## 동적 하중을 받는 3점 굽힘 시험편들에서의 J와 CMOD와의 관계 이 억 섭\*. 차 일 남\*\* .조 재 응\*\*\*

Relation between J and CMOD in Dynamic Loaded 3-Point Bend Specimens

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### **ABSTRACT**

Numerical calculations are made in order to find a possible relation between the J-integral and the crack mouth opening displacement (CMOD) in dynamic nonlinear fracture experiments. Both elastic-plastic and elastic-viscoplastic materials are considered at different impact velocities. The J-integral may be estimated from the crack mouth opening displacement which can be measured directly from photographs taken during dynamic experiments.

Key Words: Dynamic Fracture(동적파괴), Crack Mouth Opening Displacement(크랙개구변위), Visco-plastic Property(점소성), J-integral(J-적분)

### 1. Introduction

The J-Integral used as a ductile crack initiation criterion for dynamically loaded elastic-plastic 3-point bend specimens has been discussed in  $(1\sim5)$ . Some experimental methods to measure or estimate the J-integral history history under dynamical loading conditions have been investigated and compared to theoretically obtained values  $(1,6\sim8)$ . For example, a caustic method has been successfully applied in (1). Another method is to use the multiple strain gauge measurements and then to estimate the J-integral value near the crack tip (2).

It is well known that a correlation between

the J-integral and CMOD exists under static and small scale yield conditions (9 $\sim$ 11). In this paper, numerical calculations are performed in order to find a relation between the J-integral and CMOD for the dynamic nonlinear stationary crack. Then, from the relation between CMOD and the J-integral, the dynamical J-integral history has been estimated at different impact velocities (Vo=15, 30, 45, 60m/s). Both elastic-plastic and elastic-viscoplastic materials are considered.

### 2. Finite Element Model

The geometry of the specimen and the finite

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element model is shown in Fig. 1.

Due to symmetry, only half a specimen is modeled. A two-dimensional mesh including 92 eight node plane stress elements with 2\*2 Gauss points, i. e. with reduced integration, is chosen. The mesh near the crack tip is concentrated by using degenerated eight node elements. In order to model a possible loss of contact at the load point A and at the support point B as discussed in (12), gap elements with one degree of freedom are introduced. Furthermore, a lumped mass element is used to model the impact head, see Fig. 1.

No crack propagation is taken into account in the calculations. The dynamical J-integral and CMOD are calculated using the commercial finite element method code ABAQUS(13). In this code, the virtual crack extension method is successfully used to evaluate the J-integral in the nonlinear case(14, 15) and the dynamic case (2, 16).

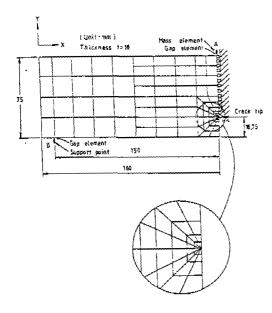


Fig. 1 Finite element model for the three point bend specimen with a quarter notch

# 3. Results From Elastic-Plastic Calculation With a Quarter Notched Specimen

An isotropic elastic-plastic hardening von Mises material is modeled with Young's modulus E=206 GPa, Poisson's ratio v=0.3, density  $\phi=7800$  Kg/m² and yield stress  $\sigma_Y=360$  MPa. The static stress to strain curve is shown in Fig. 2.

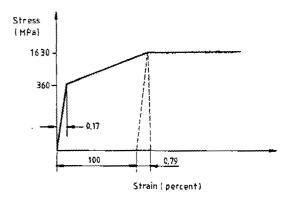


Fig. 2 Static stress-strain curve of the material

The specimen is impact loaded at the middle point A by an impact head with a weight of M=1.96 KN. A comparison of the dynamic behavior for impact loading at side point B or at middle point A can be found in (12). Four different impact velocities are chosen for the simulations.

 $V_o = 15m/s$ 

 $V_0 = 30 \text{m/s}$ 

 $V_0 = 45 \text{m/s}$ 

 $V_0 = 60 \text{m/s}$ 

Calculations are run up to 600  $\mu$ s after impact. The J-integral and CMOD history can be found at every time step. Then, it is found that the relation between  $J=J(V_0,t)$  and CMOD= $\delta_M(V_0,t)$  can be written as (9, 11):

$$J(V_o, t) = \beta(V_o) \sigma_Y \delta_H(V_o, t)$$
 (1)

where  $\beta(V_o)$  is estimated by the least squre method.

With k being the number of time increments in the finite element calculations, we obtain:

$$\beta (V_o) = \frac{\sum_{i=1}^{k} (V_o) \delta_{\Pi i} (V_o)}{\sigma_Y \sum_{i=1}^{k} \delta_{\Pi i} (V_o)^2}$$
(2)

where  $J_i$  and  $\delta_{M_i}$  are the calculated values of J and  $\delta_{M}$  at the time i.

