The Korean Journal of Rheology,
Vol. 6, No. 1, June 1994 pp. 1-9

〈총　　설〉

쉬트 다이 설계의 기초

곽 태 훈

한화그룹종합연구소

(1994년 6월 8일 접수)

Basics of Sheet Die Design

Tae Hoon Kwack

Research & Engineering Center, Hanwha Group
6 Shinsung-dong, Yusung-ku, Taejon 305-345, Korea

(Received June 8, 1994)

1. Introduction

One important piece of equipment in a polymeric film production line is a die. There are two kinds of die for producting film. They are an annular die for a film blowing process and a sheet die for casting or coating process. The die design technology of die manufacturers have been gradually advanced over the past years. The internal design of feedblock, manifold, and slit channels became more and more sophisticated to meet the requirements of producing high valued products. Three-layer die is very common and five-layer and even seven-layer die are being utilized in film producing industry. However, film extrusion industry appear to have limited knowledge in the area of die design due to the lack of proper analytical means. Apparently, a trial and error approach has been dominant in solving die related problems. A need to enhance the die design understanding is more demanding than ever as film products become more and more diversified with exquisite resins and as more than three dissimilar resins are involved in film products.

In this paper we will review the basics that are necessary in designing a sheet die and confine our discussions in a single-layer die only. Before we proceed further, let us take a look at the typical sheet dies that are commercially well-employed. They are known as T-die, coathanger die, and the inverted preland die depending on their internal shapes. Regardless of the shape of a die, the purpose of die design is to generate a uniform flow of molten polymer that is transferred from an extruder, and then, deliver it to a subsequent take-off equipment.

The T-die shown in Fig. 1 is distinguished by its relatively large and generally circular manifold along with its distinct sliding lips. This die provides excellent gauge thickness when used for processing relatively low viscosity polymers which

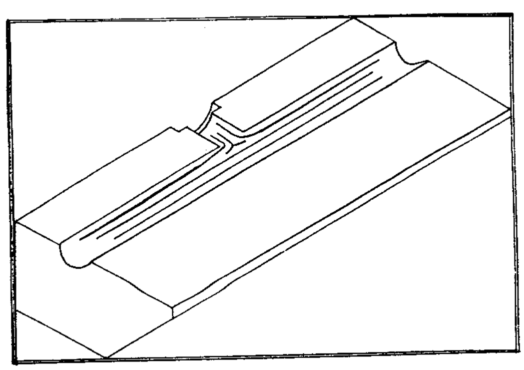

Fig. 1. Schematic view of a typical T-die.

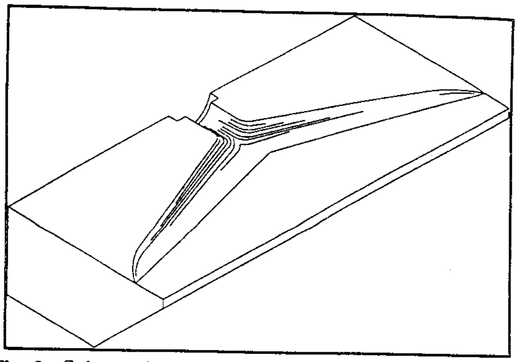

Fig. 2. Schematic view of a typical coat-hanger die.

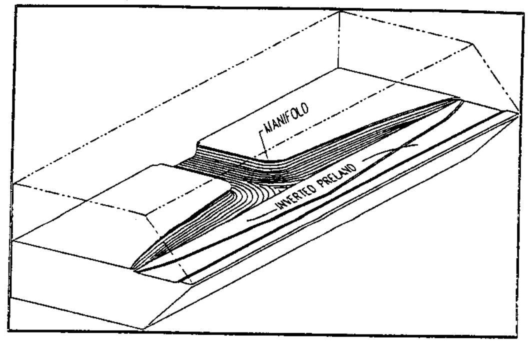

Fig. 3. Schematic view of a the EPOCH^R die.

are typical of the extrusion coating industry.

