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The Effect of The Initial Phase Angles of The Large-Scale Coherent Structures in a Spatially Developing Viscous Shear Layer

공간적으로 발전하는 점성 전단층에서 Lage-Scale 구조의 초기 위상각의 효과

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요 약

이 논문에서 우리는 발전하는 전단층의 2차원 Wave Mode에 대한 비선형 상호작용에 대한 문제를 다루었다. 총 위상각은 Wave Mode ij와 kℓ의 상응하는 에너지와 위상 상호작용을 조절한다.

그러므로 이 논문의 목적은 전단층에서 Lange-Scale 구조의 초기 위상각의 효과를 조사하고자 하는 것이다. 이 연구에서 우리는 Subharmonic의 존재는 전단층의 성장에 상당한 영향을 준다는 것을 알았고 Entrainment에서도 증가하는데 영향을 준다는 것을 알았다. 우리는 또한 Mean Flaw와 Fundamental의 다른 초기 위상각의 효과는 Subharmonic이 성장하는 먼 Downstream 영역에서 보여지기 시작한다는 것을 알았다.

1. Introduction

The integral energy method is applied to solve the nonlinear development of the large-scale coherent structures. The integral energy method was first given by Stuart(1962) for the nonlinear stability analysis. This method was further developed by Ko, Kubota&Lees(1970) in the analysis of the nonlinear development of a laminar wake. Nikitopoulos&Liu(1987, 1989) studied the nonlinear two- and three-mode interactions in a developing mixing layer. In recent Seo(1993) and Seo&Nikitopoulos (1993) also studied three-dimensional wave mode interactions in a spatially developing plane mixing layer by using energy method.

The integral energy equations coupled with the shape assumptions are used to derive the nonlinear ordinary differential equations that describe the energy exchange between the mean flow and large-scale wave modes. The shape assumptions for the mean flow and wave modes are required in order to determine the integral coefficients in the energy equations. In this study we assumed that the large-scale wave modes are decomposed into fundamental and subharmonic wave modes.

In here, we derived the energy equations of the mean flow and wave modes, and solved those equations numerically.

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2. Governing Equations

An arbitrary quantity $q(\vec{x}, t)$ is expressed as

$$q(\vec{x}, t)=Q(\vec{x})+\tilde{q}(\vec{x}, t) \tag{1}$$

where $Q(\vec{x})$ is the time-averaged mean quantity and $\tilde{q}(\vec{x}, t)$ represents the large-scale coherent structure.

For the purposes of this study the large-scale velocity components $\tilde{u}_j$ and pressure $\tilde{p}$ are assumed to be Fourier analyzable, and are decomposed into two wave modes which are periodic in time t. Thus the decomposition of the large-scale structure yields :

$$\tilde{u}=\tilde{u}_{10}+\tilde{u}_{20} \tag{2}$$

$$\tilde{v}=\tilde{v}_{10}+\tilde{v}_{20} \tag{3}$$

$$\tilde{p}=\tilde{p}_{10}+\tilde{p}_{20} \tag{4}$$

with

$$\begin{bmatrix}\tilde{u}_{mn}\\ \tilde{v}_{mn}\\ \tilde{p}_{mn}\end{bmatrix}=\begin{bmatrix}u'_{mn}\\ v'_{mn}\\ p'_{mn}\end{bmatrix}e^{-i\beta_{mn}t}+c.c. \tag{5}$$

where u'_{mn}, v'_{mn} and p'_{mn} and are complex amplitudes, and β_{mn} is the frequency of wave mode mn. The large-scale wave mode with mn=10 will be called *subharmonics* and one with mn=20 will be called *fundamental*. The fundamental frequency is 2 times of the subharmonic.

Following work by Nikitopoulos&Liu(1987) and Seo(1993), the Fourier amplitudes and are assumed to be separable into an unknown finite complex amplitude and corresponding vertical shape functions :

$$\begin{bmatrix}u'_{mn}(x, \eta)\\ v'_{mn}(x, \eta)\\ p'_{mn}(x, \eta)\end{bmatrix}=A(x)\begin{bmatrix}\hat{u}_{mn}(\eta)\\ \hat{v}_{mn}(\eta)\\ \hat{p}_{mn}(\eta)\end{bmatrix} \tag{6}$$

Following earlier work by Stuart(1962), Ko, Kubota&Lees(1970) and Nikitopoulos&Liu(1987, 1989), the shape assumption in the form of traveling wave for the large-scale structures is given by assuming the separable form of the product of an unknown finite amplitude A(x) with a vertical shape function given by the local linear stability theory.

