Dynamic Behaviour of Granular Meterial during the Rapid Motion

급속운동을 하는 입자물질의 동적거동

Hwang, Hak*
황 학

요 지

입자물질의 빠른 움직임을 미시적으로 관찰하고 이를 연속체이론에 적용하였다. 두 입자의 상호충돌현상에서 두 종류의 시간 즉 비행시간과 접촉시간을 나누었다. 전자는 시간에 따라 변하는 움직임과 체적의 변화를 나타내며, 후자는 두 입자가 접촉시에 걸린 시간을 뜻하며 이 크기는 입자의 탄성성질을 나타낸다. 이러한 두 종류의 시간을 이용하여 4개의 상태변수 즉 압축응력, 점성, 에너지전달률 그리고 에너지손실률로서 동력학적 구조거동식을 세웠다. 연속체이론의 질량, 운동량 및 에너지방정식을 위의 상태변수로서 표기하고 이를 다시 두 가지의 모델에 적용한 결과 탄성성질로 인한 이완 및 에너지 흡수현상이 나타났다.

Abstract

The rapid motion of granular material is microscopically observed, and investigated by continuum theory. From the binary collision phenomenon two different times are introduced: flying time and contact time. The former says the non−stationary motion and at a same time the variation of bulk volume. The latter is operative by a delayed time during the contact and describes the elastic properties of granular material. With both times a dynamic constitutive equation is postulated for four state variables; dispersive pressure, viscosity, thermal diffusivity and energy annihilation rate. The balance laws of mass, momentum and energy which are represented through above four variabls, are applied to the model, in which due to the elastic property the relaxation and energy absorption are explained.

1. Introduction

It is the purpose of this paper to outline a theory of grain flow which is based upon the description of continuous matter fields. The thesis is that, depending upon the agitation of

*정회원, 대림산업 서해대교 지원팀 차장

the particles, their mean free path may vary so that the bulk material under the rapid motion of the particles is not density preserving. We close this system of equations by phenomenological relationships for (i) the dispersive pressure σ, (ii) the viscosity of the "viscous" stresses η, (iii) the diffusivity of the flux of fluctuation energy k and (iv) the annihilation rate of the fluctuation energy i. These qualities are coupled with the other field variables, primarily the fluctuation energy. This model is simple because it is in large parts based on dimensional arguments, but it will be demonstrated that it captures the essential physics to a very high degree of satisfaction.

Up−to−date surways of the state of the art are given by Campbell[1] and Hutter & Rajagopal[2] which contain a large number of references.

We still restrict considerations to binary collisions but define the time of an encounter between two particles to consist of the small but finite contact time plus the time of free flight prior to the next collision. The duration of a collision depends on the deformation of the interacting particles and how long it takes for this information to be transmitted forth and back through the particle when the particles rebounce from one another. With this contact time being included in the formal definition of the duration of an encounter the proposed model is able to avoid the often mentioned singularities in the transport coefficients.

2. The binary collision phenomenon

Ensuing theoretical developments will concentrate upon rapid motions of granular materials in close analogy to Hafff's original theory[3], however with the physically significant difference that

(i) the(visco)−elastic properties of the material of the particles is accounted for by e. g. assuming a finite non zero Young's modulus($0<E<\infty$) and

(ii) the bulk density is assumed to vary with the state of agitation of the particles.

Consider the collision of two smooth spheres that experience a centric encounter. The binary collision is the following phenomenon: *The two spheres approach each other with a speed, encounter with one another, remain in contact for some short duration, separate afterwards and subsequently move apart from each other with relative velocity.* This impulse phenomenon can essentially be divided into two phases: the flying phase and the contact phase. These can be interpreted as follows:

Prior to any encounter, i. e. during the flying phase, the two spheres possess only *kinetic energy*. During collision they completely give away their kinetic energy which is transferred within the spherical particle into *vibrational* or *wave energy*. Once the elastic wave, that has been produced within a particle by this wave energy, is reflected and refracted at the sphere's boundaries and has returned to the contact point of the two particles, kinetic energy will again be created, but it will be less than the kinetic energy of the two particles before the encounter. The remaining energy, i. e. the difference between the kinetic energy

of the approaching and repulsing particle is dissipated in the plastic deformation during collision and the eigenoscillations within the particle. In reality, the wave will essentially dissipate away and be transferred into heat in a finite time, ideally a time that, roughly, corresponds to the free flying time of the particles. The particles then move until they encounter the next collision. During this flying episode they are subject to kinetic energy and vibrational energy, but during the contact time they primarily experience vibrational energy. All this is valid for an observer moving with the mean speed of the two particles.