The coathanger die, probably the most widely used die throughout the extrusion industry, is shown in Fig. 2. It is being offered by Extrusion Dies, Inc. (EDI), one of the major sheet die manufacturers in U.S.A.. The flow uniformity is controlled mainly by the diminishing manifold cross-section along the manifold direction. The profile of the manifold cross-section is designed in such that a constant pressure drop is provided for each melt flow path. Thus, as the melt flows along the manifold, a small portion of the melt leaks from the manifold and flows along the machine direction. In this way the volumetric flow of each melt stream remains uniform across the die.

The other die shown in Fig. 3 is employed by the Cloeren Company and is called as 'Epoch' die. Instead of changing the manifold diameter, the Cloeren introduced an inverted preland whose length along the machine direction change across the die. The preland length decreases while it moves from the center of a die to the edges. A profile of the preland length is designed to restrict the flow in the center more than the flow in the edges. One aspect of the Epoch die is a uniform wetted surface across the width of a die. This offers a minimum clamshelling effects providing more uniformity across broad processing conditions. The clamshelling is a phenomenon that the die gap in the center tends to be wider than both edges due to a difference in force at the center versus the edges of a die.

2. Basic Equations for Die Design

In the first part of this section, simple descriptions on non-Newtonian fluid will be reviewed briefly. After the review, mathematical formulars for a fluid flowing in a circular pipe and between two plates will be introduced following a power-law model. This is based on the fact that the melt flow inside a manifold can be described as a Poiseuille flow in a long circular pipe and other sections in a die as a Poiseuille flow between two parallel plates. These simplified flow models actually serve as bases for die design in industry. Later in this section, a clear explanation will be given on the utilization of flow models in describing the melt flow in a sheet die. One will see how to determine the geometry of a sheet die to ensure a uniform flow across a die.

2.1. Simple descriptions on non-Newtonian fluid

A fluid whose viscosity varies as the shear rate changes is called non-Newtonian fluid. Or, any fluid whose shear stress deviates from a linearity with respect to shear rate is defined as non-Newtonian fluid. Polymeric materials are known to exhibit non-Newtonian characteristics. Shear viscosity of most molten polymers decreases as the shear rate increases at a constant temperature. The extent of changes in the slope of a viscosity curve in the log(viscosity) vs. log(shear rate) plot determines one of the non-Newtonian characteristics of material. There exist numerous viscosity models that describe this viscosity vs. shear rate curve. A very popular choice for a long time, especially in industry, has been the power-law model:

$$\eta = m\dot{\gamma}^{n-1} \quad (1)$$

Where the temperature dependency of m, consistency index, is given as

$$m = m_o Exp(-b(T - T_{ref})) \quad (2)$$

In order to determine the power-law parameters, m and n, one can plot the viscosity data on a log-log paper. The slope of a straight line that best fits the experimental data is n and the viscosity at a shear rate of 1.0 s^{-1} is m. Typical values of n for some common thermoplastics are: polyethylene, 0.3~0.6; polypropylene, 0.3~0.4; PVC, 0.2~0.5; and nylon, 0.6~0.9.

There is one problem with the power-law model: At low shear rates polymer melt viscosity tends to be constant, while the power-law model predicts infinite viscosity. Another viscosity correlation that is popularly used in calculations involving polymer melt flows through extruder channels and dies is the following six parameter model:

$$\log\eta = A_0 + A_1T + A_2T^2 + A_3T\log(\dot{\gamma}) + A_4\log(\dot{\gamma}) + A_5(\log\dot{\gamma})^2 \quad (3)$$

The coefficient of each term is obtained by the method of regression analysis on viscosity vs. shear rate data measured at several different temperatures. This expression fits the actual viscosity data better than the power-law model. Most resin producers have their own data files describing the values of the six parameters for each grade of resins they produce.

Let us now proceed to review mathematical formulars for a power-law fluid flowing in a circular pipe and between two parallel plates.

