $\delta^b(X)$ is a generalized Bayes rule. Thus, for estimating θ, it is natural to obtain estimators improved upon $\delta^b(X)$ and hence we propose $\delta(X) = \delta^b(X) + \phi(X)$ as an alternative to $\delta^b(X)$, where $\phi(X) = (\phi_1(X), \ldots, \phi_p(X))$.

In section 2, a difference inequality involving the risk difference between the improved estimators and the unbiased estimator is obtained and solved. In section 3, two different classes of improved estimators are proposed, which are a class of shrinkage estimators and a class of their trimmed versions for the Poisson case. In section 4, risk simulation results in Poisson distribution are presented.

2. OBTANING THE DIFFERENCE INEQUALITY

Let $x = (x_1, \ldots, x_p)$ be a vector of observations of the random vector $X = (X_1, \ldots, X_p)$, where X_i, $i = 1, \ldots, p$, are mutually independent random variables with probability function $p_{\theta_i}(x_i)$ as given in (1.1). Then for any real valued function $\Phi_i(x)$ such that $E_\theta|\Phi_i(X)|$ is finite and $\Phi_i(x) = 0$ for $x < -1$, the following identity holds;

$$E_\theta[\theta_i \Phi_i(X)] = E_\theta[\Phi_i(X - e_i)\delta_i^0(X)], \tag{2.1}$$

where $\delta_i^0(X)$ is given by (1.4) and e_i is the p row vector whose ith coordinate is 1 and the other coordinates are 0. For a proof, see Hwang(1982).

The following theorem gives an unbiased estimator of the risk difference.

Theorem 2.1. Let $\delta(X) = \delta^b(X) + \phi(X)$ be an estimator of θ, where $\delta^b(X) = \delta^0(X) + b, b = (b_1, \ldots, b_p), b_i > 0, i = 1, \ldots, p$, $\delta^0(X)$ given by (1.4) and $\phi(X) = (\phi_1(X), \ldots, \phi_p(X))$ with $\phi_i(x)$ satisfying $E_\theta|\phi_i(X)| < \infty$ and $\phi_i(x) = 0$ for $x < -1$. Let $\Delta(\theta) = R(\delta, \theta) - R(\delta^b, \theta)$ be the risk difference. Then under the loss (1.2),

$$\Delta(\theta) \le E[\Delta(X)],$$

where

$$\Delta(X) = \sum_{i=1}^{p} c_i\{\delta_i^b(X)\psi_i(X) - \delta_i^0(X) ln[1 + \psi_i(X - e_i)] I[\delta_i^0(X) \ge a]\} \tag{2.2}$$

with $\psi_i(X) = \phi_i(X)/\delta_i^b(X)$. Here a denotes a suitable positive integer for defining $ln[1 + \psi_i(X - e_i)] \ge 0$ and $I(A)$ denotes the usual indicator function of the set A.

Proof. It follows from (2.1) that

$$\begin{aligned}
\Delta(\theta) &= R(\delta, \theta) - R(\delta^b, \theta) \\
&= E[L(\theta, \delta(X)) - L(\theta, \delta^b(X))] \\
&= \sum_{i=1}^{p} c_i E\left[\phi_i(X) - \theta_i ln(1 + \frac{\phi_i(X)}{\delta_i^b(X)})\right] \\
&\le \sum_{i=1}^{p} c_i E\left\{\phi_i(X) - \delta_i^0(X) ln[1 + \frac{\phi_i(X - e_i)}{\delta_i^b(X - e_i)}] I[\delta^0(X) \ge a]\right\} \\
&\le \sum_{i=1}^{p} c_i E\left\{\delta_i^b(X)\psi_i(X) - \delta_i^0(X) ln[1 + \psi_i(X - e_i)] I[\delta^0(X) \ge a]\right\}.
\end{aligned}$$

Now we need the following lemma to prove the main results. The proof is given in Dey and Srinivasan (1985).

