카오스 이론을 이용한 시뮬레이션 출력 분석

Simulation Output Analysis using Chaos Theory

오 형 술* 이 영 해**
Hyung-Sool Oh and Young-Hae Lee

Abstract

In the steady-state simulation, it is important to identify initialization bias for the correct estimates of the simulation model under study. In this paper, the methods from chaos theory are applied to the determination of truncation points in the simulation data for controlling the initial bias. Two methods are proposed and evaluated based on their effectiveness for estimating the average waiting time in M/M/1(∞) queueing model.

1. Introduction

A simulation method has been used broadly to evaluate the performance of the system under study, especially when the quantitative analysis using mathematical models is not applicable. The initial conditions of the stochastic simulated system may be different each time the simulation is run. And the estimation of true responses in the steady-state simulation is so complicated because of the possible presence of initial bias. So, in studies of the steady-state characteristics of a simulation model, it is important to identify initialization bias. Many researchers have proposed the methods for determining the truncation points(or warm up period) to control initial bias, but these methods seem not to work well as intended.

Variations of the system observations in the transient state are large and irregular due to initial bias as compared to those in the steady state. In this paper, we first propose how to measure the difference between the system variation in the transient state and the variation in the steady state and then develop two methods to determine truncation points that can be used in eliminatinng initial bias of the system.

We evaluate the performances of the two proposed methods based on their effectiveness in estimating the average waiting time using an M/M/1(∞) queueing

* 수원전문대학 공업경영과
** 한양대학교 산업공학과

model. These results are also compared with the method suggested in the literature.

2. Reconstruction of State Space

Until recently, the motions of determinism and randomness were seen as opposites, and were studied as seperate subjects with little or no overlap. Complicated phenomena were assumed to result from complicated physics among the processes. Simple dynamic systems were assumed to produce simple phenomena, so only simple phenomena were modeled deterministically[2,4,5].

Chaos provides a link between deterministic systems and random processes. The chaotic dynamics in a deterministic system can amplify small differences. Also, chaos implies that not all random-looking behavior is the product of complicated physics. An important consideration in chaos theory is the dimension of the dynamics, which is to estimate the fractal dimension of a hypothesized 'strange attractor', to define the asymptotic solution of a dynamic system, in a reconstructed state space. And a dimension of the dynamics counts the minimum number of degrees of freedom necessary to describe this motion. To estimate a dimension for a time series, we must first reconstruct a state space. The past behavior of a time series contains information about the present state x(t). And so, if the delay time τ is assumed uniform, the state at time t x(t) can be reconstructed as a delay vector of dimension m,

$$X(t) = (x(t), x(t-1), \ldots, x(t-(m-1)\cdot\tau)) \quad (2.1)$$

where m is called the embedding dimension.

If a time series is deterministic and of finite dimension, the estimated dimension of the reconstructed attractor should converge to the dimension of the strange attractor as the embedding dimension is increased. On the contrary, if a time series is random, the estimated dimension should be equal to the embedding dimension[1,6]. The Dimension that is considered in this paper is expressed as follows[1] :

$$C(r) = \lim_{r \to 0} \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} H(r - |x_i - x_j|) \quad (2.2)$$

where

$$H(s) = \begin{cases} 1, & \text{if } s > 0 \\ 0, & \text{if } s < 0 \end{cases}$$

N : total number of points x_i in the recontructed state space.

r : length of side for small cubes.

3. Lyapunov Exponents

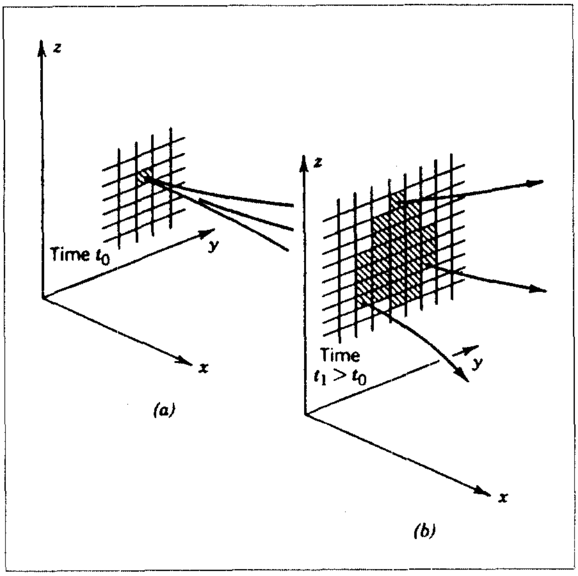
Suppose one has the ability to measure a position with accuracy Δx and a velocity with accuracy Δv. Then in the position-velocity plane (known as the phase space) we can divide up the space into areas of size $\Delta x \cdot \Delta v$ as shown in Figure 3.1. If we are given initial conditions to the stated accuracy, we know the system is somewhere in the shaded box in the phase plane. But if the system is chaotic, this uncertainty grows in time to N(t) boxes as shown in Figure 3.1. b[1].