Two phases of motion are typical for the phenomenon: First the *flying phase* with its kinetic energy. (The vibrational energy the particle transports with it during this phase is a dead parameter and is of no concern here). We shall view it as a pure translation by which momentum and energy is transported, i. e. rotational inertia of the particles is regarded as insignificant. This translational motion of the flying particles is responsible for the size of the mean free path between the particles that is established by the dispersive pressure it produces and corresponds to the fluctuation energy of "granular" temperature known from kinetic models. Second, in the *contact phase* the particles experience material deformations. The particles in this phase will react to the external loads or contact forces. It is apparent, that both phases are characterized by different, mutually(nearly) independent, physical effects; the flying phase is responsible for the translation of the kinetic properties and governs the particle concentration(dnsity) under the fluctuating motion, while the contact phase responds to the material properties of the particles.

3. Parameterizations

Our aim is to lay down suitable parameters by which each of the two states of the granular material can be described. Of particular interest hereby are the interactions of translational momentum and energy.

Let t_f and t_c denote the *flying time* and the *contact time,* respectively; they may be defined by

$$t_f = \frac{s}{v}, \; t_c = \alpha\frac{d}{c} \tag{3.1}$$

in which s is the mean separation distance of the spherical particles, v their speed of approach, d their common diameter and $c = \sqrt{E/\delta}$ the primary elastic wave speed of their material, respectively, see Fig. 1. If the granular material consists of sand, gravel or other natural grains, s and d will be a typical mean separation and a typical diameter of the grains. Moreover, α in (3.1) is a dimensionless number of order unity that can be treated as an adjustable parameter of the theory, or may simply be set equal to two. t_f is an important parameter for time-dependent non-stationary motions. Its temporal variation is accompanied by variation of the specific volume and thus encompasses dilation and con-

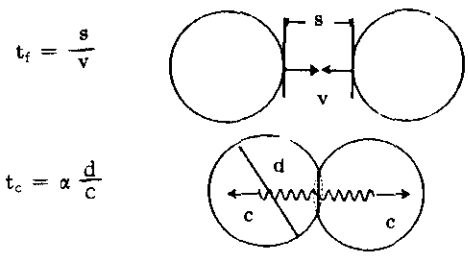

Fig.1 Flying time and contact time

traction. The elastic properties of the grains are operative by a delayed time t_c. Haff in his work has ignored these elasticity effects so $t_c = 0$ or $c \to \infty$, $E \to \infty$ in his limit that corresponds to *rigid* particles.

If s is the mean free path between spherical particles of diameter d and density ρ_s in which s is zero, the mass density of the array of particles is

$$\rho(s) = \left(\frac{d}{d+s} \right)^3 \rho_s \cdot \tag{3.2}$$

Given a diameter d the density of the granular array is therefore a function of s, the mean free path length. Unlike Haff who sets $\rho \sim m/d^3$ we shall not ignore s and thus incorporate in our model effects of the variation of the density.

We define an encounter period of two particles to consist of their collision plus the processes in between two collisions. Thus the time t_e between two encounters is the sum of the flying time and the contact time.

$$t_e = t_c + t_f = \frac{s}{v} + \alpha \frac{d}{c} = \frac{d}{v} \left[\left\{ \left(\frac{\rho}{\rho_s} \right)^{1/3} - 1 \right\} + \alpha \frac{v}{c} \right], \tag{3.3}$$

in which (3.2) has been used.

4. The microscopic model

To close the above system phenomenological statements must be proposed for the state variables σ, η, k and i. Our approach will be simple minded, as we only employ arguments of dimensional analysis. In order to determine the relationships that govern σ, η, k and i, we make use of the simple picture provided by a cell model[4]. The central grain is imagined to vibrate with an average speed v (in the gas dynamics it is called fluctuation velocity) in a random fashion and its effect upon the surrounding grains is described.