Lemma 2.1. For $|x| \leq u < 1$,

$$ln(1+x) \geq x - \frac{3-u}{6(1-u)}x^2. \tag{2.3}$$

Thus, in order to get estimators improved upon $\delta^b(X)$, it is sufficient to find the solution of the difference inequality $\Delta(X) \leq 0$, with strict inequality for some set of X with positive measure. The following section gives some improved estimators.

3. CLASSES OF IMPROVED ESTIMATORS

In this section we will propose three different classes of improved estimators which are all of shrinkage types. The following theorem gives a class of shrinkage estimators based on all the observations.

Theorem 3.1. Suppose that $\delta(X) = \delta^b(X) + \phi(X)$, where for $d > 0$ and $c_i \geq 1$, $\phi(X)$ is given componentwise as

$$\phi_i(X) = -\frac{C(X)\delta_i^b(X)}{c_i \sum_{j=1}^p [\delta_j^0(X) + d]}, i = 1, \ldots, p. \tag{3.1}$$

Assume that the following conditions hold for some positive integer a :

1) $C(X)$ is nondecreasing in each coordinate.
2) $\delta_i^0(X - e_i) - \delta_i^0(X) \geq -a, \quad i = 1, \ldots, p.$
3) $\sum b_i = K > a$ and $pd > a$.
4) $0 < C(X) < \frac{2(K-a)pd}{pd+a+2(K-a)} = G(\text{say})$.

Then $\delta(X)$ dominates $\delta^b(X)$ in terms of risk.

Proof. First defining $T = \sum_{i=1}^p \delta_i^0(X)$ and using our assumptions, it follows that

$$|\frac{\phi_i(X-e_i)}{\delta_i^b(X-e_i)}| = \frac{C(X-e_i)}{c_i[\sum_{j=1}^p(\delta_j^0(X)+d) + \delta_i^0(X-e_i) - \delta_i^0(X)]}. \tag{3.2}$$

Thus, defining $I(A)$ as the usual indicator function of the set A, it follows from (3.2) that

$$\left|\frac{\phi_i(X-e_i)}{\delta_i^b(X-e_i)}\right| I[\delta_i^0(X) \geq a] \leq \frac{C(X-e_i)}{c_i(T+pd-a)} I[\delta_i^0(X) \geq a] < \frac{G}{c_i pd} < \frac{G}{pd} < 1. \tag{3.3}$$

Next using (3.3) and Lemma 2.1, it follows immediately that

$$\begin{aligned} & ln\left[1+\frac{\phi_i(X-e_i)}{\delta_i^b(X-e_i)}\right] I[\delta_i^0(X) \geq a] \qquad (3.4)\\ \geq & \left\{\frac{\phi_i(X-e_i)}{\delta_i^b(X-e_i)} - \frac{3-G(pd)^{-1}}{6(1-G(pd)^{-1})}\frac{\phi_i^2(X-e_i)}{[\delta_i^b(X-e_i)]^2}\right\} I[\delta_i^0(X) \geq a] \\ \geq & \left\{-\frac{C(X-e_i)}{c_i(T+pd-a)} - \frac{1}{2}pd(pd-G)^{-1}\frac{C^2(X-e_i)}{c_i^2(T+pd-a)^2}\right\} I[\delta_i^0(X) \geq a]. \end{aligned}$$

Using the fact that $C(X)$ is nondecreasing in each coordinate it follows from (2.2) and (3.4) that

