### Random Response Analysis of Base Isolated Nuclear Container System

기초분리된 원전 격납구조물의 무작위 반응해석

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요 약

고무받침등을 이용한 기초분리공법은 상부구조물의 고유주기를 기초분리가 되지 않은 구조물보다 길게 하여줌으로써 지진에 의해서 발생하는 밑면 전단력을 감소시키는 원리를 이용하고 있다. 이 원리는 지진지역에 있는 일반 건물구조물에 세계각국에서 성공적으로 사용되고 있으며, 특히 교량구조물에도 그 역할이 입증되어 미국, 일본 등을 중심으로 적용이 급증하고 있다. 본 논문은 동일한 원리를 우리나라에서 건설되고 있는 원전구조물에 적용하여 기초분리된 원전 격납구조물의 거동을 고찰하고자 한다. 이와 같은 거동해석을 실시하는데 있어서, 시간영역 해석은 많은 시간과 경비를 요하게 되어 현실적으로 사용하기에 여러 어려움이 존재하게 되는데 반해, 주파수영역 해석은 이러한 단점을 극복하게 되어 실용적이며 효과적인 결과를 제공하게 된다. 즉, 입력 지진파에 의한 기초분리 원전 격납구조물의 거동을 예측함에 있어서 시스템 복소주파 응답함수 및 지진파의 파워스펙트럼 계산을 통하여 보다 합리적인 접근이 가능함을 보이고자 한다.

### Abstract

Seismic isolation in ordinary buildings has been successively adapted to provide flexibility for the reduction of base shear forces and its concept is accepting wide agreement in lengthening the natural period to lessen the spectral acceleration transmitted into the structure. However, one of difficulties in implementing the innovative concept to nuclear structures is due to more severe requirements in both understanding and predicting the characteristics of isolators and the behavior of cushioned structures, Stochastic analysis has been carried out to investigate the response of base isolated nuclear containers to the random earthquake ground motion.

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#### 1. INTRODUCTION

An analytical model of a containment superstructure has been created based on one of the nuclear power plants in Korea. The equations of motion of containment with a resilient-friction base isolator are modified using stochastic linearization technique to calculate transfer functions. From these transfer functions the responses of interest including mean square relative displacements of lumped masses with respect to foundation are calculated as a function of system parameters for both fixed and isolated base conditions. Mean square accelerations of lumped degrees of freedom are also calculated for the realistic spectral shape of the site of nuclear structures. The responses from fixed and isolated bases are compared to demonstrate that Coulomb-type friction isolator be able to limit the relative responses of the superstructure. The response reduction of the superstructure in terms of deflections and accelerations then causes the floor response spectrum to diminish. Another important aspect which must be considered in isolating nuclear structures is to seek the reliability of performance of the damper to estimate probability based reduction in the elastic deformation and acceleration of the superstructure. The seismic isolation of nuclear power plants is an excellent concept. This paper presents an analytical solution of frequency domain for the stochastic response to uncover the effectiveness of isolated systems in mitigating seismic load while enhancing the overall structural safety.

## 2. EQUATIONS OF MOTION AND STOCHASTIC RESPONSE

An analytical model is used for a base isol-

ated nuclear container as shown in Fig. 1.

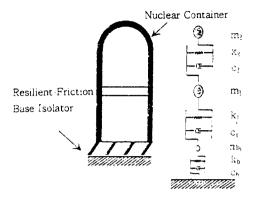


Fig. 1. Model of nuclear container with isolated base

The equations of motion for the superstructure and the base isolator can be written as in Eq. 1. and Eq. 2. The resilient-friction base isolator will be used to provide the system with flexibility and the equation of motion of the isolator is rewritten in Eq. 3[1].

