Simplification of Monte Carlo Techniques for the Estimation of Expected Benefits in Stochastic Analysis of Multiple Reservoir System

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ABSTRACT/For the system benefit optimization by considering risk or reliability from a multiple reservoir system using the Monte Carlo Technique, many stochastically generated inflow series have to be used for the system analysis. In this study, the stochastically generated inflow series for the multiple reservoir system operation are preprocessed according to the considering system objectives and operating time periods. Through this procedure, several representative inflow series which have discrete probability levels and operation horizons are selected among the thousands of generated inflows. Then a deterministic optimization technique is applied to the hydropower energy estimation from the Han River Reservoir System which considers five reservoirs in this study. It took much less computational requirements than using the original Monte Carlo Technique, even though estimated result was almost similar.

1. Introduction

In the optimal planning or operation of water resources facilities such as multiple reservoir system, deterministic analysis can be utilized by regarding the stochastic variables like inflows, demands for water or power as known values like historical data or as random sets of equally likely sequences. Then, commonly utilizing optimization techniques such as linear programming or dynamic programming can be applied as the most desirable method in this kind of deterministic analysis. However, when the stochastic input data such as reservoir inflows have strong random characteristics, these kinds of optimization techniques cannot be used directly in reservoir operation. Moreover, deterministic models based on the average inflows or inflows of equally distributed probability are tend to be optimistic in a sense that system benefits are overestimated while costs and losses are underestimated (Loucks et al., 1981).

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Therefore, it is hard to get reliable estimation of expected benefits from multiple reservoir system operation by using conventional deterministic approach because of uncertainties in predicting future states of input data such as stream inflows or demands which have strong stochastic properties. Especially, the estimation of hydropower energy benefits from a multiple reservoir system which includes many reservoirs and hydroelectric power plants requires a lot of efforts because of the differences in storage states and size of plant facilities with different constraint conditions on the releases from each reservoir.

To deal with these kind of uncertainties, several stochastic analysis techniques such as Chance Constrained Programming (CCP) using Linear Decision Rule (ReVelle et al., 1969), Stochastic Dynamic Programming (Askew, 1974), and Reliability Analysis Technique (Colorni and Fronza, 1976; Croley and Rao, 1979) have been developed and employed in actual system. Among these techniques, the Chance Constrained Programming has been utilized most often because of its convenience to combine the stochastic properties with the other optimization algorithms. However, there is criticism that some of the CCP models have conservative characteristics by which more active storage capacity was specified by the solution than was actually needed to meet the reliability requirements defined by the chance constraints (Loucks and Dorfman, 1975). Moreover, it is almost impossible to apply the Stochastic Dynamic Programming to the practical problems in which multiple reservoirs have to be considered at the same time except the case of single reservoir problem due to the tremendous computational requirement with the complexity of formulation of transition probability.

In order to compensate these kinds of deficits, Monte Carlo Techniques have been applied to estimate the benefits from reservoir systems operation since Fiering (1961), Hufschmidt and Fiering (1966). Through these techniques, expected benefits or operational rules according to the assessed reliability levels can be estimated from reservoir systems by using significantly long periods of stochastically generated data which include bigger and smaller data than the historically recorded ones. Furthermore, Monte Carlo Techniques are still being widely utilized in the stochastic analyses because it is possible to estimate the reliability or risk levels for design or operation standard which are frequently required in the planning or operational stages of the water resources facilities. Young (1967) applied the Monte Carlo Techniques in developing the operation rules to determining the optimal annual discharge from a single reservoir by utilizing the deterministic dynamic programming technique. Askew et al. (1971) applied the Monte Carlo Techniques in determining the optimal annual contract level of water supply from a multiple reservoir system by considering risk and reliability levels. Using this technique, Willis et al. (1984) suggested a method which determines the optimal releases according to the assessed return periods. However, Monte Carlo Techniques generally require too much computational efforts, because the techniques require as many solutions from simulating reservoir system operation which usually adopts an optimization technique such as linear, non-linear or dynamic programming as the numbers of generated inflows. It usually requires hundred to thousand times of repeated operation by using the inflow series which are synthesized from a stochastic hydrologic model. As McGrath and Irving (1973) pointed out that the method requires iterating simulations at least more than 1,000 times to estimate the mean or variance of the considering state variable to converge to a constant value. Therefore, immense computational time is required in multiple reservoir system operation problem for the application of an optimization algorithm as the solution technique.