$$\begin{aligned} \Delta(X) &= \sum_{i=1}^{p} c_i[\phi_i(X) - \delta_i^0(X) ln(1+\frac{\phi_i(X-e_i)}{\delta_i^b(X-e_i)}) I[\delta_i^0(X) \geq a]] \qquad (3.5)\\ &\leq -\frac{C(X)\sum_{i=1}^p \delta_i^b(X)}{\sum_{i=1}^p(\delta_i^0(X)+d)} + \sum_{i=1}^{p}\left\{\frac{C(X-e_i)\delta_i^0(X)}{T+pd-a}\right. \\ &\quad \left. +\frac{1}{2}\frac{pd}{pd-G}\frac{C^2(X-e_i)\delta_i^0(X)}{c_i(T+pd-a)^2}\right\} I[T \geq a] \\ &\leq \left[-\frac{C(X)(T+K)}{T+pd} + \frac{C(X)T}{T+pd-a} + \frac{pd}{2(pd-G)}\frac{TC^2(X)}{(T+pd-a)^2}\right] I[T \geq a] \\ &\leq \left[-\frac{C(X)T(K-a)}{(T+pd)(T+pd-a)} + \frac{pdTC^2(X)}{2(pd-G)(T+pd-a)^2}\right] I[T \geq a] \\ &\leq -\frac{C(X)T}{(T+pd)(T+pd-a)}\left[(K-a) - \frac{pd+a}{2(pd-G)}C(X)\right] I[T \geq a] \\ &\leq -\frac{C(X)T(pd+a)}{(T+pd)(T+pd-a)2(pd-G)}[G-C(X)] I[T \geq a] \\ &\leq 0. \end{aligned}$$

This completes the proof of the theorem.

Remark 3.1. Note that in Theorem 3.1, we have the condition that $\delta_i^0(X - e_i) - \delta_i^0(X)$ is bounded below by some fixed negative number, say $-a$, where $a > 0$. It can be easily checked that both Poisson and negative binomial distributions satisfy such a condition.

Next, we consider the two important special cases, which are the Poisson and the negative binomial distributions.

Example 3.1. Let X_i's be independently distributed with probability mass function

$$p_{\theta_i}(x_i) = exp(-\theta_i)\frac{\theta_i^{x_i}}{x_i!}, \quad x_i = 0, 1, \ldots. \tag{3.6}$$

It follows that the minimum variance unbiased estimator of θ_i is

$$\delta_i^0(x) = \frac{t_i(x_i - 1)}{t_i(x_i)} = x_i, \quad x_i = 0, 1, \ldots.$$

Since $\delta_i^0(x - e_i) - \delta_i^0(x) = -1$, the Poisson distribution satisfies the condition 2) of Theorem 3.1 and so under the loss (1.2) with all $c_i = 1$, for all $b_i > 0$ and $\sum_{i=1}^p b_i > 1, \delta^b(X) = X + b$ is dominated by

$$\delta(X) = (1 - \frac{C(X)}{\sum_{i=1}^p (X_i + d)})(X + b).$$

Thus Ghosh and Yang's(1988) estimator is one of our improved estimators with suitable choice of a (that is, $a = 1$).

Example 3.2. Let X_i's be independently distributed with probability mass function

$$p_{\theta_i}(x_i) = \frac{(x_i + r_i - 1)!}{x_i!(r_i - 1)!}(\theta_i)^{x_i}(1 - \theta_i)^{r_i}, x_i = 0, 1, \ldots$$

and $r_i \geq 2, i = 1, \ldots, p$.

The best unbiased estimator in this case is given componentwise as $\delta_i^0(x) = \frac{x_i}{x_i + r_i - 1}$. Since $\delta_i^0(x - e_i) - \delta_i^0(x) > -(r_i - 1), a = r_i - 1 \geq 1$. Therefore for all $b_i > 0$ and $\sum_{i=1}^p b_i > a, \delta^b(x)$ is dominated by an estimator in Theorem 3.1.