$$[M]{\ddot{y}}+[C]{\dot{y}}+[K]{y} = -[M]{1}(\ddot{y}_b+\ddot{y}_g)$$
(1)

where

$$[M] = \begin{bmatrix} m_2 \\ m_1 \end{bmatrix}, [K] = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 + k_1 \end{bmatrix},$$

$$[C] = \frac{2\xi_j}{\omega_j} [K], j = 1, 2$$

$$m_b(\ddot{y}_b + \ddot{y}_g) + c_b \dot{y}_b - k_b y_b - c_1 \dot{y}_1 - k_1 y_1 = 0 \qquad (2)$$

$$m_b(\ddot{y}_b + \ddot{y}_g) + c_b \dot{y}_b + k_b y_b + (m_b + m_1 + m_2) \mu' g$$

$$sgn(\dot{y}_b) - c_1 \dot{y}_1 - k_1 y_1 = 0 \qquad (3)$$

The subscripts 1, 2 and b indicate the first, second and the base isolator degrees of freedom, respectively[3]; therefore,  $m_1$ ,  $m_2$  and  $m_b$  are the masses at the first, second and base isolator degree of freedom, respectively.

In order to avoid mathematical complexity, stochastic linearization thechnique is applied to find the equivalent linear expression of Eq. 4. Upon introducing the error and letting this mean square error function Eq. 5 be minimal with respect to  $\xi_e$ , the equivalent damping of the system Eq. 6 is obtained.

$$\ddot{\mathbf{y}}_{b}+2(\xi_{b}+\xi_{e})\omega_{b}\dot{\mathbf{y}}_{b}+\omega_{b}^{2}\mathbf{y}_{b}-2\xi_{1}\omega_{1}ab\dot{\mathbf{y}}-\omega_{1}^{2}$$

$$ab\mathbf{y}=-\ddot{\mathbf{y}}_{g}$$
(4)

error = 
$$(1+N\alpha)\mu' g \operatorname{sgn}(\dot{y}_b) - 2\xi_e \omega_b \dot{y}_b$$
 (5)

$$\xi_{e} = \frac{(1+N\alpha)\mu' g}{2\omega_{b}} \frac{E[y_{b}sgn(\dot{y}_{b})]}{E[\dot{y}_{b}]}$$
(6)

Eq. 6 can be reduced to Eq. 7 when the assumption of zero mean Gaussian process is valid on the ground motion, displacement and velocity response of the structure.

$$\xi_{e} = \frac{(1+N\alpha)\mu'g}{2\omega_{b}} \frac{2\int_{0}^{\infty} \dot{y}_{b} f_{\dot{y}_{b}}(\dot{y})dy}{\sigma_{\dot{y}_{b}}}$$
(7)

where  $f_{\dot{y}_b}$  and  $\sigma_{\dot{y}_b}$  are the probability density function and the standard deviation of the velocity  $\dot{y}_b$ , respectively.

Integrating Eq. 7 yields the following Eq. 8 while standard deviation is shown in Eq. 9.

$$\xi_{e} = \frac{(1+N\alpha)\mu' g}{\sqrt{2}\omega_{b}\sigma_{\dot{y}_{b}}}$$
(8)
$$\sigma_{yb} = \frac{\left[\frac{2}{\pi}(1+N\alpha)^{2}\mu'^{2}g^{2}+8\xi_{b}\omega_{b}\pi G_{0}\right]^{1/2}-\left(\frac{2}{\pi}\right)(1+N\alpha)\mu' g}{4\xi_{b}\omega_{b}}$$
(9)

N and  $\alpha$  are the number of degrees of freedom and the ratio of mass at first degree of freedom to the mass of base isolator  $(m_1/m_b)$  respectively, and  $\mu'$  is the coulomb friction coefficient. Since standard deviation  $\sigma_{\hat{y}_b}$  is time dependent,  $\xi_e$  is also time dependent.  $\xi_e$  becomes constant, however, when the standard deviation  $\sigma_{\hat{y}_b}$  reaches stationary in a very short

time[2].