The Size of uncertainty at time t, N(t) can be expressed as follows :

$$N(t) \approx N(0)e^{ht} \quad (3.1)$$

where constant h is related to the concept of entropy in information theory and will also be related to another concept called the Lyapunov exponent[1]. The test using Lyapunov exponent measures the sensitivity of the system to the change in the initial conditions.

Conceptually, one imagines a small ball of initial conditions in phase space and looks at its deformation into an ellipsoid under the dynamics of the system. If d(t) is the maximum length of the ellipsoid at time t and d(0) is the initial size of the initial condition sphere, the Lyapunov exponent λ is interpreted by the



〈Figure 3.1〉 The growth of uncertainty in a dynamic system

equation (3.2) in the reconstructed state space[1].

$$d(t) = d(0)2^{\lambda t}$$

$$\lambda = \frac{1}{t}\log_2 \frac{d(t)}{d(0)} \qquad (3.2)$$

There is a relationship between Lyapunov exponent which test the stability of chaotic system and eigenvalue which test the stability of dynamic system, and also a relationship between Lyapunov exponent which test the sensitive dependence on initial conditions and entropy which measure the growth of uncertainty.

The sign of Lyapunov exponent provides a qualitative picture of a system's dynamics such as

$\lambda > 0$: chaotic motion

$\lambda \leq 0$: regular motion. (3.3)

A chaotic system must have nonlinear elements or properties. A linear system cannot exhibit chaotic vibrations. The notion of a Lyapunov exponent is a generalizaiton of the idea of an eigenvalue as a measure of the stability of a fixed point. For a chaotic trajectory, it is not sensible to examine the instantaneous eigenvalue of a trajectory. The best quantity, therefore, is an eigenvalue averaged over the whole trajectory. The idea of measuring the average stability of a trajectory leads to the formal notion of a Lyapunov exponent.

Mathematically, Lyapunov exponent λ is defined by the equation (3.4)[3].

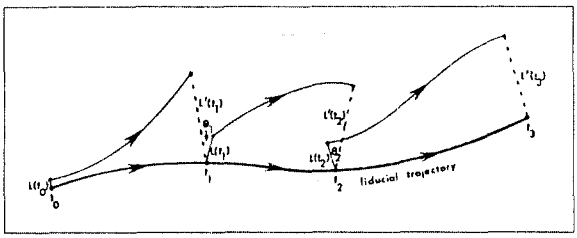
$$\lambda = \frac{1}{t_M - t_0}\sum_{k=1}^{M}\log_2 \frac{L'(t_k)}{L(t_{k-1})} \qquad (3.4)$$

where

$L(t_{k-1})$: Euclidean distance between the initial point and the nearest neighbor.

$L'(t_k)$: Euclidean distance between the initial point and the nearest neighbor at a later time t_k.

A schematic representation of the evolution and replacement procedure is shown in Fig. 3.2[3].



〈Figure 3.2〉 A schematic representation of the evolution and replacement procedure

The magnitudes of the Lyapunov exponent quantify an attractor's dynamics in information theoretic terms. The exponent λ measures the rate at which system processes create or destroy information[12] ; thus the exponents are expressed in bits of information per unit time or bits per orbit for a continuous system and bits per iteration for a discrete system.

4. Simulation Output Analysis

When the goal of a simulation experiment is to estimate the value of steady state parameters, the initial conditions of the simulation usually bias the estimators. This problem is particularly troublesome when several independently seeded runs of the simulation are made and the results are used to construct confidence intervals. The frequency with which confidence intervals based on biased outputs include the true performance value generally decreases as more runs are made. This is caused by the intervals shrinking about an inaccurate point estimator. The literature on simulation methodology contains some techniques for controlling initialization bias as follow. These techniques are often too elaborate and offer no assurance that initialization bias will be effectively controlled.