Now it is clear from (3.1) that if $\delta_i^0(X)$ is large, the estimator $\delta(X)$ collapses back to $\delta^b(X)$. Thus in the presence of observations, we should look for trimmed versions of the improved estimators. In view of that, we obtain another set of solutions to this difference inequality (2.2) which lead to a similar-trimmed version of the estimators obtained earlier. Related results on trimmed estimators under weighted squared error loss are given in Ghosh and Dey (1984). We will restrict the trimmed estimators only to Poisson distributions. First of all, We need to develop a few notations before starting the results concerning trimmed estimators for the Poisson case. Let us define

$$h_i(x_i) = \sum_{k=1}^{\delta_i^0(x)} k^{-1} (= \sum_{k=1}^{x_i} k^{-1}).$$

Suppose that $x_{(1)} \le x_{(2)} \le \ldots \le x_{(l)} \le \ldots \le x_{(p)}$ are the ordered x_i's, where $x_{(l)}$ is the lth ordered statistic. Define

$$d_i(x_i) = Ah_i(x_i)h_i(x_i+1) + d_0$$

where $A = \max(\{x_i\}_{i=1}^p, 1)$ and $d_0 > 0$ is a constant. Let

$$D = D(X) = \sum_1 d_j(X_j) + \sum_2 d_j(X_{(l)})$$

$$D_i = D_i(X) = \sum_3 d_j(X_j) + \sum_4 d_j(X_{(l)}) + d_i(X_i+1)$$

$$D_{(l)} = D_{(l)}(X) = \sum_3 d_j(X_j) + \sum_4 d_j(X_{(l)}) + d_i(X_{(l)})$$

where $\sum_1, \sum_2, \sum_3$ and $\sum_4$ denote the summations over $\{j : X_j \le X_{(l)}\}$, $\{j : X_j > X_{(l)}\}$, $\{j \ne i : X_j \le X_{(l)}\}$ and $\{j \ne i : X_j > X_{(l)}\}$ respectively.

Also define $q^+ = \max(q, 0)$. Now, the following theorem gives new class of trimmed shrinkage estimator which takes only l smallest observations and largest observation among the p observations if $p \ge 3$ and $l \ge 3$.

Theorem 3.2. Suppose that $\delta^{(l)}(X) = \delta^b(X)(1 + \Phi(X))$ where $\Phi(X)$ is given componentwise as

$$\Phi_i(X) = \begin{cases} \frac{-aC(X)h_i(X_i)}{c_iD_i} & X_i < X_{(l)} \\ \frac{-aC(X)h_i(X_{(l)})}{c_iD_{(l)}} & X_i \ge X_{(l)}. \end{cases} \tag{3.7}$$

Assume that the following conditions hold:

1) $0 < a < c' \min(\frac{u}{l-2}, \frac{6(1-u)}{3-u}), c' = \min(c_i), l \ge 3$ and $0 < u < 1$.
2) $C(X)$ is nondecreasing in each coordinate.
3) $0 < C(X) \le l-2$.

Then for $p \geq 3$, $\delta^{(l)}(X)$ will dominate $\delta^b(X)$ in terms of risk.

Proof. First consider $|\Phi_i(X-e_i)|$. If $X_i > X_{(l)}$, then $X_i - 1 \geq X_{(l)}$. So,

$$|\Phi_i(X-e_i)| = |\frac{aC(X-e_i)h_i(X_{(l)})}{c_i D_{(l)}}| \leq |\frac{aC(X)}{c_i}| \leq u < 1.$$

Similarly, if $X_i \leq X_{(l)}$, then $|\Phi_i(X-e_i)| \leq u < 1$. Using Lemma 2.1, since $u < 1$, it follows from (2.2) that

$$\begin{aligned}\Delta(X) &\leq \sum_{i=1}^{p} c_i[\delta_i^0(X)(\Phi_i(X) - \Phi_i(X-e_i)) + k\delta_i^0(X)\Phi_i^2(X-e_i)] \\ &+ \sum_{i=1}^{p} c_i[\delta_i^b(X) - \delta_i^0(X)]\Phi_i(X)\end{aligned}$$

where $k = \frac{3-u}{6(1-u)}$. Since $\delta_i^b(X) - \delta_i^0(X) = b_i > 0$, it is sufficient to show that

$$\sum_{i=1}^{p} c_i[\delta_i^0(X)\{\Phi_i(X) - \Phi_i(X-e_i)\} + k\delta_i^0(X)\Phi_i^2(X-e_i)] \leq 0. \tag{3.8}$$