4.1 The equation of state for the dispersive pressure

One effect is certainly dispersive pressure that is established. Dimensionally, it is "force per area" or "mass times acceleration per area". The mass of a cell is $m=\rho(d+s)^3$, a typical acceleration is the translational speed divided by the time of encounter, v/t_e and the area of a cell having side length $(d+s)$ is $(d+s)^2$. Hence

$$\sigma = p\,\frac{\rho(d+s)^3\,\frac{v}{t_e}}{(d+s)^2} = p\,\rho(d+s)\,\frac{v}{\frac{s}{v}+\alpha\frac{d}{c}}, \tag{4.1}$$

where p is a dimensionless constant.

4.2 Viscosity

The dynamic viscosity is dimensionally given by "density times area divided by time", if the three quantities are taken to be ρ, $(d+s)^2$ and t_e, respectively, then

$$\eta = q\rho(d+s)^2\,\frac{1}{\frac{s}{v}+\alpha\frac{d}{c}}, \tag{4.2}$$

in which q is a dimensionless coefficient. A more appropriate motivation follows Prandtl's[5] seminal work on turbulent eddy viscosity by word-by-word translation to the present situation. Imagine a shear flow of a granular system, see Fig. 2, When grains collide between two neighbouring layers, an average net momentum of magnitude $m\Delta u=\rho(d+s)^3\,\Delta u$ in the flow direction is transferred. With the collision rate being t_e^{-1} the shear stress exerted by the upper layer, on the lower layer, is

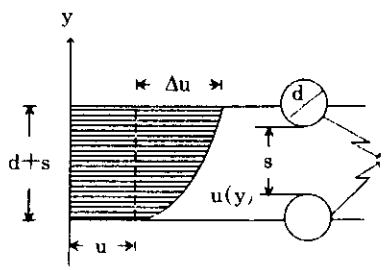

Fig.2 Two neighbouring layers at a distance d+s with a different velocity Δu

$$\tau \sim \frac{m\Delta u}{t_e(d+s)^2} = \frac{\rho(d+s)\Delta u}{\frac{s}{v}+\alpha\frac{d}{c}} = \eta\,\frac{du}{dy}, \tag{4.3}$$

where $\Delta u/(d+s) \sim du/dy$ has been used.

4.3 Thermal diffusivity

The kinematic viscosity(η/ρ) and the thermal diffusivity have the same dimension and so one might be tempted to simply postulate $k \sim \eta/\rho$ with η given in (4.2). However, this would imply that for an infinitely dilute system of particles ($s \to \infty$) k tends to infinity, which is unphysical. It is more appropriate to choose $k \sim \eta/\rho_s$ which now yields

$$k = r\frac{\rho}{\rho_s}(d+s)^2 \frac{1}{\frac{s}{v} + \alpha\frac{d}{c}}, \tag{4.4}$$

in which r is again a dimensionless coefficient. Note that for $s \to \infty$ (4.4) implies $k \to 0$ as it should be. Incidentally these considerations suggest that also the kinematic viscosity should be defined as η/ρ_s.

4.4 The collisional energy annihilation rate

By formulating the momentum and energy balances of two colliding spheres it can be shown that the energy lost in such a collision can be expressed as $\Delta E = (1-e^2)m v^2/2$, where e is the coefficient of restitution and m the mass of the colliding particles. Multiplying this by the collision rate, t_c^{-1} and the number density of the particles, n, and using $n m = \rho$ yields

$$i = \gamma\rho \frac{V^2}{\frac{s}{v} + \alpha\frac{d}{c}}, \tag{4.6}$$

where γ is a dimensionless factor proportional to $(1-e^2)$.