The survey by Gafarian et al.[7] indicated that published procedures for indentifying a truncation points appear not to exhibit very good behavior. Kelton and Law[8,9] investigated the deletion effect of initialization bias for the three types of point estimator criteria using a particular stochastic model. Schruben et al.[10,11] presented a family of tests for detecting initialization bias in the mean of a simulation output data using a hypothesis testing framework. Cash et al.[13] evaluated the power of a family of tests for initialization bias.

Variations of the system observations in the transient state are large and irregular due to initial bias as compared to those in the steady state. In this research, we first propose how to measure the difference between the system variation in the transient state and the variation in the steady state and then develop two methods to determine truncation points that can be used in eliminatinng initial bias of the system.

The equations like (4.1) or (4.2) in Table 4.1 measures the level of variations of the system observations over time. The level of varitions is then analyzed using the chaos theory to ascertain whether or not the difference in the varitions is significant. The chaos theory used for this purpose has some good features. One good feature is that it can classify dynamic characteristics of the system into the regular motion, the chaotic motion, and the random motion without knowing any information about the input parameters. This classification process is done according to reconstruction state space, dimension, and Lyapunov exponent that are based on the time series of the system of under study.

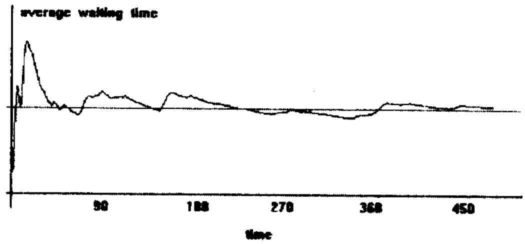
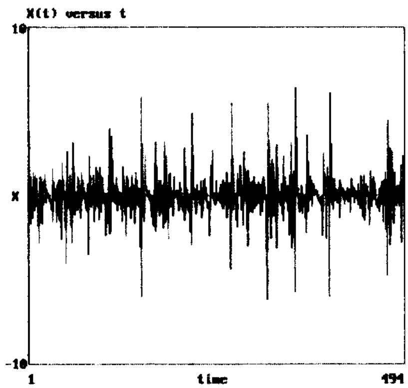
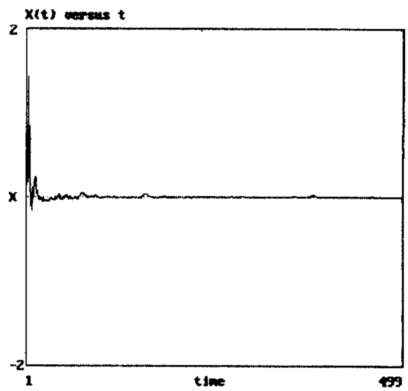
To test the effectiveness of equation (4.1) and (4.2), the equation are applied to given simulation output data in Table 4.1. The obtained time series data, reconstructed state space, dimension using equation (2.2), and Lyapunov exponent using equation (3.4) for two equations are also shown in the Table 4.1.

As seen in the time series of the equation in Table 4.1, equation (4.1) does not illustrate the system varition sufficiently. Also, since the reconstruction attractor has an infinite dimension and Lyapunov exponent results in a positive value, the variation rate evaluted using equation (4.1) is under the random motion. This implies that equation (4.1) fails to distinguish the transient state from the steady state.

On the contrary, when equation (4.2) is used, the time series of the equation indicates the system variation correctly. Since the reconstruction attractor has a finite dimension and Lyapunov exponent results in a negative value, the varition rate seems under the regular motion. This also states that the difference of variation rates between the transient state and the steady state is significant.

Based on these results, we propose two methods M1 and M2 using equation (4.2) for the determination of truncation points. The underlying concept of the two methods is that they determine truncation points based on the system variations in order to control initial bias. This is quite different from the concept of the existing methods in which truncation points are decided using the mean and the variance of the output data.

〈Table 4.1〉 An example of a dynamic characteristics for given simulation output data.