Consider the second term on the left hand side of (3.8). If $X_j \leq X_{(l)}$, then

$$\begin{aligned}\delta_j^0(X)h_j^2(X_j) &\leq \delta_j^0(X)h_j(X_j)h_j(X_j+1) \\ &\leq Ah_j(X_j)h_j(X_j+1) < d_j(X_j).\end{aligned}$$

Similarily $\delta_j^0(X)h_j^2(X_{(l)}) < d_j(X_{(l)})$ if $X_j > X_{(l)}$,since $A > 1$. Thus,

$$\sum_1 \delta_j^0(X)h_j^2(X_j) + \sum_2 \delta_j^0(X)h_j^2(X_{(l)}) < D.$$

Therefore, it follows that

$$\begin{aligned}\sum_{i=1}^{p} c_i k\delta_i^0(X)\Phi_i^2(X-e_i) &\leq \frac{ka^2}{c'}C^2(X)\frac{\sum_1 \delta_j^0(X)h_j^2(X_j) + \sum_2 \delta_j^0(X)h_j^2(X_{(l)})}{D^2} \\ &\leq k\frac{a^2C^2(X)}{c'D}.\end{aligned} \tag{3.9}$$

Next, consider $\sum_{i=1}^{p} c_i\delta_i^0(X)[\Phi_i(X) - \Phi_i(X-e_i)]$. If $X_i > X_{(l)}$, then $\Phi_i(X) - \Phi_i(X-e_i) = 0$. Suppose that $X_i \leq X_{(l)}$. Then, it can be shown that

$$c_i[\Phi_i(X) - \Phi_i(X-e_i)] \leq aC(X)\frac{h_i(X_i)(D_i - D) - D_i(h_i(X_i) - h_i(X_i-1))}{DD_i}. \tag{3.10}$$

Finally, using the definition of $h_i(X_i)$ and D_i's, it follows that

$$\begin{aligned}
\sum_{i=1}^{p} c_i\delta_i^0(X)[\Phi_i(X) \quad - \quad \Phi_i(X-e_i)] &= \sum_{1} c_i\delta_i^0(X)[\Phi_i(X)-\Phi_i(X-e_i)] \\
&\le \sum_{1} aC(X)\delta_i^0(X)[\frac{\frac{2}{\delta_i^0(X)}d_i(X_i)}{DD_i} - \frac{1}{D\delta_i^0(X)}] \\
&\le \sum_{1} aC(X)[\frac{2d_i(X_i)}{DD_i} - \frac{1}{D}] \\
&\le -aC(X)\frac{(N_l(X)-2)^+}{D} \\
&\le -\frac{aC^2(X)}{D}, \qquad (3.11)
\end{aligned}$$

where $N_l(X) = \#\{j : X_j \le X_{(l)}\}$.
Combining (3.9) and (3.11), we have

$$\begin{aligned}
\sum_{i=1}^{p} c_i\delta_i^0(X)[\Phi_i(X) \quad - \quad \Phi_i(X-e_i) + k\Phi_i^2(X-e_i)] \\
&\le [\frac{-aC^2(X)}{D} + k\frac{a^2C^2(X)}{c'D}] \\
&\le \frac{aC^2(X)}{D}(\frac{ka}{c'}-1) \\
&\le 0.
\end{aligned}$$

This completes the proof of the theorem.

Next, the following theorem gives another class of trimmed shrinkage estimators which takes only $p-l+1$ largest observations among the p observations . Let us define

$$S = S(X) = \sum_{2} d_j(X_j) + \sum_{1} d_j(X_{(l)}),$$

$$S_i = S_i(X) = \sum_{4} d_j(X_j) + \sum_{3} d_j(X_{(l)}) + d_i(X_i+1)$$

$$S_{(l)} = S_{(l)}(X) = \sum_{4} d_j(X_j) + \sum_{3} d_j(X_{(l)}) + d_i(X_{(l)})$$

Note that $N(X) = \#\{i : X_i > X_{(l)}\}$ is nondecreasing function in each coordinate.