4.5 Disscussion

To discuss the proposed phenomenological relations for σ, η, k and i it is advantageous to write them in dimensionless form. To this end, let

$$S = \frac{s}{d}, \quad V = \alpha\frac{v}{c}, \tag{4.7}$$

and

$$\bar{\sigma} = \frac{\sigma}{p\rho_s v^2}, \bar{\eta} = \frac{\eta}{q\rho_s dv}, \bar{k} = \frac{k}{rdv}, \bar{i} = \frac{i}{\gamma\rho_s\alpha\frac{v}{d}} \tag{4.8}$$

With these, it is straightforward to show that

$$\bar{\sigma} = \frac{1}{(S+V)(S+1)^2}, \bar{\eta} = \bar{k} = \frac{1}{(S+V)(S+1)}, \bar{i} = \frac{1}{(S+V)(S+1)^3} \tag{4.9}$$

Note that in the definitions of the dimensionless quantities $\bar{\sigma}$, $\bar{\eta}$, $\bar{k}$, $\bar{i}$, we do not make use of the elastic properties of the granular material, this is important if results are being compared with those of Haff in which V=0(or $c \to \infty$) at fixed ρ_s. Fig. 3 displays the dependencies of $\bar{\sigma}$, $\bar{\eta}$ or $\bar{k}$ and $\bar{i}$ upon the dimensionless mean free path S and the dimensionless elasticity V of the granules, V = 0 corresponding to the rigid particle of Haff. Evidently, all state variables $\bar{\sigma}$, $\bar{\eta}$ $\bar{k}$ and $\bar{i}$ are monotonically decreasing functions of S.

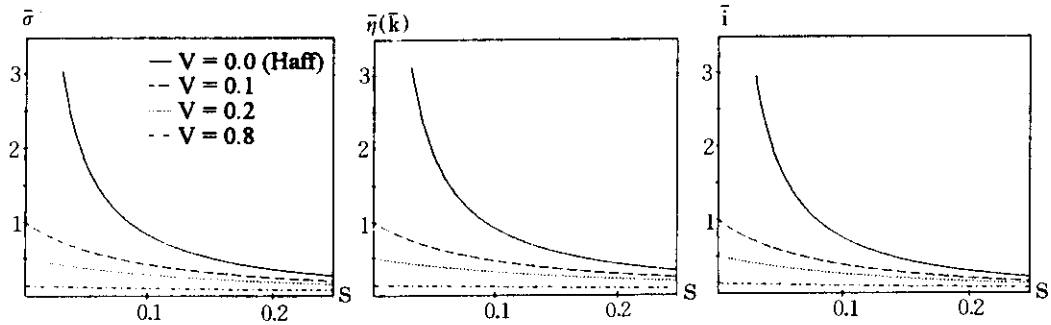

Fig.3 Developments of the dimensionless state variables σ, η, k and i against the free path S, parameterized for four different values V, measuring the role of elasticity.

Furthermore,

- the more elastic the particle is, i. e. the larger V is, the smaller will be the value of the state variable and the less will its value vary with S,
- the larger the compaction of the particles, i. e. the smaller S, is, the larger will be the values of the state variables.

Alternatively, the elastic particle absorbs after each collision a small portion of the energy, stores it as vibrational energy and dissipates it eventually as heat. The rigid particles do not so; such particles transport more fluctuation energy, the agitation and thus the fluctuation velocity v of the granular assemblage with rigid particles is larger than for deformable particles, and this must be so independent of the actual value of the mean free path length: $|v_{elast}| < |v_{rigid}|$. This causes the values of $\bar{\sigma}$, $\bar{\eta}$, $\bar{k}$ and $\bar{i}$ to be smaller for elastic particles than for rigid particles and gives rise to the relaxation effects that can become effective when the vibrational energy is lost by dissipation.

5. Some solutions of the field equations

We shall in this capital study the behaviour of the model equations under very simple

but typical flow conditions. In particular,

- closed systems will be analyzed, i. e. neither material nor energy is assumed to be exchangeable with the surroundings,
- the macroscopic or mean particle motion vanishes, i. e. no material is transported,
- the effects of gravity are ignored (g=0).