For our nonlinear analysis the amplitude A_{mn} can be witten as

$$A_{mn}(x)=|A_{mn}(x)|\, e^{i\psi_{mn}(x)} \tag{7}$$

in terms of its magnitude $|A_{mn}(x)|$ and phase angle $\psi_{mn}(x)$.

Each large-scale wave mode thus has the form :

$$\begin{bmatrix}\tilde{u}_{mn}\\ \tilde{v}_{mn}\\ \tilde{p}_{mn}\end{bmatrix}=A(x)\begin{bmatrix}\hat{u}_{mn}(\eta)\\ \hat{v}_{mn}(\eta)\\ \hat{p}_{mn}(\eta)\end{bmatrix}e^{i\int_0^x \alpha rm d\xi\; + i\psi_{mn}(x)\; - i\beta_{mn}t}+C.C. \tag{8}$$

where α_{mn} is complex wave number, $\eta=y/\delta(x)$ is the rescaled vertical variable and $\delta(x)$ is defined as the local shear layer maximum slope thickness. c.c. represents the complex conjugate. The shape functions $\hat{u}_{mn}$, $\hat{v}_{mn}$ and $\hat{p}_{mn}$ and are assumed to be identical to the eigenfunctions of the local linear stability solutions for the wave mode mn(Seo 1993 ; Seo&Nikitopoulos 1993).

For the mean flow, we will assume to have the shape of a hyperbolic tangent profile. In the developed mixing region, the mean velocity profile is (Ho&Huang 1982)

$$U=1-R\ tanh(\eta) \tag{9}$$

where $R=(U_{-\infty}-U_{\infty})/(U_{-\infty}+U_{\infty})$

We begin with the continuity and Navier-Stokes equations for an incompressible homogeneous fluid.

Using the energy content of the wave mode mn defined as $E_{mn}(x) = |A_{mn}(x)|^2\delta(x)$, the following set of equations are obtained :

1. Mean Flow :

$$I_{aM}\frac{d\delta}{dx} = \frac{1}{Re_\delta}\ \Phi_M - \frac{1}{\delta}\left(E_{10}I_{MW10} + E_{20}I_{MW20}\right) \quad (10)$$

where $Re_\delta = \frac{\delta\bar{U}}{\nu}$ is the Reynolds number of the shear layer mean flow.

2. Wave mode energies :

• 10 mode ;

$$I_{apW10}\frac{dE_{10}}{dx} = -\frac{1}{\delta}E_{10}I_{MW10}\frac{1}{\delta Re_\delta}\ E_{10}I_{vW10} - \frac{1}{\delta^{\frac{3}{2}}}E_{10}\sqrt{E_{20}}I^{10}_{1020} \quad (11)$$

• 20 mode ;

$$I_{apW20}\frac{dE_{20}}{dx} = -\frac{1}{\delta}E_{20}I_{MW20}\frac{1}{\delta Re_\delta}\ E_{20}I_{vW20} - \frac{1}{\delta^{\frac{3}{2}}}E_{10}\sqrt{E_{20}}I^{10}_{1020} \quad (12)$$

3. Wave phase angles :

• 10 mode ;

$$P_{aW10}\frac{d\psi_{10}}{dx} = \beta_{10} + \frac{P_{WM10}}{\delta} + \frac{1}{E_{10}}\frac{dE_{10}}{dx}P_{pW10} + \frac{1}{\delta^{\frac{3}{2}}}\sqrt{E_{20}}P^{10}_{1020} \quad (13)$$

• 20 mode ;

$$P_{aW20}\frac{d\psi_{20}}{dx} = \beta_{20} + \frac{P_{WM20}}{\delta} + \frac{1}{E_{20}}\frac{dE_{20}}{dx}P_{pW20} + \frac{1}{\delta^{\frac{3}{2}}}\frac{E_{10}}{\sqrt{E_{20}}}P^{10}_{1020} \quad (14)$$