<table>
<tr><td>Simulation Output Data</td><td colspan="2">
</td></tr>
<tr><td>Equations</td><td>$\log_2 \frac{|x_{i+1}-x_i|}{|x_i-x_{i-1}|}$ (4.1)</td><td>$\log_2 \frac{x_{i+1}}{x_i}$ (4.2)</td></tr>
<tr><td>Modified Time Series by Equations</td><td>
</td><td>
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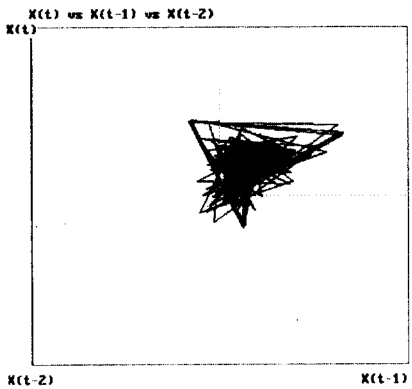
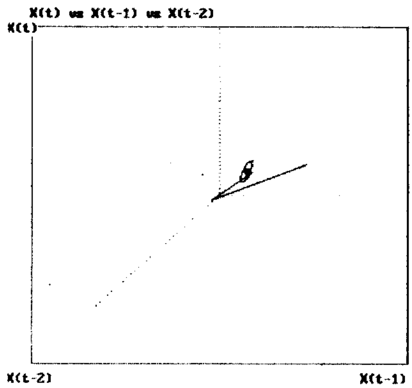
<tr><td>Reconstructed State Space</td><td>
</td><td>
</td></tr>
<tr><td>Dimension</td><td>∞</td><td>4</td></tr>
<tr><td>Lyapunov Exponent</td><td>0.587</td><td>-0.002</td></tr>
</table>

5. The deletion Strategies

Initialization bias can be a major source of error in estimating the steady state value of a simulated system performance measure. Lyapunov exponent λ which decides whether convergence or divergence of a trajectory is applied to deciding how much data to be deleted which is called truncation points in simulation output data. The methods to be used in the experiments are as follow.

1) Method 1(M1)

The first method uses λ_k for deciding truncation point represented by equation (5.1).

$$\lambda_k = \frac{1}{k}\sum_{i=1}^{k}\log_2\frac{x_{i+1}}{x_i} \quad , k = 1,2,\ldots,n-1. \quad (5.1)$$

where

x_i : average waiting time in queue for up to the i-th customer from the first customer in M/M/1(∞) with arrival rate λ, service rate μ.

n : run length (total number of observations in simulation output data).

The criteria for λ_k to decide the truncation points are as follow.

· criterion 1(C_1) : $|\lambda_k|$ must be less than the specified value.

The specified values for average variation rate 5% and 10% are obtained to be 0.036 and 0.07 respectively as follows,

$$\log_2[\frac{a(1\pm0.025)}{a}] = 0.036$$

$$\log_2[\frac{a(1\pm0.05)}{a}] = 0.07.$$

· criterion 2(C_2) : λ_k must be decreased continuously.

A variation is decreased if system's behavior comes to be the steady state, and the average variation rate can be decreased gradually.

2) Method 2 (M2)

Partition n simulation data $x_1, x_2, \ldots, x_n$ into b nonoverlapping batches in which each batch has m observations such that $n = b \cdot m$, and define the following functions of the original data for i=1, 2, ..., b.

$$\lambda_i = \frac{1}{m}\sum_{j=1}^{m}\log_2\frac{x_{(i-1)m+j}}{x_{(i-1)m+1}} \quad (5.2)$$

where

$x_{(i-1)m+j}$: average waiting time in queue for up to the j-th customer in i-th batch.

· criterion 3(C_3) : $|\lambda_i|$ must be less than the specified value.

This criterion C_3 is equal to the criterion C_1.

3) Method 3 (M3)

The deletion strategy suggested by Cash et al.[13] is as follow. First, compute the below values for i=1,2,...,b and j=1,2,...,m. And perform the F test with f=0.25, the fraction of the batches used to compute the variance estimator ; if the null hypothesis of no initial condition bias is rejected, delete the first 25% of the data and apply the test again to the remaining data. If the null hypothesis is accepted, retest at f=0.5 and next at f=0.75.