The following  $\beta(V_0)$  values were found:

$$\beta$$
 (15)=0. 72  $\beta$  (30)=0. 78   
 $\beta$  (45)=0. 79  $\beta$  (60)=0. 76 (3)

These results suggest that  $\beta(V_o)$  is insensitive to the impact velocity  $V_o$  and we may therefore take  $\beta = \beta(V_o) = 0.76$ , i. e. the mean value of the above results.

Indeed, the maximum error using this  $\beta$ -value is less than 6% when compared with the  $\beta$ -values given by eq. (3).

Figs. 3~6 show the J-integral and  $\beta \cdot \sigma_Y \cdot$  CMOD history at the four different impact

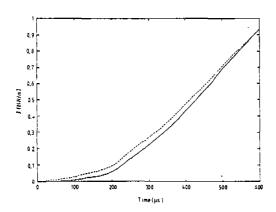


Fig. 3 J-integral (solid line) and  $\beta$  ( $V_o$ )  $\sigma_Y \delta_M$  (dashed line) history at 15 m/s, ( $\beta$  is chosen to 0.76)

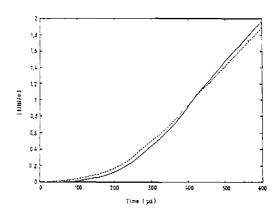


Fig. 4 J-integral (solid line) and  $\beta$  (V<sub>o</sub>)  $\sigma_Y \delta_M$  (dashed line) history at 30 m/s, ( $\beta$  is chosen to 0.76)

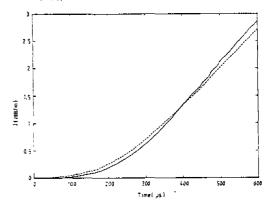


Fig. 5 J-integral (solid line) and  $\beta$  (V<sub>o</sub>)  $\sigma_{Y}\delta_{M}$  (dashed line) history at 45 m/s, ( $\beta$  is chosen to 0.76)

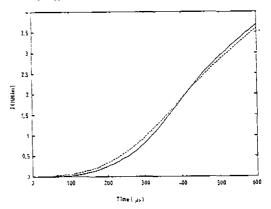


Fig. 6 J-integral (solid line) and  $\beta$  (V<sub>o</sub>)  $\sigma_{Y}\delta_{M}$  (dashed line) history at 60 m/s, ( $\beta$  is chosen to 0.76)

velocities of 15, 30, 45, and 60 m/s with  $\beta$ = 0.76.

In Figs. 3~6 the J-integral and  $\beta \cdot \sigma_Y \cdot \text{CMOD}$  history are found to be in a good agreement. By this result, as soon as the CMOD history  $\delta_M(V_o,t)$  is measured from experiments at a specific impact velocity, the J-integral can be calculated according to the relation  $J(V_o,t) = \beta(V_o) \sigma_X \delta_M(V_o,t)$ 

### 4. Inclusion of Viscoplastic Material Behavior

Since the impact velocities considered above are rather high, rate dependent properties might have a marked effect. In order to investigate these phenomena, the viscoplastic behavior is introduced in the model. To provide a short exposition of this theory, we shall consider small strains for the moment. In this case,

the total strain rate  $\dot{\epsilon}_{ij}$  is written as:

where & is linearly related to the stress rate according to Hooke's law:

$$\varepsilon = \frac{1}{2 \mu} \cdot \frac{1-2 \nu}{s_{13}} + \frac{1-2 \nu}{E} s \delta_{13}$$
 (5)

With 
$$S_{i,j} = \dot{\sigma}_{i,j} - \dot{S} \delta_{i,j}$$
,  $\dot{S} = \frac{1}{2} \dot{\sigma}_{i,j}$ .

 $\delta_{ij}$ =Kronecker's delta and  $\mu$ =shear modulus.

 $\varepsilon^{\bullet\bullet}$  represents combined viscous and plastic effects:

$$\varepsilon_{i,j}^{op} = r \varphi(F) \frac{\partial f}{\partial \sigma_{i,j}}$$
 (6)

where,

$$F = \frac{f(\sigma_{3,3})}{-1} - 1 \tag{7}$$

and 
$$\varphi(F) = \begin{cases} 0 & \text{if } F \le 0 \\ \varphi(F) & \text{if } F > 0 \end{cases}$$
 (8)

In the above equations,  $\gamma$  is a viscosity constant of the material and x is a strain hardening parameter. f is the potential function that depends on the state of stress  $\sigma_{ij}$  for an isotropic work-hardening material. F is the yield function and  $\phi$  is a function of F. All these quantities may be determined from tests of the material under dynamic loading.