The integral coefficients in the above equations are given below. The mean flow energy advection integral coefficient I_{aM} is

$$I_{aM} = -\frac{1}{2}\left[\int_{-\infty}^{\infty}(1-R\ \tanh\eta)((1-R\ \tanh\eta)^2 - (1+R)^2)d\eta + \int_0^{\infty}(1-R\ \tanh\eta)((1-R\tanh\eta)^2-(1-R)^2)d\eta\right] = r^2(3-2\ln 2) \quad (15)$$

and the mean viscous dissipation integral coefficient Φ_M is

$$\Phi_M = \int_{-\infty}^{\infty}\frac{R^2}{\cosh^2\eta}d\eta = \frac{4R^2}{3} \quad (16)$$

where R is the velocity ratio.

The integral coefficient I_{apWmn} consists of a wave mode energy advection and pressure transport component :

$$I_{apWmn} = I_{aWmn} + I_{pWmn} \quad (17)$$

where the wave mode energy advection integral coefficient I_{aWmn} is

$$I_{aWmn} = \int_{-\infty}^{\infty}U(|\hat{u}_{mn}|^2 + |\hat{v}_{mn}|^2)d\eta \quad (18)$$

and the wave mode pressure work integral coefficient I_{pWmn} is

$$I_{pWmn} = \int_{-\infty}^{\infty}2Rel(\hat{u}_{mn}\hat{p}^*_{mn})d\eta \quad (19)$$

where Rel represents the real part of the eigenfunction and $(\ )^*$ complex conjugate.

The wave mode production integral coefficient I_{MWmn} is

$$I_{MWmn}=\int_{-\infty}^{\infty} 2\mathrm{Rel}(\hat{u}_{mn}\hat{v}_{mn})\frac{\partial U}{\partial \eta}d\eta \quad (20)$$

and the wave mode viscous dissipation integral coefficient I_{vWmn} is

$$I_{vWmn}=2|\alpha_{mn}|^2+2\int_{-\infty}^{\infty}(|\frac{\partial \hat{u}_{mn}}{\partial \eta}|^2+|\frac{\partial \hat{v}_{mn}}{\partial \eta}|^2)d\eta \quad (21)$$

In the phase angle equations, P_{aWmn} is equal to the wave mode advection integral I_{aWmn}. The integral influencing the mode phase shift from its interaction with the mean flow is

$$P_{WMmn}=\int_{-\infty}^{\infty} Im(\hat{u}_{mn}\hat{v}_{mn})\frac{\partial U}{\partial \eta}d\eta \quad (22)$$

where Im represents the imaginary part of the eigenfunctions and the one influencing the phase induced by the pressure field is

$$P_{pMmn}=\int_{-\infty}^{\infty} Im(\hat{u}_{mn}\hat{p}^*_{mn})d\eta \quad (23)$$

The remaining terms of each wave energy and phase angle equations represent mode-mode energy exchange and the phase shift induced by interaction between wave modes respectively.

$$I^{pq}_{ijkl}=2Rel(\sum^{pq}_{ij}\Delta_{kl}e^{-i((-)^p\psi_{pq}+(-)^i\psi_{ij}+(-)^k\psi_{kl})}) \quad (24)$$

$$P^{pq}_{ijkl}=\mathrm{Im}(\sum^{pq}_{ij}\Delta_{kl}e^{-i((-)^p\psi_{pq}+(-)^i\psi_{ij}+(-)^k\psi_{kl})}) \quad (25)$$

where

$$\sum^{10}_{10}\Delta_{20}=\int_{-\infty}^{\infty} -i\alpha^*_{20}(\hat{u}^2_{10}\,\hat{u}^*_{20}-\hat{v}_{10}\,\hat{u}_{10}\,\hat{v}^*_{20})d\eta$$

$$+\int_{-\infty}^{\infty}\left(\hat{u}_{10}\,\hat{v}_{10}\frac{\partial \hat{u}^*_{20}}{\partial \eta}+\hat{v}^2_{10}\frac{\partial \hat{v}^*_{20}}{\partial \eta}\right)d\eta \quad (26)$$