# 3. RESPONSE OF BASE ISOLATED NUCLEAR CONTAINER TO STATIONARY RANDOM EXCI-

The excitation here is the motion of the foundation which we consider to be described by its acceleration  $\ddot{x}_0(t)$ . Nine different responses will be examined: the relative displacement  $y_1=x_1-x_0$  and  $y_2=x_2-x_1$  and  $y_b=x_b-x_0$  and the absolute accelerations of the three masses,  $\ddot{y}_1$ ,  $\ddot{y}_2$  and  $\ddot{y}_b$ .

There are nine complex frequency responses,  $H_{y_1}(\omega)$ ,  $H_{\ddot{y}_1}(\omega)$ ,  $H_{\ddot{y}_1}(\omega)$ ,  $H_{y_2}(\omega)$ ,  $H_{\dot{y}_2}(\omega)$ ,  $H_{\ddot{y}_b}(\omega)$ ,  $H_{\ddot{y}_b}(\omega)$  and  $H_{\ddot{y}_b}(\omega)$ , as appeared in Eqs. 10-18.

$$y_1 = e^{i\omega x} H_{y_1}(\omega) \tag{10}$$

$$\dot{\mathbf{y}}_1 = (i\omega)e^{i\omega \mathbf{x}} \mathbf{H}_{\dot{\mathbf{y}}_1}(\omega) \tag{11}$$

$$\ddot{\mathbf{y}}_{1} = -\omega^{2} e^{i\omega \mathbf{x}} \mathbf{H}_{\ddot{\mathbf{y}}_{1}}(\omega) \tag{12}$$

$$\mathbf{v}_2 = \mathbf{e}^{\mathrm{i}\omega\mathbf{x}} \, \mathbf{H}_{\mathbf{v}} \, (\omega) \tag{13}$$

$$\dot{\mathbf{y}}_2 = (\mathrm{i}\omega)\mathrm{e}^{\mathrm{i}\omega\mathbf{x}}\,\mathbf{H}_{\dot{\mathbf{y}}_\alpha}(\omega) \tag{14}$$

$$\ddot{\mathbf{y}}_2 = -\omega^2 \mathbf{e}^{\mathbf{i}\omega \mathbf{x}} \,\mathbf{H}_{\ddot{\mathbf{y}}_3}(\omega) \tag{15}$$

$$y_h = e^{i\omega x} H_{y_h}(\omega) \tag{16}$$

$$\dot{\mathbf{v}}_{b} = (i\omega)e^{i\omega\mathbf{x}} \mathbf{H}_{\dot{\mathbf{v}}_{c}}(\omega) \tag{17}$$

$$\ddot{\mathbf{y}}_{b} = -\omega^{2} \mathbf{e}^{i\omega \mathbf{x}} \mathbf{H}_{\ddot{\mathbf{y}}_{c}}(\omega) \tag{18}$$

Substituting these input-output relationships in terms of complex frequency responses into the equations of motion, Eqs. 1, 2 and 4, the following linearized Eqs. 19-21 are established.

$$-\omega^{2}H_{y_{2}}(\omega) + 2\xi_{2}\omega_{2}(i\omega)H_{y_{2}}(\omega) - 2\xi_{2}\omega_{2}(i\omega)H_{y_{1}}(\omega)$$

$$+\omega^{2}H_{y_{2}}(\omega) - \omega^{2}H_{y_{1}}(\omega) = -(-\omega^{2}H_{y_{1}}(\omega) + 1)$$
(19)

$$-\omega^{2}H_{y_{1}}(\omega) - 2\xi_{2}\omega_{2}(i\omega)\mu H_{y_{2}}(\omega) + (2\xi_{1}\omega_{1} + 2\xi_{2}\omega_{2}\mu)(i\omega)H_{y_{1}}(\omega) - \mu\omega_{2}H_{y_{2}}(\omega) + (\omega_{1}^{2} + \omega_{2}^{2}\mu)H_{y_{1}}(\omega) = -(-\omega^{2}H_{y_{R}}(\omega) + 1) (20)$$

$$\begin{aligned} &-\omega^2 H_{y_y}(\omega) + 2(\xi_b + \xi_e)\omega_b(i\omega)H_{y_b}(\omega) + \omega_b^2 H_{y_b}(\omega) \\ &-2\xi_1\omega_1\alpha(i\omega)H_{y_y}(\omega) - \omega_1^2\alpha H_{y_1}(\omega) = -1 \end{aligned} (21)$$

where  $\mu = m_1/m_2$  and  $\alpha = 1/m_b$ .