2.2. Flow in a circular pipe

For a steady and fully-developed flow in a circular pipe, the equation of motions in cylindrical coordinates reduce to the following equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau) + \frac{\partial P}{\partial z} = 0 \quad (4)$$

Upon integration of equation (4), we obtain

$$\tau = \frac{r}{2}\left(\frac{\partial P}{\partial z}\right) \quad (5)$$

Whilst for a power law fluid

$$\tau = m(\dot{\gamma})^n \quad (6)$$

Combining these two equations and remembering that for a tube, $\dot{\gamma} = (dV_z/dr)$, one obtains

$$\frac{dV_z}{dr} = \left[\frac{r}{2m}\left(\frac{\partial P}{\partial z}\right)\right]^{1/n} \quad (7)$$

Which yields on integration

$$V_z = \left(\frac{n}{n+1}\right)\left[\frac{1}{2m}\left(\frac{\partial P}{\partial z}\right)\right]^{1/n} r^{(n+1)/n+C} \quad (8)$$

The constant C may be calculated by bearing in mind that when r=R, $V_z=0$, from which one obtains that

$$V_z = \left(\frac{n}{n+1}\right)\left[\frac{1}{2m}\left(\frac{\partial P}{\partial z}\right)\right]^{1/n} R^{(n+1)/n}[(r/R)^{(n+1)/n} - L] \quad (9)$$

When r=0, $V_z = V_{max}$, thus the maximum velocity is

$$V_{max} = -\left(\frac{1}{n+1}\right)\left[\frac{1}{2m}\left(\frac{\partial P}{\partial z}\right)\right]^{1/n} R^{(n+1)/n} \quad (10)$$

From which one obtains a general expression for a velocity profile in a circular pipe for a power-law fluid.

$$V_z = V_{max}\left[1 - \left(\frac{r}{R}\right)^{(n+1)/n}\right] \quad (11)$$

The equation for the overall output Q is

$$Q = \left(\frac{\pi n}{3n+1}\right)\left(\frac{1}{2m}\right)^{1/n} \cdot R^{(3n+1)/n} \cdot \left[\frac{\partial P(z)}{\partial z}\right]^{1/n} \quad (12)$$

Thus, the pressure drop in a tube is

$$\frac{\partial P}{\partial z} = 2m\left(\frac{3n+1}{n}\right)^n\left(\frac{Q}{\pi R^3}\right)^n R^{-1} \quad (13)$$

It is also possible to relate the average velocity

$\langle V \rangle_z$ to V_{max} and V_z. Since the average velocity $\langle V \rangle_z$ is Q/R^2, rearranging in terms of V_z.

$$\langle V \rangle_z = -(\frac{n}{3n+1})(\frac{1}{2m}\frac{\partial P}{\partial z})^{1/n} R^{(n+1)/n} \qquad (14)$$

On comparison with the equation for V_{max}, it is seen that

$$\langle V \rangle_z = (\frac{n+1}{3n+1})\ V_{max} \qquad (15)$$

Substituting for V_{max}, one obtains

$$\frac{V_z}{\langle V \rangle_z} = (\frac{3n+1}{n+1})\left[1-(\frac{r}{R})^{(n+1)/n}\right] \qquad (16)$$

If the radius of a tube is linearly decreasing, like a truncated cone, the pressure drop for a tube of length L is

$$\frac{\Delta P}{L} = \frac{2m}{3n}\left[\frac{Q}{\pi}(\frac{1}{n}+3)\right]^n (\frac{R_L^{-3n}-R_O^{-3n}}{R_O-R_L}) \qquad (17)$$

However, for a small tube length, dz, one can use equation (13) for a pressure drop, using the following expression for R,

$$R = \frac{R_L(z)+R_O(z)}{2} \qquad (18)$$

Where $R_O(z)$ and $R_L(z)$ are the radii at the entrance and at length L, respectively.