If we rewrite the mode-mode interaction integral $\sum^{pq}_{ij}\Delta_{kl}$,

$$\sum^{pq}_{ij}\Delta_{kl}=|\sum^{pq}_{ij}\Delta_{kl}|e^{i\theta^{pq}_{ijkl}} \quad (27)$$

then the total phase angle difference involved in the relevant interaction term will be

$$\Psi^{pq}_{ijkl}=(-)^{p+1}\psi_{pq}+(-)^{i+1}\psi_{ij}+(-)^{k+1}\psi_{kl}+\theta^{pq}_{ijkl} \quad (28)$$

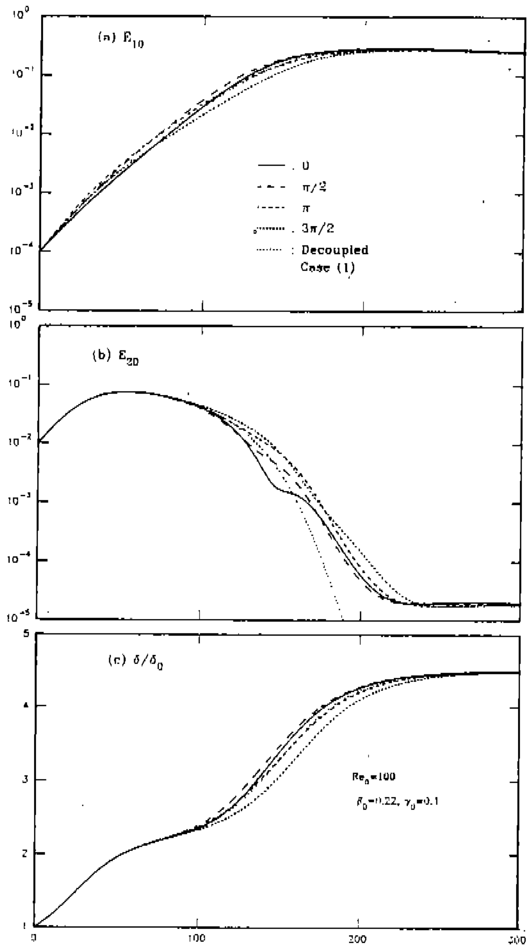
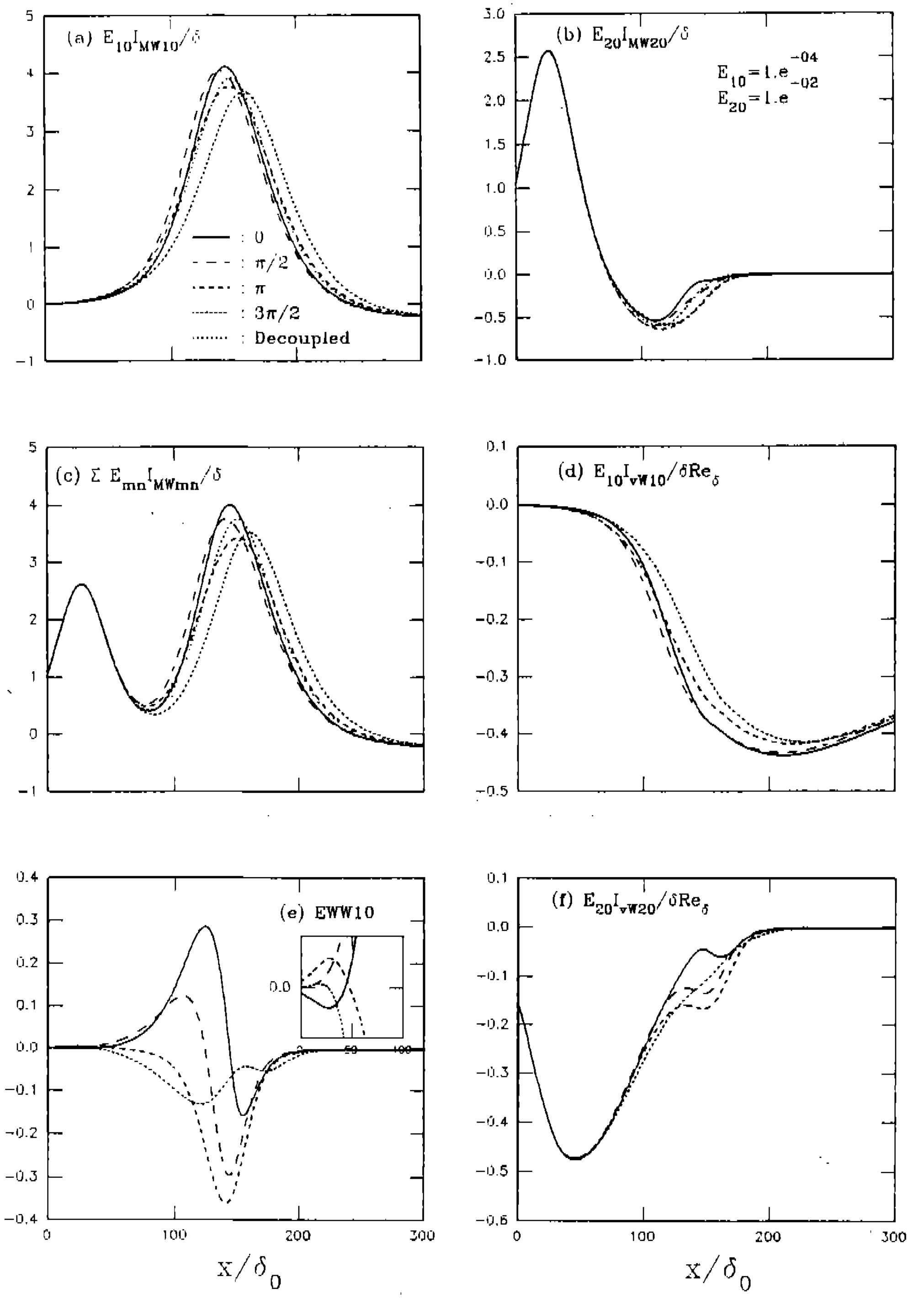