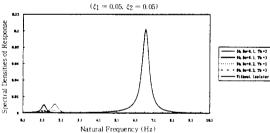
Solving for the complex frequency responses at the first D.O.F. as well as the isolator and plotting them in Fig. 2 as the function of system parameters explicitly identify the influences of these parameters on the behavior of the base isolated nuclear container.

Rearranging the linearized system equations in terms of acceleration and combining them with complex frequency responses yield Eqs. 22-24.

$$H_{\ddot{y}_{2}}(\omega) + 2\xi_{2}\omega_{2}(i\omega)H_{y_{2}}(\omega) - 2\xi_{2}\omega_{2}(i\omega)H_{y_{1}}(\omega) + \omega_{2}^{2}H_{y_{2}}(\omega) - \omega_{2}^{2}H_{y_{1}}(\omega) = 0$$
 (22)

$$H_{\ddot{y}_{1}}(\omega) - 2\xi_{2}\omega_{2}(i\omega)\mu H_{y_{2}}(\omega) + (2\xi_{1}\omega_{1} + 2\xi_{2}\omega_{2}\mu)H_{y_{1}}(\omega) - \mu\omega_{2}^{2}H_{y_{2}}(\omega) + (\omega_{1}^{2} + \omega_{2}^{2}\mu)H_{y_{1}}(\omega) = 0$$
 (23)

Relative Displacement Response Spectral Density for Ideal White Noise Excitation (1st D.O.F.)



Relative Displacement Response Spectral Density for Ideal White Noise Excitation (Base Isolator)

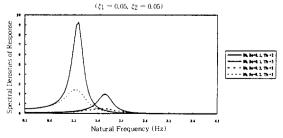
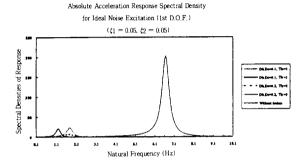


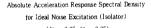
Fig. 2. Relative displacement response spectral density for ideal white noise excitation

$$H_{y_a}^{..}(\omega) + 2(\xi_b + \xi_e)\omega_b(i\omega)H_{y_b}(\omega) + \omega_b^2 H_{y_b}(\omega) -2\xi_1\omega_1\alpha(i\omega)H_{y_a}(\omega) - \omega_1^2\alpha H_{y_a}(\omega) = 0$$
 (24)

In Figure 2 to Figure 5.  $D_b$  and  $D_e$  represent the damping ratio of base isolator and equivalent linear damping, respectively. And  $T_b$  is the natural period of base isolator.

The transfer functions are solved for both at the first D.O.F. and the isolator. They are shown in Fig. 3.





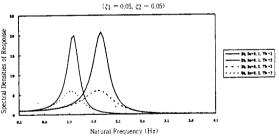


Fig. 3. Absolute acceleration response spectral density for ideal white noise excitation

### 4. RESULTS AND CONCLUSIONS

Consider Wiener-Khintchine relation

$$S_{x}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{x}(\xi) e^{(-i\omega\xi)} d\xi$$
 (25)

The second integral is recognized as the Fourier transform  $H(\omega)$  of the impulse response. The first integral has identical form

except for the sign in the exponent. It can be written as  $H(-\omega)$  and it may be seen that it will be the complex conjugate of  $H(\omega)$ . With these interpretations,

$$S_{v}(\omega) = H(-\omega)H(\omega)S_{v}(\omega) \tag{26}$$

A slightly more compact form is achieved by noting that the product of  $H(\omega)$  and its complex conjugate may be written as the square of the magnitude of  $H(\omega)$ ;