$$\bar{x}_{i,j} = \frac{1}{j}\sum_{t=1}^{j}x_{(i-1)m+t}$$

$$S_{i,j} = \bar{x}_{i,m} - \bar{x}_{i,j}$$

$$\hat{K}_i = \max\{j \cdot S_{i,j}\}, \text{ for } 1 \leq j \leq m$$

$$\hat{S}_i = \hat{K}_i \times \hat{S}_i, \hat{K}_i$$

$$Q_{max} = \sum_{i=1}^{b}\frac{m \cdot \hat{S}_i^2}{\hat{K}_i \cdot (m-\hat{K}_i)}$$

$$V_{max} = \frac{Q_{max}}{3b}$$

6. Experimental Results

$M/M/1(\infty)$ queueing model was used to evaluate the performance of methods. In order to gather the data, we executed the simulation using the following conditions. Simulations are conducted with the utilities $\rho=0.7$ ($1/\lambda=14.29$, $1/\mu=10.00$) and $\rho=0.9$ ($1/\lambda=11.11$, $1/\mu=10.00$). Each experiment involved 10 independently seeded replications in which each has 500 observations of customer waiting times in the queue. The decreasing length for C_2 considered in this experiment would be 195 times and 200 times.

To evaluate the effectiveness of the methods, the criterian in equation (6.1) was used.

$$\frac{|\mathbf{W}_q - W_q|}{\mathbf{W}_q} \qquad (6.1)$$

where

$$\hat{W}_q = \frac{1}{k}\sum_{j=1}^{k}\bar{x}_j$$

$$\bar{x}_j = \frac{1}{n-l}\sum_{i=l+1}^{n} x_{ij}, \qquad j = 1,2,\ldots,k.$$

W_q : theory value of average waiting time.

x_{ij} : average waiting time of i-th customer in j-th replication.

n : run length of each run.

l : truncation point in each run.

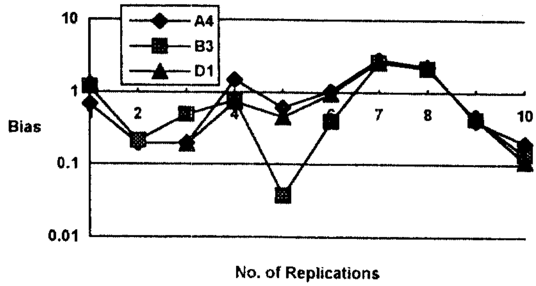
k : number of replications.

The cases of experimentals for the deletion strategies are shown in Table 6.1 and their experimental results are shown in Table 6.2.

Experimental results can be summarized as follow.

1) In case of $\rho=0.7$

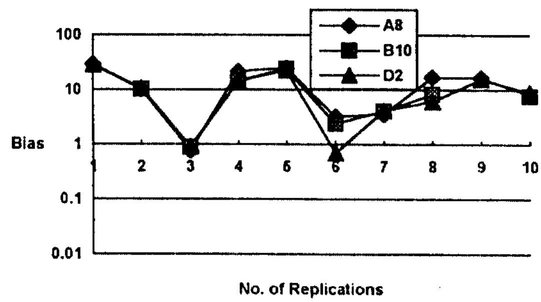
With respect to point estimators and equation (6.1), A_4 and B_3 produced good results among the cases of the method M1 and M2 respectively. By the comparison of the three methods, we could conclude that the method M2 shows the best result with respect to point estimator. Comparisons of A_4(M1), B_3(M2), D_1(M3) for each run are shown in Fig. 6.1.



〈Figure 6.1〉 Comparisions of Deletion Strategies for p=0.7.

2) In case of $\rho=0.9$

With respect to point estimator and equation (6.1), the case A_8 of the method M1 produced the best outcomes. For the method M2, we did not consider the results by B_7, B_9 and B_{11} since they failed to find truncation points. Among the rest three cases of B_8, B_{10} and B_{12}, B_{10} yielded the best outcome regarding to point estimator and equation (6.1). Throughout the evaluation of the three methods, we observed that the method M2 had the best result. Comparisons of A_8(M1), B_{10}(M2), D_2(M3) for each run are shown in Fig. 6.2.



〈Figure 6.2〉 Comparisions of Deletion Strategies for p=0.9.