More complex typical motions of granular material in which the formulation of boundary conditions becomes crucial are deferred to follow-up papers. We regard our granular system to be describable by the balance laws of mass, momentum and energy of classical continuum physics. Thus, if $u=(u_1, u_2, u_3)$ is the macroscopic flow velocity of the grain system, then conservation of mass leads, as in fluid mechanics, to

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0, \tag{5.1}$$

in which $i = 1, 2, 3$ indicates the Cartesian co-ordinate x_i, Repeated indices are summed from 1 to 3. As a momentum equation thought to be appropriate we take

$$\frac{\partial}{\partial_t}(\rho u_i) + \frac{\partial}{\partial x_k}(\rho u_i u_k) = -\frac{\partial \sigma}{\partial x_i} + \frac{\partial t_{ij}}{\partial x_j} + \rho g_i , \tag{5.2}$$

in which σ is the dispersive pressure, t_{ij} the stress tensor and ρg_i the specific gravity force. The conservation of the energy is taken here in the form.

$$\frac{\partial}{\partial_t}(\frac{1}{2}\rho u^2 + \frac{1}{2}\rho v^2) = -\frac{\partial}{\partial x_k}[\rho u_k(\frac{\sigma}{\rho} + \frac{1}{2}u^2 + \frac{1}{2}v^2) - u_i\eta(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}) - k\frac{\partial}{\partial x_k}(\frac{1}{2}\rho v^2)] + \rho u_i g_i - i, \tag{5.3}$$

in which v is the fluctuating speed, and k functions like a thermal diffusivity. We have partitioned in (5.3) the specific energy into the overall flow kinetic energy $pu^2/2$ and the "internal" energy of the fluctuations $pv^2/2$ and have ignored a rotational energy in conformity will the earlier assumption that rotational inertia of the grains is negligible.

5.1 Uniformly excited system in a closed box of fixed volume

Consider a granular material in a rigid rectangular box. Imagine that this system has for a long time been subject to random vibrations about its center at rest so that the particles inside the box are uniformly distributed and perform a fluctuating motion(that is affected by repetitive ongoing collisions) about their constant mean position. Assume that at time $t = 0$ the shaking motion is suddenly stopped. The fluctuation velocity $v(0) = v_0$ is then allowed to decay with time. Because of the constancy of the volume of the box

the particle density or the mean free path length remain constant, $s(t) = s_0$. Of the field equations the balance laws of mass and momentum are trivially satisfied, while the energy equation reduces to

$$\frac{1}{2}\frac{d}{dt}\rho v^2 + \gamma\rho\frac{v^2}{s + \alpha\frac{v}{c}d} = 0 \tag{5.4}$$

integration of (5.4) under the condition that $s = s_0$ and subject to the initial condition $v(0) = v_0$ yields

$$(1-\frac{v_0}{v}) + \alpha\frac{v_0}{c}\frac{d}{s_0}\ln\frac{v}{v_0} + \gamma\frac{v_0}{s_0}t = 0 \tag{5.5}$$

with the dimensionless quantities

$$Z = \alpha\frac{v_0 d}{c}\frac{d}{s_0} = \frac{\alpha\frac{d}{c}}{\frac{s_0}{v_0}} = \frac{t_c}{t_{f0}}, \qquad T := \gamma\frac{v_0}{s_0}t := \gamma\frac{t}{t_{f0}} \tag{5.6}$$

this can be written as

$$(1-\frac{v_0}{v}) + Z\ln\frac{v}{v_0} + T = 0 \tag{5.7}$$

in the above, $t_{f0} = s_0/v_0$ is the flying time at $t = 0$, so Z is a measure of the influence of

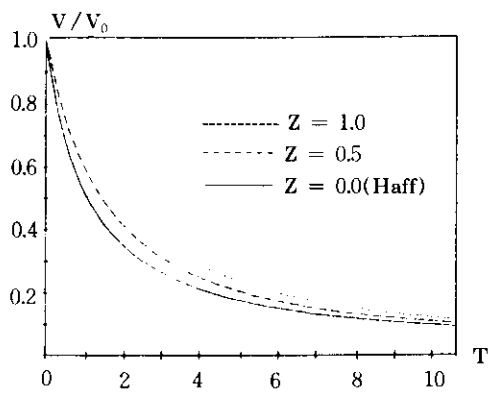

Fig.4 Relaxation of the mean fluctuation velocity v for various values of the elasticities of the particle in the constant−volume box experiment.

the elastic properties on the temporal evolution of the scaled fluctuation energy and thus measures its relaxation time, while T is a dimensionless time. Fig. 4 displays the decay of the fluctuation velocity v with time for various values of the variable Z.