Figure 1 Effect of the initial phase angle ψ_{20i} on the development of the shear layer for two wave mode when E^{-04}_{10i} and E^{-02}_{20i}



Terms have been multiplied by a factor 10^3

Figure 2 Effect of the initial phase angle ψ_{20i} on the development of the energy integral terms for two wave mode when E^{-04}_{10i} and E^{-02}_{20i}.

3. Results and Discussion

We will present the results of the effects of the initial phase of mode 20, ψ_{20i}, for the purely two-dimensional wave mode interactions. In doing this we will set $\psi_{10i}=0$, using the phase of mode 10 as a reference. Figure 1 shows the reeesults of the nonlinear analysis for the effect of the initial total phase angle difference on the development of the shear layer when the large-scale coherent structures are assumed to be composed of two wave modes(10 and 20), and the initial energy densities of the subharmonic 10 and fundamental 20 wave modes are 10^{-04} and 10^{-02} respectively. In the initial region, where E_{10} is much smaller than E_{20}, the shear layer thickness grows mainly because of energy transfer from the mean flow to the fundamental as shown in Figure 2. The growth rate of the shear layer due to energy drain of the mean flow, is depicted in Figure 2(c) and it can be seen that growth rate reaches a first pick as the energy drain from the fundamental is maximized. Figure 2(c) is also indicative of the entrainment ignoring the effect of viscosity in the growth rate of the shear layer. The growth of the fundamental in the initial region is thus responsible for increased entrainment and mixing. The fundamental energy grows first, reaches the maximum point where it becomes *neutral*, and starts to decay because of loss of energy to the mean flow(see Figures 1(b) and 2(b). The subharmonic also follows approximately the same course of the fundamental but peaks further downstream. As the fundamental saturates, the growth of the shear layer declines(Figure 2(c)) steadily until the subharmonic becomes strong enough to counteract the trend by extracting enough energy from the mean flow. As the subharmonic grows stronger, it extracts an increasing amount of energy from the mean flow leading to the second peak in Figure 2(c). Thus the presence of the subharmonic is responsible for a significant increase in the growth of the shear layer and the subsequent increase in entrainment. The peak of the fundamental wave mode energy is related with the first plateau of the shear layer thickness, and the peak of the subharmonic energy is related with the second plateau in the shear layer thickness further downstream(see Figure 1 (c)).