7. Conclusions

Initialization bias is the most troublesome problem

⟨Table 6.1⟩ The cases of experiments for deletion strategy

Deletion strategy	cases	
Method 1	A_1	$\rho=0.7$, $C_1=0.036$, $C_2=195$
	A_2	$\rho=0.7$, $C_1=0.036$, $C_2=200$
	A_3	$\rho=0.7$, $C_1=0.07$, $C_2=195$
	A_4	$\rho=0.7$, $C_1=0.07$, $C_2=200$
	A_5	$\rho=0.9$, $C_1=0.036$, $C_2=195$
	A_6	$\rho=0.9$, $C_1=0.036$, $C_2=200$
	A_7	$\rho=0.9$, $C_1=0.07$, $C_2=195$
	A_8	$\rho=0.9$, $C_1=0.07$, $C_2=200$
Method 2	B_1	$\rho=0.7$, $C_1=0.036$, $C_3=30$
	B_2	$\rho=0.7$, $C_1=0.07$, $C_3=30$
	B_3	$\rho=0.7$, $C_1=0.036$, $C_3=40$
	B_4	$\rho=0.7$, $C_1=0.07$, $C_3=40$
	B_5	$\rho=0.7$, $C_1=0.036$, $C_3=50$
	B_6	$\rho=0.7$, $C_1=0.07$, $C_3=50$
	B_7	$\rho=0.9$, $C_1=0.036$, $C_3=30$
	B_8	$\rho=0.9$, $C_1=0.07$, $C_3=30$
	B_9	$\rho=0.9$, $C_1=0.036$, $C_3=40$
	B_{10}	$\rho=0.9$, $C_1=0.07$, $C_3=40$
	B_{11}	$\rho=0.9$, $C_1=0.036$, $C_3=50$
	B_{12}	$\rho=0.9$, $C_1=0.07$, $C_3=50$
Method 3	D_1	$\rho=0.7$
	D_2	$\rho=0.9$

C_1 : Condition for variation rate in M1.
C_2 : Condition for decreasing length in M1.
C_3 : Batch size for M2.

in evaluating system's responses correctly using simulation output data. Two methods have been proposed to handle this problem using chaos theory.

The suggested methods have been compared with the method by Cash et al.[13] using M/M/1(∞). We could find that the method M2 with $C_1=0.07$ and $C_3=40$ among the three methods produced the best results with respect to $\rho=0.7$ and $\rho=0.9$. Truncation point cannot be found in some cases. This means that system behavior fails to reach steady state, with the given the present run length. For these cases, we could natually think that the run length should be increased to get more data.

Further researches are needed to determine the truncation points and the appropriate of the given run length simultaneously. Some statistical work is also required to determine the most appropriate criteria to be used in the proposed methods.

〈Table 6.2〉 **Performances of Deletion Strategies**

Deletion Strategy		Input Data	$\hat{W}_q$	$\lvert\hat{W}_q - W_q\rvert / W_q$	
Method 1	$\rho=0.7$	A_1	22.733	23.310	0.0248
		A_2	22.971		0.0145
		A_3	22.846		0.0199
		A_4	23.084		0.0097
	$\rho=0.9$	A_5	85.248	90.090	0.0537
		A_6	86.525		0.0396
		A_7	85.676		0.0490
		A_8	87.074		0.0335
Method 2	$\rho=0.7$	B_1	23.064	23.310	0.0106
		B_2	22.820		0.0210
		B_3	23.203		0.0046
		B_4	23.019		0.0125
		B_5	23.071		0.0103
		B_6	22.926		0.0165
	$\rho=0.9$	B_8	90.493	90.090	0.0045
		B_{10}	90.120		0.0003
		B_{12}	88.097		0.0221
Method 3	$\rho=0.7$	D_1	23.028	23.310	0.0121
	$\rho=0.9$	D_2	93.966	90.090	0.0430

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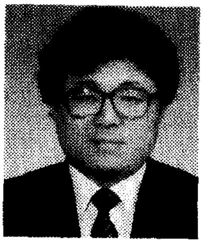
● 저자소개 ●



이영해

1977년 고려대학교 산업공학 학사

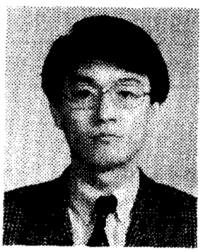
1983년 미국 Univ. of Illinois, 산업공학 석사

1986년 미국 Univ. of Illinois, 산업공학 박사

일본 오사카대학 전자제어기계공학과 객원교수(1990년)

현재 한국시뮬레이션학회 이사, 대한산업공학회 이사, 한양대학교 산업공학과 부교수

관심분야: Simulation in Manufacturing, Simulation Output Analysis, Simulation Optimization.



오형술

1987년 한양대학교 산업공학 학사

1989년 한양대학교 산업공학 석사

현재 한양대학교 산업공학 박사과정, 수원전문대학 공업경영과 전임강사

관심분야: Simulation Output Analysis, Chaos Theory, Cybernetics.