When $Z \neq 0$, as can be seen from Fig. 4 these relaxation times are larger, i. e., the more elastic the material is, the slower the particles will relax in this experiment.

5.2 Uniformly excited system in a closed box at constant pressure

Consider that the box now possesses a rigid lid, a frictionless moveable piston which carries a constant load, so that the inside mean pressure is constant in time $\sigma(t) = \sigma_0$, see Fig. 5. As a result, the volume of the box will adjust itself to the existing pressure. For uniform and steady state conditions of the macroscopic system the balance laws of mass and momentum are trivially satisfied, if gravity forces are ignored. We shall confine attention first to such a Gedankenexperiment and later in a second experiment stop the shaking and investigate the decay rates at constant pressure.

Using

$$S_0 = \frac{s_0}{d}, \quad S = \frac{s}{d}, \quad V_0 = \alpha \frac{v_0}{c}, \quad V = \alpha \frac{v}{c}. \tag{5.8}$$

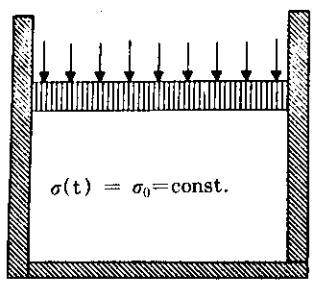

Fig.5 Uniformly excited system with a constant pressure.

(the indexed quantities are referred to the initial reference state) as previously defined in (4.7), and introducing the scaled fluctuation energy

$$\Delta_t^2 = \frac{\rho v^2}{\rho_s v_0^2} = \frac{1}{(1+S)^3} \left(\frac{V}{V_0} \right)^2 \tag{5.9}$$

whose value in the reference state is $\Delta_{t0} = 1/(1+S_0)^3$ and writting

$$P_0 = \frac{1}{p} \frac{\sigma_0}{\rho_s v_0^2} \tag{5.10}$$

for the constant dimensionless pressure, it is easily shown that the expression for the pressure(4.1) can be written as

$$P_0 = \frac{(1+S)\Delta_t^2}{S + V_0(1+S)^{3/2}\Delta_t} = \text{const.} \tag{5.11}$$

These equations have been derived for the (relaxed) conditions that the mean free path length and the fluctuation energy may vary with time. In initial state in which energy source and sink are balanced $S = S_0$ and $\Delta_t^2 = \Delta_{to}^2$ so that equations (5.11) become two possible forms of the "thermal equation of state". The relation says that pressure, density and "granular temperature" are related to one another, and demonstrates the isobars, (P_0 fixed), isotherms (Δ_{to}^2 fixed) and isochores (S_0 fixed). In a gas the time scales of the relaxation of the microscopic processes are very much smaller than the time scales of the macroscopic processes implying that on the macroscopic level the thermal equation of state is a meaningful concept. This is not necessarily so for a granular system, so that the more general equation system that incorporates relaxation effects must be solved.

Further physical insight is gained from (5.11), if we introduce the state of closest packing, corresponding to $s = 0$ or $S = 0$. We shall refer to this state as the *solid body state*. This state need not be motionless, even when $u=0$, because rearrangements of particles at $s = 0$ seem to be possible. Thus, the fluctuation energy corresponding to this state is not unique but lies in the interval $0 \leqslant \Delta_t^2 \leqslant \Delta_{ts}^2$ where Δ_{ts}^2 is that minimal energy beyond which the granular assemblage gives away the densest packing. We call this energy the *transition energy*, or *solid body energy*

$$\Delta_{ts} = P_0 V_0. \tag{5.12}$$

Because it is difficult to find an order of magnitude for P_0, estimates for (5.12) cannot easily be found. However, the quantity

$$\Psi = P_0 V_0^2 = \frac{1}{p} \frac{\sigma_0}{\rho_s v_0^2} \alpha^2 \frac{v_0^2}{c^2} = \frac{\alpha^2}{p} \frac{\sigma_0}{E_s} = P_0 V_0 V_0 = \Delta_{ts}^2 \alpha \frac{v_0}{c} << 1, \tag{5.13}$$