In the initial region of the shear layer, as shown in Figure 1 and 2, the different initial phase angles

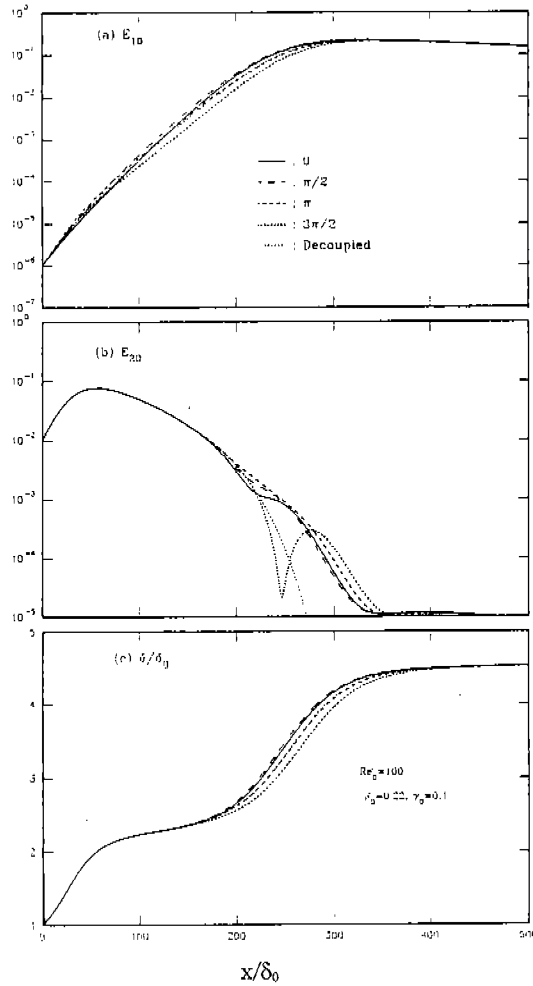
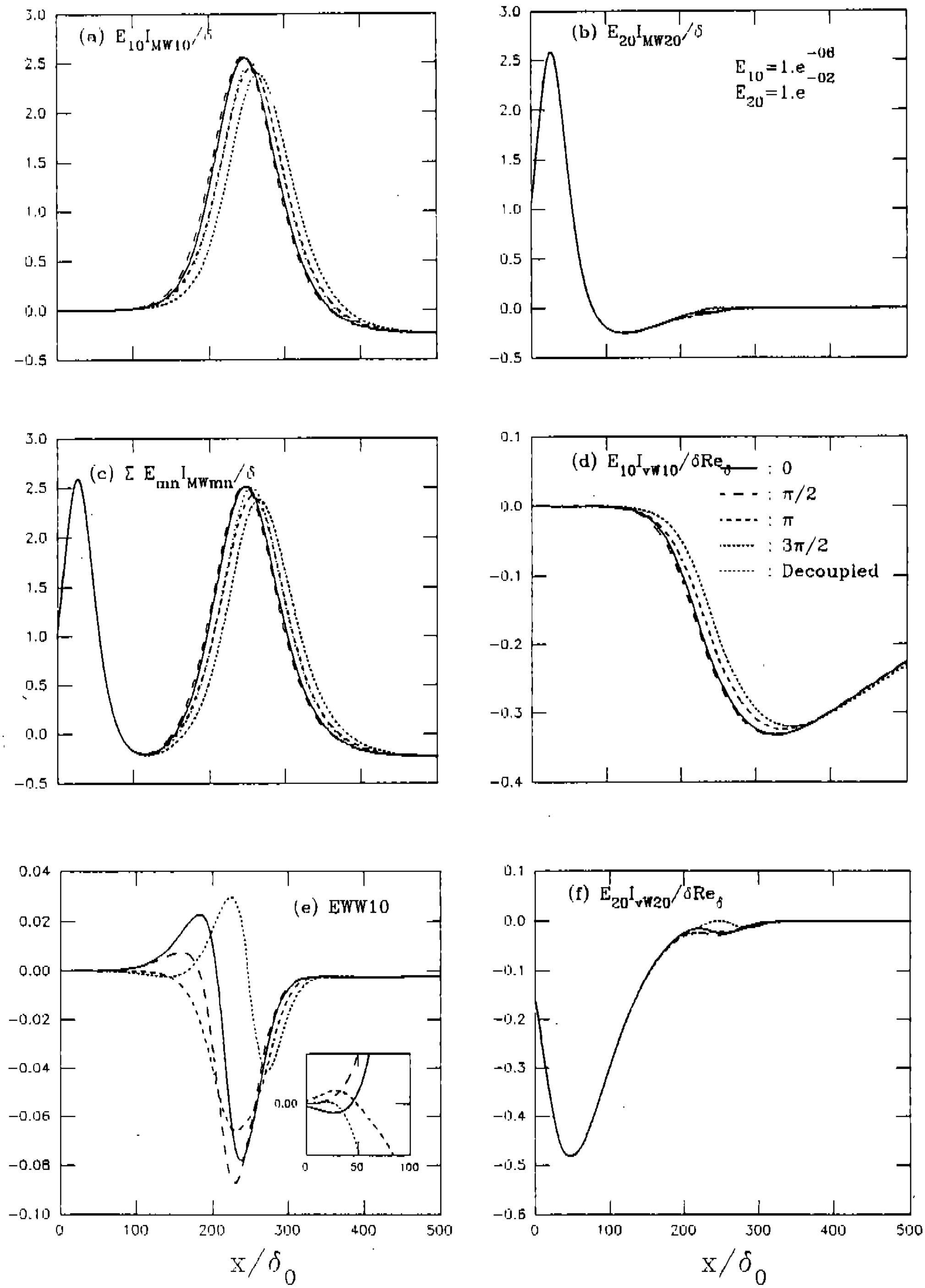