When the von Mises yield condition is assumed, the one dimensional form of (6) becomes:

$$\varepsilon^{\varphi} = {2/\sqrt{3}} \gamma \varphi \left( \frac{\sigma}{\sigma \gamma} - 1 \right)$$
 (9)

where  $\sigma_Y$  is the current yield stress. By introducing the choice  $\Phi(F) = F^P$ ,

We obtain:

$$\varepsilon^{\text{vp}} = D(\frac{\sigma}{\sigma_{\text{Y}}} - 1)^{\text{P}}$$
(10)

where  $D = (^2/\sqrt{3})\gamma$ 

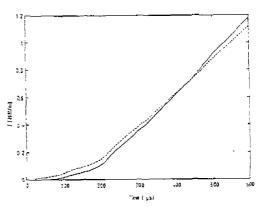


Fig. 7 J-integral (solid line) and  $\beta(V_o) \sigma_Y \delta_M$  (dashed line) history at 15 m/s with viscoplasticity ( $\beta$  is chosen to 0.85).

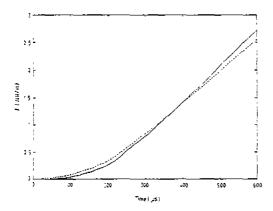


Fig. 8 J-integral (solid line) and  $\beta$  (V<sub>o</sub>)  $\sigma_Y \delta_M$  (dashed line) history at 30 m/s with viscoplasticity ( $\beta$  is chosen to 0.94).

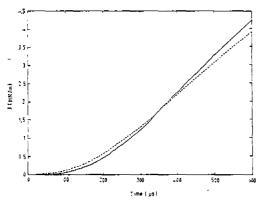


Fig. 9 J-integral (solid line) and  $\beta(V_o) \sigma_Y \delta_M$  (dashed line) history at 45 m/s with viscoplasticity ( $\beta$  is chosen to 0.98).

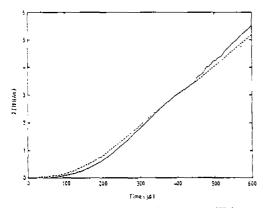


Fig. 10 J-integral (solid line) and  $\beta$  (V<sub>o</sub>)  $\sigma_Y \delta_M$  (dashed line) history at 60 m/s with viscoplasticity ( $\beta$  is chosen to 1.03).

In the calculations, the data of  $^{(17)}$  are adopted, i. e. D=4100 1/s and p=2.

For the four impact velocities, the development with time of the J-integral and the CMOD-value is calculated. Again, the relation (1) between  $J=J(V_o,t)$  and  $CMOD=\delta_M(V_o,t)$  is adopted.

Figs.  $7{\sim}10$  show the J-integral and  $\beta \cdot y \cdot$  CMOD history at the four different impact velocities of 15, 30, 45 and 60 m/s with the viscoplasticity.

In this case,  $\beta$  varies according to different velocities. Therefore  $\beta$  is no longer same at the different velocities.

Using (2) we obtain:

$$\beta$$
 (15)=0. 85  $\beta$  (30)=0. 94  
 $\beta$  (45)=0. 98  $\beta$  (60)=1. 03

In Figs. 7~10, the J-integral and  $\beta \cdot \sigma_Y \cdot$  CMOD history are found to indicate a good agreement.

From the  $\beta$ -values given by (11), a  $\beta$ -Vo curve as shown in Fig. 11 can be established.

With this curve and relation (1), the CMOD-value can be then used to estimate the J-integral for each impact velocity  $V_{\rm o}$ .

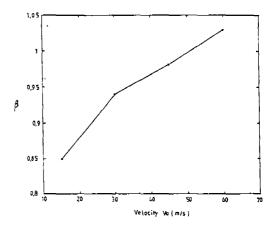


Fig. 11 Estimated  $\beta$ -V<sub>o</sub> curve for the viscoplastic model.

### Conclusions

From the numerical analysis for a ductile crack initiation criterion with dynamically loaded 3-point bend specimens, the following results are obtained.

- 1. The possibility relating the J-integral and the crack mouth opening displacement at dynamically loaded three point bend specimens has been investigated. The J-integral can be the yielding stress multiplied by crack mouth opening displacement times  $\beta(V_o)$ .
- In the calculations, the impact velocities vared from 15 m/s up to 60 m/s. Two different material properties, i. e. elastic-plastic and elastic-viscoplastic properties have been considered.
- 3. In case of elastic-plastic material, it was found that the parameter  $\beta(V_o)$  was independent on the impact velocity  $V_o$  in the impact velocity region studied. Thus, once  $\beta$  is determined by a finite element calculation for a specific material and geometry, the J-integral can be calculated from CMOD experiments.
- 4. For an elastic-viscoplastic material, a linear relation between the J-integral and the crack mouth opening displacement is found. But the coefficient  $\beta(V_0)$  now varies according to different impact velocities. However, once  $\beta(V_0)$  is determined, the value of the J-integral can be calculated from CMOD experiments.

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