2.3. Flow between two parallel plates

By analogy with the derivation for a velocity distribution in a tube, the following equation is obtained for a flow between two plates.

$$\tau = h(\frac{\partial P}{\partial z}) \qquad (19)$$

By combining with equation (6), one obtains

$$\dot{\gamma} = \frac{dV_z}{dh} = \left[\frac{h}{m}(\frac{\partial P}{\partial z})\right]^{1/n} \qquad (20)$$

On integration this yields

$$V_z = (\frac{n}{n+1})\left[\frac{1}{m}(\frac{\partial P}{\partial z})\right]^{1/n} h^{(n+1)/n} + C \qquad (21)$$

When $h=H/2$, $V_z=0$
Hence

$$C = -(\frac{n}{n+1})\left[\frac{1}{m}(\frac{\partial P}{\partial z})\right]^{1/n}\left[\frac{H}{2}\right]^{(n+1)/n} \qquad (22)$$

When $h=0$, $V_z=V_{max}$, so that

$$V_{max} = -(\frac{n}{n+1})\left[\frac{1}{m}(\frac{\partial P}{\partial z})\right]^{1/n}\left[\frac{H}{2}\right]^{(n+1)/n} \qquad (23)$$

Combining the equation for V_z and V_{max} yields

$$V_z = V_{max}\left[1-(\frac{2h}{H})^{(n+1)/n}\right] \qquad (24)$$

The overall volumetric flow rate can be described as

$$Q = \frac{1}{2}(\frac{n}{2n+1})(\frac{1}{2m})^{1/n} W \cdot H^{(2n+1)/n}(\frac{dP(z)}{dz})^{1/n} \qquad (25)$$

Then, the pressure drop between two parallel plates of length L is

$$\frac{\Delta P}{L} = 2^{n+1} m (2+\frac{1}{n})^n \cdot \frac{Q^n}{W^n H^{2n+1}} \qquad (26)$$

It is also possible to express V_{max} and V_z in terms of $\langle V \rangle_z$, the average velocity, Q/WH. Rearranging equation (25) in terms of V_z

$$\langle V \rangle_z = -\frac{H}{2}\left[\frac{n}{2n+1}\right]\left[\frac{H}{2m}(\frac{\partial P}{\partial z})\right]^{1/n} \qquad (27)$$

or

$$\langle V \rangle_z = -\frac{1}{2}\left[\frac{n}{2n+1}\right]\left[\frac{1}{2}\right]^{1/n}\left[\frac{1}{m}(\frac{\partial P}{\partial Z})\right]^{1/n} H^{(n+1)/n} \qquad (28)$$

Combining this with the equation for V_{max} provides

$$\langle V \rangle_z = V_{max}\left[\frac{n+1}{2n+1}\right] \qquad (29)$$

and

$$V_z = \langle V \rangle_z (\frac{2n+1}{n+1})\left[1-(\frac{2h}{H})^{(n+1)/n}\right] \qquad (30)$$

For a Newtonian fluid (n=1), $\langle V \rangle_z = 2/3\ V_{max}$ for a slit compared with $\langle V \rangle_z = 1/2\ V_{max}$ for a pipe.

Pressure drop in a tapered slit of length L can be expressed as

$$\frac{\Delta P}{L}=\frac{m}{2^{n+1}n}\left[\frac{Q}{W}\left(\frac{1}{n}+2\right)\right]\left(\frac{H_L^{-2n}-H_O^{-2n}}{H_O-H_L}\right) \tag{31}$$

The shear rate can be determined by differentiating the velocity profile, as is shown in equation (7) for a tube flow and equation (20) for a slit flow. For a tube flow the shear rate at the wall is:

$$\dot{\gamma}_w=\left(\frac{\partial V_z}{\partial r}\right)W=\frac{n+1}{n}\frac{V_{max}}{R} \tag{32}$$

For a flow between two parallel plates the shear rate at the wall is:

$$\dot{\gamma}_w=\left(\frac{\partial V_z}{\partial r}\right)_w=\frac{n+1}{n}\frac{V_{max}}{H/2} \tag{33}$$

The shear stress can be calculated from equation (7) and the maximum shear stress at the wall for a tube flow is

$$\tau_w=m\dot{\gamma}_w^n=m\left[\frac{n+1}{n}\frac{V_{max}}{R}\right]^n \tag{34}$$

and for a flow between two parallel plates is

$$\tau_w=m\dot{\gamma}_w^n=m\left[\frac{n+1}{n}\frac{V_{max}}{H/2}\right]^n \tag{35}$$

It should be noted that the onset of melt fracture or melt flow instability occurs when the magnitude of stress reaches a certain level.