Figure 3 Effect of the initial phase angle ψ_{20i} on the development of the shear layer for two wave mode when E^{-06}_{10i} and E^{-02}_{20i}.



Terms have been multiplied by a factor 10^3

Figure 4 Effect of the initial phase angle ψ_{20i} on the development of the energy integral terms for two wave mode when E^{-06}_{10i} and E^{-02}_{20i}.

and the interaction between modes do not affect the development of the shear layer thickness δ and the fundamental energy density E_{20}, because the fundamental energy production $E_{20}I_{MW20}/\delta$ is much greater than both the subharmonic energy production $E_{10}I_{MW10}/\delta$, and the modal interaction term, because of the small energy content of the subharmonic. In the same region the development of the subharmonic energy density E_{10} is affected by the value of the initial total phase angle because the nonlinear mode-mode interaction can be a sizable fraction of the subharmonic energy production. The total phase angle difference Ψ^{10}_{1020} controls the direction of the energy transfer of the nonlinear wave mode interaction. The initial values 0 and π of total phase angle difference maximize the initial energy exchange between wave modes while they do not affect the initial variation of the phases. In similar manner, the initial values $\pi/2$ and $3\pi/2$ maximize the nonlinear phase shift and eliminate the initial energy exchange between wave modes. As the subharmonic grows, the effects of the different initial phase angle on the mean flow and the fundamental begin to show(see Figure 1).

In Figures 3 and 4 we show the results for different initial values of the phase angle when $E_{10i}=10^{-06}$ and $E_{20i}=10^{-02}$ i.e. when the subharmonic is initially weaker. It is obvious that in the initial region where the fundamental extracts most of the energy from the mean flow, because the initial energy density of the fundamental is much stronger than that of the subharmonic, the trends are in general the same as in the previous case where the subharmonic was stronger. The fundamental persists further downstream after if saturates and is overtaken by the subharmonic at a much later location than before. This essentially moves vortex-merging and the associated step-like growth of the shear layer downstream, and results in an overall much slower growth of the shear layer and locally reduced entrainment. Between the peaks attributed to the fundamental(first) and the subharmonic(second) in Figure 4 it is seen that energy is returned to the mean flow by the fundamental and in this region a slow growth of the shear layer Figure 3(c) is sustained by viscous diffusion.

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