Theorem 3.3. Suppose that $\delta^*(X) = \delta^b(X)(1 + \Phi(X))$ where $\Phi(X)$ is given componentwise as

$$\Phi_i(X) = \begin{cases} \frac{-a(N(X)-2)^+ h_i(X_i)}{c_i S_i} & X_i > X_{(l)} \\ \frac{-a(N(X)-2)^+ h_i(X_{(l)})}{c_i S_{(l)}} & X_i \leq X_{(l)} \end{cases} \tag{3.12}$$

where $0 < a < c' min(\frac{u}{p-l-2}, \frac{6(1-u)}{3-u}), c' = min\{c_i\}$ with $0 < u < 1$ and $1 \leq l \leq p-2$. Then for $p \geq 3$, $\delta^*(X)$ will dominates $\delta^b(X)$ in terms of risk.

Proof. The proof is omitted because of its similarity to the proof of the Theorem 3.2.

4. RISK SIMULATION STUDY

In this section we will compute the risk of the shrinkage and the corresponding trimmed estimators for the simultaneous estimation of the Poisson means under the loss(1.2) with different values of c_i. First, the number p of independent Poisson random variables is chosen. Next, p parameters θ_i are generated randomly within certain range (a, b). Finally, one observation of each the p distributions with θ_i obtained in the first and second step is generated. Estimates of the parameters are calculated using the estimators $\delta(X)$ and $\delta^{(l)}(X)$, which are given by, respectively

$$\delta_i(X) = \left(1 - \frac{2(p-1)pd}{c_i(pd + 2p - 1)\sum_{j=1}^{p}(X_j + d)}\right)(X_i + 1) \tag{4.1}$$

and

$$\delta^{(l)}(X) = (1 + \Phi(X))(X + 1)$$

where

$$\Phi_i(X) = \begin{cases} -\frac{(l-2)h_i(X_i)}{c_i D_i} & X_i < X_{(l)} \\ -\frac{(l-2)h_i(X_{(l)})}{c_i D_{(l)}} & X_i \geq X_{(l)}. \end{cases} \tag{4.2}$$

The entire steps are repeated 1000 times and the risks under the loss function (1.2) for the estimators $\delta(X)$ and $\delta^{(l)}(X)$ are calculated. The percentage of the savings in the risk using $\delta^{(l)}(X)$ are compared to the generalized Bayes estimator $\delta^{b}(X)$, using the formula

$$PRI(\delta^{(l)}(X)) = \frac{R(\theta, \delta^{b}(X)) - R(\theta, \delta^{(l)}(X))}{R(\theta, \delta^{b}(X))} * 100. \tag{4.3}$$

In table 1 and 2, we calculate the percentage improvement of $\delta(X)$ in (4.1) over $\delta^1(X) = X + 1$ for the different values of d in $\delta(x)$ when $p = 2$ and $p = 5$ respectively. The percentage improvement decreases as the value of d increases. That means when the magnitude of d's increases, the denominator of $\delta(X)$ becomes larger and so there is high possibility for $\delta(X)$ to collapse back to $\delta^1(X)$. Therefore it is reasonable to consider the trimmed version estimator like $\delta^{(l)}(X)$ and $\delta^*(X)$ in (3.7) and (3.12), respectively. In table 3 and 4, the percentage improvement over $\delta^1(X) = X + 1$ are computed when the estimators are trimmed at specified(lth) order statistics for the different values of c_i. The estimator $\delta^{(l)}(X)$ as defined in (4.2) stands for the shrinkage estimator which is trimmed at lth order statistic($l = 3, 4, 5, 6, 7, 8$). In table 5, the percentage improvement of $\delta^{(l)}(X)$ over $\delta^1(X)$ is computed for certain values of $\theta_1, \ldots, \theta_5$. We observe that the percentage improvement decreases as the magnitude of the θ_i's increases, which conforms to our intuition since $\delta(X)$ and $\delta^{(l)}(X)$ shrink $\delta^b(X)$ toward zero. It is also observed that the improvement is always positive, which indicates that $\delta^{(l)}(X)$ is minimax.