2.4. Equations for sheet die design for a power-law fluid

After an introduction of the basic equations for a flow in a pipe and between two plates, let us now review past work on die design equations. Many investigators discussed on the design equations (1-9). Carley [1] was known to be the first to propose a design criterion for a sheet die. He developed a unformity index, UI, which is the rate of extrusion at the far end to that at the feed end, which is shown in equation (36).

$$UI=\left[1-\left(\frac{n}{1+n}\right)\left(\frac{L}{n}\right)^{1+n}\alpha^{2n}\right]^{1/n} \tag{36}$$

Where $\alpha^2=\dfrac{3n+1}{\pi}\;\dfrac{n}{2(2n+1)}\;\dfrac{H^{(2n+1)/n}}{R^{(3n+1)/n}}\;\dfrac{1}{t^{1/n}}$

R and L are the radius and the length of the tube, respectively, H is the size of the slit opening, t is the length of land. A plot, UI against flow index, n, with geometric values as parameters, indicates that the uniformity of flow can be improved by increasing R and t, but by decreasing H and L. Also, according to the index, a center fed die will always be better than an end-fed die. However, in practice, both the radius of the manifold and the slit opening have their own limits.

McKelvey and Ito [2] proposed other uniformity function which enables die dimensions to be established at a specified uniformity at particular flow rates and pressure drops. The analyses showed that the flow index, n, of the power-law was the key parameter determining the flow uniformity. Their work is noted to be the one which provided a concept of varying the manifold radius and slit thickness.

Apparently, Chung and Lohkamp [3] developed an equation for the varying radius of the manifold. Let us review their work in detail here. The relationship between pressure along the machine and manifold axis can be expressed as

$$P_x=P_z\cos\theta \tag{37}$$

Where P_z is the pressure gradient along the machine direction, P_x along the manifold axis, and θ is the manifold angle.

The substitution of equation (37) into equations (12) gives,

$$Q(x)=\left(\frac{\pi n}{3n+1}\right)\left(\frac{1}{2m}\right)^{1/n}(\Delta P_z\cos\theta)^{1/n}R(x)^{(3n+1)/n} \tag{38}$$

Now we can obtain the inlet radius of the manifold at x=0 from equation (38) using the boundary condition of at x=0, $Q_o=Q/2$ and equation (25).

$$R_O=\left[\frac{(3n+1)}{4\pi(2n+1)}W\right]^{n/(3n+1)}\cdot H^{(2n+1)/(3n+1)}\cdot\cos\theta^{-1/(3n+1)} \tag{39}$$

It should be noted that the inlet manifold size is a function of n, W, H, and θ.

The mass balance over a differential manifold length dx is,

$$dQ(x) = -\sin\theta \frac{1}{2W} \frac{n}{2n+1} (\frac{1}{2m})^{1/n} \cdot H^{(2n+1)/n} \cdot P_z^{1/n} \cdot dx \quad (40)$$

The integration of equation (40) gives,

$$Q(x) = Q(0) - \sin\theta \frac{1}{2W} \frac{n}{2n+1} (\frac{1}{2m})^{1/n} \cdot H^{(2n+1)/n} \cdot P_z^{1/n} \quad (41)$$

Substituting equation (26) and (38) into equation (41) with the boundary condition of $R=R_O$ at $x=0$, one obtains,

$$(\frac{\pi n}{3n+1})(\frac{1}{2m})^{1/n}(\Delta P_z \cos\theta)^{1/n} R(x)^{(3n+1)/n} = (\frac{\pi n}{3n+1})(\frac{1}{2m})^{1/n}(\Delta P_z \cos\theta)^{1/n} \cdot R_O^{(3n+1)/n} - \sin\theta(\frac{1}{2W})(\frac{n}{2n+1})(\frac{1}{2m}) \cdot H^{(2n+1)/n} \cdot \Delta P_z^{1/n} \quad (42)$$