in which E_s is the modulus of elasticity in the solid body state, is easily estimated. With (5.12) and (5.13) and by scaling the fluctuation energy with Δ_{ts}, i. e., $\bar{\Delta}_t = \Delta_t / \Delta_{ts}$, it is readily shown that (5.11) implies

$$\bar{\Delta}_t^2 - (1+S)^{1/2} \bar{\Delta}_t - \frac{S}{\Psi(1+S)} = 0, \tag{5.14}$$

whose positive root (the only physically relevant one) is

$$\bar{\Delta}_t = \frac{1}{2}(1+S)^{1/2} \left[1 + \left\{ 1 + \frac{4}{\Psi} \frac{S}{(1+S)^2} \right\}^{1/2} \right] \tag{5.15}$$

Qualitatively, the normalized energy $\bar{\Delta}_t^2$ grows monotonically with growing S see Fig. 6.: indeed it assumes a minimum for S = 0. The Panel b) shows an enlarged closeup of the neighbourhood of the origin, where $\bar{\Delta}_t(S=0)=1$. This non−zero value of the transition energy in our theory is the true physical reason that no singularities arise in the expressions for the state variables σ, η, k and i when s → 0. It also gives rise to the following interpretation. The material can sustain a certain amount of energy and stay in the rigid−body state without any sign of loosening−up. Only when an amount of energy corresponding to the transition energy has been established, the material will loosen−up. Incidentally, our concept has found a well known application in soil mechanics in methods of soil compaction. In order to enlarge the load capacity of a soil, the soil material is compacted by repetitive beats of a vibrating machine. If the energy that is driven into the soil is too small, the soil will hardly react. If the energy is above the threshold value of the transitional energy the soil material will become loose according to Fig, 6. This energy will be stored in the form of elastic waves which eventually will die away owing to the anelasticities that are present.

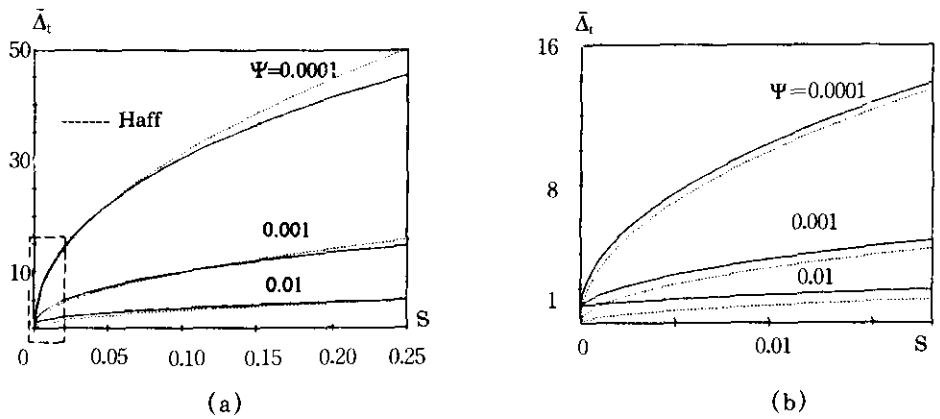

Fig.6 Non−dimensionalized fluctuation energy Δ_t for different values of Ψ.

We next study the relaxation process that sets in, once the shaking motion of the constant−pressure box is stopped. Then the energy equation applies in the form (5.4), which upon substitution of the scales(5.8)−(5.10) takes the form

$$\frac{1}{2}\frac{d\Delta_t^2}{dT} + S_0(1+S)^{3/2}\frac{\Delta_t^3}{S+V_0(1+S)^{3/2}\Delta_t} = 0 \tag{5.16}$$

and, when the constant pressure formula is incorporated, becomes

$$\frac{1}{2}\frac{d\Delta_t^2}{dT} + P_0S_0(1+S)^{1/2}\Delta_t = 0 \tag{5.17}$$

When the scales

$$\bar{\Delta}_t = \frac{\Delta_t}{\Delta_{ts}} \text{ and } \bar{T} = 2\frac{S_0}{V_0} T \tag{5.18}$$

are used, we finally get

$$\frac{d\bar{\Delta}_t^2}{d\bar{T}} + [(1+S)\bar{\Delta}_t^2]^{1/2} = 0 \tag{5.19}$$