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Table 1. $PRI(\delta(X))$ **with** $p=2$

range of the parameters θ_i	$c_i = 1$				
	$d=1$	$d=2$	$d=3$	$d=5$	$d=10$
(0,1)	37.60	29.26	23.65	16.97	9.9
(0,2)	32.37	27.00	22.56	16.75	10.08
(0,4)	28.04	24.61	21.16	16.23	10.13
(4,8)	4.94	6.52	7.42	8.19	7.87
range of the parameters θ_i	$c_i = i$				
	$d=1$	$d=2$	$d=3$	$d=5$	$d=10$
(0,1)	25.83	20.06	16.19	11.61	6.76
(0,2)	22.39	18.63	15.54	11.52	6.92
(0,4)	19.48	17.07	14.66	11.22	6.89
(4,8)	3.75	5.01	5.63	6.03	5.57

Table 2. $PRI(\delta(X))$ **with** $p=5$

range of the parameters θ_i	$c_i = 1$				
	$d=1$	$d=2$	$d=3$	$d=5$	$d=10$
(0,1)	45.58	41.13	34.52	25.71	15.50
(0,2)	39.75	36.85	32.28	25.08	15.73
(0,4)	29.20	30.03	27.80	22.97	15.35
(4,8)	10.78	12.56	13.20	13.62	12.73
range of the parameters θ_i	$c_i = i$				
	$d=1$	$d=2$	$d=3$	$d=5$	$d=10$
(0,1)	17.09	14.39	12.03	8.92	5.35
(0,2)	14.15	13.06	11.39	8.79	5.48
(0,4)	10.42	10.74	9.96	8.12	5.37
(4,8)	4.44	5.60	5.96	5.95	5.09

Table 3. $PRI(\delta^{(l)}(X))$ **with** $d_0 = 1$ **and** $c_i = 1$

range of the parameters θ_i	p=5		p=10			
	$l = 3$	$l = 4$	$l = 5$	$l = 6$	$l = 7$	$l = 8$
(0 , 1)	3.55	5.11	1.43	2.07	2.39	3.09
(0 , 2)	2.16	4.97	1.39	1.09	2.18	2.56
(0 , 4)	2.57	2.80	1.26	1.30	1.28	1.24
(4 , 8)	0.70	0.78	0.37	0.37	0.36	0.35

Table 4. $PRI(\delta^{(l)}(X))$ **with** $d_0 = 1$ **and** $c_i = i$

range of the parameters θ_i	p=5		p=10			
	$l = 3$	$l = 4$	$l = 5$	$l = 6$	$l = 7$	$l = 8$
(0 , 1)	1.69	1.69	0.25	0.37	0.44	0.56
(0 , 2)	1.22	1.73	0.08	0.22	0.37	0.46
(0 , 4)	0.91	0.92	0.23	0.23	0.23	0.22
(4 , 8)	0.24	0.24	0.06	0.06	0.06	0.06

Table 5. $PRI(\delta^{(l)}(X))$ **with** $d_0 = 1$

range of the parameters θ_i	$c_i = 1$		$c_i = i$	
	$l = 3$	$l = 4$	$l = 3$	$l = 4$
(1,1,1,1,1)	5.33	6.14	1.71	2.10
(0.8,1.6,2.4,3.2,4)	2.24	2.18	0.81	0.74
(5,5,5,5,5)	0.95	0.92	0.32	0.33
(4,5,6,7,8)	0.72	0.75	0.24	0.24