Rewriting equation (42) gives.

$$R(x)^{3n+1/n} = R_O^{(3n+1)/n} - \sin\theta \frac{\frac{1}{2W}(\frac{n}{2n+1})(\frac{1}{2m})^{1/n}}{(\frac{\pi n}{3n+1})(\frac{1}{2m})^{1/n}} \frac{H^{(2n+1)/n}}{\cos\theta^{1/n}} \quad (43)$$

Now substituting $\sin\theta = W/2Lm$ and equation (39) into equation (43) leads to

$$\frac{R(x)}{R_O} = (1 - \frac{x}{Lm})^{n/(3n+1)} \quad (44)$$

where Lm is the length of the manifold.

Equation (44) shows that manifold profile depends only on the flow index n. If the inlet radius and the profile of the manifold are designed according to equations (39) and (44), a coat-hanger die is expected to produce a uniform thickness sheet without the help of choker bars. Often the cross-section of a manifold is non-circular. In this case, the arbitrary cross-section is transformed into an equivalent circle by using the following equation

$$R_{eq} = 2\ S/C \quad (45)$$

Where R_{eq} is the equivalent radius, S the cross-sectional area and C the perimeter of its circumference.

Matsubara [4, 5] attempted to introduce methods of producing uniform melt flow residence time across the die with a curvilinearly tapered coat-hanger die. He obtained the following design equations for the manifold radius R(x) and the coat-hanger height Z(y).

$$R(x) = \frac{m^{1/3(n+1)}}{\pi^{1/3}} \left[\frac{1+3n}{2(1+2n)}\right]^{n/(3(n+1))} H^{2/3}(L-x)^{1/3} \quad (46)$$

and

$$Z(y) = \frac{3k^{1/2}}{2}\Big[(L-y)^{1/3}\{(L-y)^{2/3}-k\}^{1/2} + k\ \log[(L-y)^{1/3} + \{(L-y)^{2/3}-k\}^{1/2}]\Big]_0^y \quad (47)$$

for $y=0$ to $L-k^{3/2}$, and where k is

$$k = \left[\frac{\pi H}{m^{(1+3n)/(1+n)}}\left[\frac{1+3n}{2(1+2n)}\right]^{2n/(1+n)}\right]^{2/3} \quad (48)$$

In equation (46), m is defined as the ratio of the average residence time in the manifold to that in the slit.

The non-isothermal flow analysis of a power-law fluid in a sheet die was conducted by Vergnes et al. [6] using an iterative finite difference method. Equations of motion and energy were solved simultaneously to give pressure, temperature, stream lines, and exit flow rate distributions.

2.5. Numerical approaches in die design

The use of numerical approach became popular and commercially available die design programs are now available from such organizations as MacMaster University in Canada, Polymer Processing Institute and Scientific Process Research Inc., respectively, in U.S.A.. Here, we would like to introduce a general method of analyzing a flow in a sheet die used in Flatcad[R] program, that are developed by J. Vlachopoulos from McMaster University and his colleagues [10]. Their approach to analyze the flow in a sheet die is a control volume method. The control volume method is a variation of finite difference method in which integrated

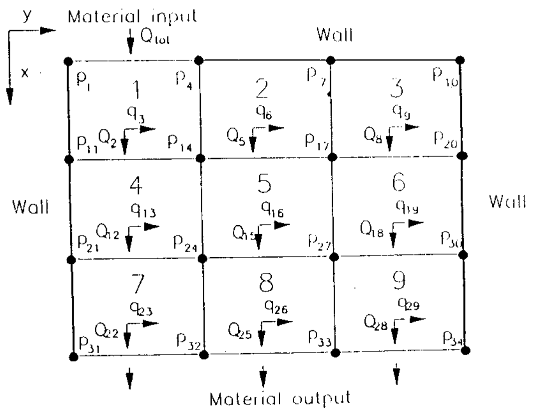

Fig. 4. A sketch of a die subdivided into control volumes.

forms of the differential equations are considered.