In our more general model (eq. (5.19)) the evolution of the scaled fluctuation energy depends upon density, which itself is given by (5.15). Let $\bar{T}_s$ be the *transition time*, i. e. the time it takes the relaxing granular system that started with an energy $\bar{\Delta}_t > 1$(larger than the transition energy) until it has relaxed to the solid body state(corresponding to $\bar{\Delta}_{ts} = 1$). Then it follows from (5.19) by integration that

$$\int_1^{\bar{\Delta}t} \frac{d(X^2)}{[(1+S(X))X^2]^{1/2}} = \bar{T}_s, \tag{5.20}$$

where S(X) is obtained from(5.15), and must be numerically integrated.

This case is graphically displayed in Fig. 7. It shows $\bar{T}_s$ plotted against $\bar{\Delta}_t^2$ in the interval $1 < \bar{\Delta}_t^2 < 10$. $\bar{\Delta}_t$ is the starting fluctuation energy and $\bar{T}_s$ the time it takes until this energy relaxes to the solid body energy. Ψ is a measure of the elasticity of the grains. It is seen that, *the more elastic the grains are, the less time it takes until the solid body state is reached.* For instance, when $\Psi = 0.001$, each particle absorbes in each encounter a relatively large amount of wave energy and keeps it so that the fluctuation energy becomes lesser and lesser.

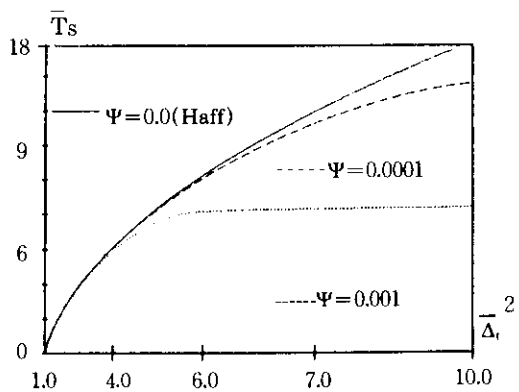

Fig.7 Relaxation of the fluctuation energy in a shaken box filled with particles at constant pressure.

6. Closing remarks

We have taken a rather simple approach and base our concept on the continuum formulation of the balances of mass, momentum and energy in which, besides the kinetic energy of the macroscopic motion, also the kinetic energy of the fluctuating motion is accounted for. These laws are complemented by constitutive relations for the dispensive pressure, shear viscosity (of a viscous stress), diffusivity of the fluctuation energy and its annihilation rate. Relationships for these are proposed on the basis of arguments of dimensional analysis by looking at the binary collision of two identical spherical particles. The mean time between encounters of particles consists of the mean free flying time in between collisions plus the mean duration of a collision. Therse differences lead to important qualitative differences of the inferences that can be drawn:

- Dispersive pressure, viscosity, diffusivity and energy annihilation rate are bounded at zero mean free path length.
- These same quantities monotonically decrease with the size of the mean free path, and the increasing elasticity, of the particle material.
- The relaxation of the settling motion at constant volume is slowed down by the elasticity of the particles.
- In a steady uniform shaking motion at constant dispersive pressure there exists a "thermal equation of state" relating pressure, mean free path length and flucutation energy(=granular temperature).
- There is a finite non−zero value of the fluctuation energy below which the particles are in contact and beyond which they are separated from one another with a nonvanishing mean free path length. This threshold is called the *solid body energy* and the granular material with fluctuation energies below it is called to be in the *solid body state*.
- The time to reach the solid body state in a relaxation experiment starting from a fluctuation energy above the threshold is finite and for fixed density the smaller, the more elastic the particles are.

These results are physically intuitively plausible and they are not borne out by other models describing the rapid flow of granular materials. To obtain them the two relaxing assumptions, i. e., variable bulk density and non−vanishing contact time are very important.

There are many areas in which further investigation is required. Among these are (i) inclusion of gravity in the above Gedankenexperiment, (ii) treatment of gravitational (shear) flow, (iii) theory of the propagation of sound and so on, some of these will be dealt with in an upcoming paper.

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