The power spectra of both displacement and acceleration are obtained through Eq. 27 and specially through Eq. 28 when ground motion can be represented by the ideal white noise spectrum of constant density S<sub>0</sub>.

$$S_{v}(\omega) = |H(\omega)|^{2}S_{x}(\omega)$$
 (27)

$$S_{v}(\omega) = |H(\omega)|^{2}S_{0} \tag{27}$$

where  $S_y(\omega)$  is the mean square response power spectrum and  $S_0$  is the mean square acceleration per rad/sec. This is the desired relation between the spectral density of the excitation x(t) and the spectral density of the response y(t). Note that this is an algebraic relation.

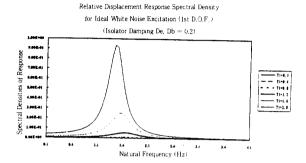
In case that the ground excitation is assumed to have zero mean stationary value, the responses are also zero mean processes with the corresponding variances, as shown in Eq. 29 and 30[4].

$$\begin{split} E[y^2] &= \int_{-\infty}^{\infty} S_y(\omega) d\omega \\ &= \int_{-\infty}^{\infty} |H(\omega)|^2 S_x(\omega) d\omega \\ &= S_0 \int_{-\infty}^{\infty} |H(\omega)|^2 d\omega \end{split} \tag{29}$$

$$\sigma = \sqrt{E[y^2]} \tag{30}$$

Fig. 4 shows spectral densities of displace-

ment and acceleration responses of the superstructure for the different natural periods of the superstructure.



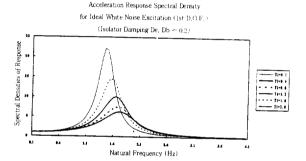
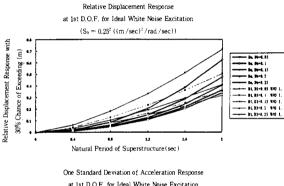


Fig. 4. Response spectral density for ideal white noise excitation for different dynamic characteristics of a superstructure

As depicted in Fig. 5, an extensive sensitivity analysis was carried out to investigate the behavior of a base isolated nuclear container. When the ratio of the frequency of the isolated superstructure to the fixed base frequency is unity, the relative displacement responses of the superstructure with the isolated mechanism become larger than those of a fixed base system for the isolator damping greater than 20%.

The differences of acceleration responses between the base isolated and the fixed base structures are becoming negligible as the superstructure becomes flexible, and the acceleration responses of the superstructure of the base isolated system with 2 second superstructure.

tural period are identical to those of a fixed base structure for the 20% critical isolator damping or higher. As the natural period of the superstructure becomes small, rigid body motion or almost rigid body motion with relatively low level of acceleration is observed in the isolated superstructure. However, significant acceleration is induced at the degree of freedom under the consideration for the conventional structures with a fixed base. As intuitively understood, it is also found that the displacement of the superstructure becomes zero, as the superstructure approaches zero natural period for the structures with both the isolated and the fixed base



One Standard Deviation of

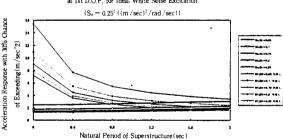


Fig. 5. Displacement and acceleration by sensitivity analysis

The effectiveness of the methodology presented in this paper has been demonstrated in understanding and predicting the behavior of base isolated nuclear containers. Providing flexibility causing structures to more deflect

results in the reduction of acceleration. This extra deflection that the structure experiences can be decreased by increasing the damping capacity of the system.

Many types of sensitivity analyses are now possible at the reasonable time and the corresponding cost. In Fig. 5 the displacement and acceleration responses of both the superstructure and the isolator are estimated in frequency domain for differenent natural periods and damping values of the superstructure. This calculation can be repeated as the dynamic characteristics of an isolator change. The analytical results performed and presented in this paper may be required to be validated through extensive shaking table tests.

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