The internal flow passages of a die is divided into regions. The regions can be represented by simple geometrical shapes like straight or tapered cylindrical channels and parallel or tapered flat plates. Each region is divided into control volumes. If the internal gap is divided into control volumes, the flow equations can be solved inside each control volume and then combined together for all the control volumes of the grid to describe the flow inside the whole die. Let us review the method using the simplest geometry, that is flow between parallel plates. A sketch of a simplified "die" is shown in Fig. 4.

Fig. 4 shows a view on the horizontal plane of the "die". It is assumed that the gap thickness h is constant everywhere in the control volume grid shown in Fig. 4. The three divisions in the x direction and the three divisions in the y direction create nine control volumes. The material enters the "die" at the top edge of control volume 1, flows inside the "die", and finally exits it from the bottom edges of control volumes 7, 8 and 9. The rest of the boundary of the "die" represents the walls at which the normal component of the velocity is assumed to vanish. Inside the die, the material flows in both the x- and y-direction and the velocity profiles are assumed to be locally fully developed. To be able to describe the flow inside the "die" two independent flows must be assumed. These two flows can be represented in each control volume by the volumetric flow rates, or the average velocities. For each control volume, these volumetric flow rates account for two independent variables, being the pressure the third independent variable present in the mathematical model. In Fig. 4, the variables associated to each of the control volumes are identified. An important assumption of the control volume method is that the volumetric flow rates Q and q are constant in each control volume and have a step change at the edges of the control volume. Since there are three variables in each control volume, three independent equations must be constructed to describe the flow. By using the above mentioned assumptions the volumetric flow rates can be related to the pressure drops for each one of the control volumes.

Stress balances for the two flow directions are

$$\frac{\partial \tau_{zx}}{\partial z} - \frac{\partial P}{\partial x} = 0$$
$$\frac{\partial \tau_{zy}}{\partial z} - \frac{\partial P}{\partial y} = 0 \qquad (49)$$

Which can be rewritten as

$$\tau_{zx} = \tau_{wzx} \frac{z}{s}$$
$$\tau_{zy} = \tau_{wzy} \frac{z}{s} \qquad (50)$$

Where $s = h/2$.

Substituting equation (50) into (49) and integrating

$$(\Delta P)_x - \frac{2\tau_{wzx}}{h} \Delta x = 0$$
$$(\Delta P)_y - \frac{2\tau_{wzy}}{h} \Delta y = 0 \qquad (51)$$

Where x and y are the dimensions of the control volume.

The shear rates for fully developed flow are

$$\dot{\gamma}_{zx} = -\frac{2n+1}{n} \frac{2Q}{\Delta y h^2}$$
$$\dot{\gamma}_{zy} = -\frac{2n+1}{n} \frac{2q}{\Delta x h^2} \qquad (52)$$

Where Q and q are the volumetric flow rates in the x- and y-direction, respectively.

The last conservation equation for a control volume is the continuity equation, that is the balance between the flow entering and exiting the control volume.

$$-Q^{*}-q^{*}+Q+q=0 \qquad (53)$$

For example, the pressure differences of control volume 5 in Fig. 4 are

$$(\Delta P)_x=(P_{14}+P_{17})/2-(P_{24}+P_{27})/2 \qquad (54)$$

for the x direction, and

$$(\Delta P)_y=(P_{14}+P_{24})/2-(P_{17}+P_{27})/2 \qquad (55)$$

for the y direction.

These pressure differences, along with the shear stress values can then, be substituted into equation (51) to give

$$\frac{P_{14}+P_{17}}{2}-\frac{P_{24}+P_{27}}{2}-\frac{2n+1}{n}\frac{2\eta(I)}{\Delta yh2}\Delta xQ_{15}=0$$
$$\frac{P_{14}+P_{24}}{2}-\frac{P_{17}+P_{27}}{2}-\frac{2n+1}{n}\frac{2\eta(I)}{\Delta yh2}\Delta xq_{16}=0 \qquad (56)$$

The continuity equation for this control volume takes the form

$$-Q_5-q_{13}+Q_{15}+q_{16}=0 \qquad (57)$$

Equation (56) and (57) describe the flow in control volume 5 in Fig. 4. Similar equations can be constructed for the other control volumes. By combining all the resulting equations, a complete set of algebraic equations is created.

To be able to solve this set, boundary conditions must be specified. The are:

$$-q_3+Q_5+q_6=0 \qquad (58)$$

$$q_9=0,\ q_{19}=0,\ q_{29}=0 \qquad (59)$$

$$P_{31}=0,\ P_{32}=0,\ P_{33}=0,\ P_{34}=0 \qquad (60)$$

$$-Q_2+Q_{12}+q_{13}=0 \qquad (61)$$

$$Q_2+q_3=Q_{tot} \qquad (62)$$

With the bove boundary conditions, equations (56) are solved using an iterative approach. A possible iteration procedure consists in using the previously calculated volumetric flow rates in each control volume to calculate the invariant I and the corresponding viscosity for the new iteration. This iterative procedure can be carried on until a small predetermined tolerance is satisfied.

Temperature effects can be incorporated into the model similarly. The viscous dissipation in each control volume, the subsequent viscosity changes of melt, and heat transfer between the melt and the body of the die are considered in the analyses. For any control volume of the grid, the energy balance can be expressed as

$$(Q+q)T-Q^{*}T_{n1}-q^{*}T_{n2}=\frac{E_V-E_T}{\rho Cp} \qquad (63)$$

Where T_{n1} and T_{n2} represent the temperatures in the top and left neighboring control volumes, respectively, and, E_V and E_T are the dissipated energy per unit time and the transferred energy per unit time between the melt and the body of the die. The set of equations for the energy balance is solved separately from the set of equations for the momentum balance. The solution of the flow problems starts assuming that the temperature in all the control volumes is the temperature of the material at the entrance of the die. The results of the flow solution are then used in the solution of the energy equations to calculate the temperature distribution which, in turn, is used to modify the viscosity of the material in each control volume for the new flow solution.

3. Conclusive remarks

We have reviewed basic equations that are required in developing die design concepts. We understood how to relate a simple power-law fluid model to the geometry of a sheet die, namely the inlet radius and the profile of the manifold. Finally we reviewed an approach of the control volume method, which is served as a base of Flatcad^{R}, one of many commercially available sheet die design programs.

References

1. J.F. Carley, *J. Appl. Phys.,* **25**(9), 1118 (1954).
2. J.M. Mckelvey and K. Ito, *Polym. Eng. and Sci.,* **11**, 256 (1971).
3. C.I. Chung and D.T. Lohkamp, SPE 33th Annual Meeting Proceeding, 363 (1975).
4. Y. Matsubara, *Polym. Eng. and Sci.,* **19**(3), 169 (1979).
5. Y. Matsubara, *Polym. Eng. and Sci.,* **20**(11), 716 (1980).
6. B. Vergnes, P. Saillard, and J.F. Agassant, *Polym. Eng. and Sci.,* **24**(12), 960 (1984).
7. R.S. Lenk, *Kunststoffe* **75**(4), 239 (1985).
8. J. Vlachopoulos and P.S. Scott, *Advances in Polymer Technology,* **5**(2), 81 (1985).
9. H. Helmy, *Advances in Polymer Technology* **7**(1), 59 (1987).
10. J. Vlacek and J. Vlachopoulos, Flatcad^R manuals (1990).

저자약력

곽 태 훈

~1975 B.S. in Chem. Eng., Hanyang University
~1983 M.S. & Ph.D. in Polym. Sci. & Eng., Polytechnic Insitute of New York
1983~ Sr. Research Engineer I, Sr. Research Engineer II, Research Associate
Technical Center, Mobil Chemical Compnay, Rochester, New York
1993~ Technical Director, HanWha Group R & E Center