In this study, lots of stochastically generated inflow data series for a multiple reservoir system operation are pre-processed according to the operational period and system objectives such as hydropower energy or water supply benefit from the system. Through this procedure, several representative inflow series which have discrete frequency levels are utilized instead of applying all the stochastically generated data series. Then a deterministic optimization technique is applied to estimate the system benefit according to the discrete uncertainty levels, and the inflow data of 1,000 years are utilized to analyze and compare with the original Monte Carlo Techniques.

Finally, a methodology on the simplification of the original Monte Carlo Technique is suggested by using the pre-processed long term synthetic data which are stochastically generated from the historical record. Also this paper shows the procedure on the pre-processing for the selection of the representative inflow series in hydropower energy benefit estimation according to the discretized reliability or risk levels. This methodology is applied to the optimal operation of the Han River Reservoirs System which has three large-scale reservoirs namely Hwacheon, Soyanggang, and Chungju Dam. The application result shows that it can dramatically save the computational efforts comparing with the original Monte Carlo Techniques, however, the estimated result of expected benefit from the multiple reservoir system is almost similar to that of original one.

2. FORMULATION OF AN OPTIMAL OPERATION MODEL FOR MULTIPLE RESERVOIR SYSTEM

Figure 1 shows the schematic representation of a multiple reservoir system which are connected by series or parallel. Following is the mathematical model for multi-objective optimization problem with ε-constraint technique. In which it maximizes hydropower energy by satisfying the given discrete water supply demand.

$$F = \mathrm{Max} \sum_{t=1}^{T} \sum_{l=1}^{n} f_l(V_{i,\,t},\ V_{l,\,t+1},\ u_{l,\,t}) \tag{1}$$

subject to,

$$\underline{V}_{t+1} = \underline{V}_t + \underline{I}_t - \underline{D}_t + C\underline{U}_t + \underline{E}_t\,\underline{A}_t^{T}(\underline{V}_t,\ \underline{V}_{t+1})\ \forall\ t \tag{2}$$

$$\underline{V}_{t\min} \leq \underline{V}_t \leq \underline{V}_{t\max}\ \forall\ t \tag{3}$$

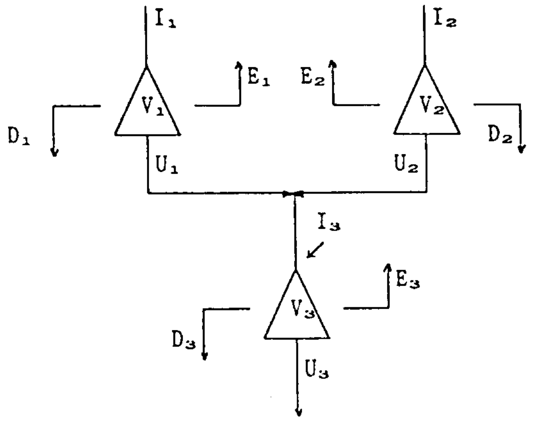
$$\underline{\varepsilon}_{t\min} \leq \underline{U}_t \leq \underline{U}_{t\max}\ \ \underline{\forall}_t \tag{4}$$

Where, $\underline{V}_t$ is the storage in each reservoir at the beginning of period t(n*1 vector) ; $\underline{V}_{t+1}$ is the storage in each reservoir at the end of period t or at the beginning of next period t+1 ; $\underline{U}_t$ is the amount of water released during the period t (n*1 vector) ; $\underline{I}_t$ is unregulated inflow during the period t (n*1 vector) ; $\underline{D}_t$ is amount of water depletion from each reservoir during the period t (n*1 vector) ; $\underline{E}_t$ is evaporation rate of period t (n*1 vector) ; $\underline{A}_t$ is average surface area of each reservoir over the current period (n*1 vector); and C is system configuration matrix which represents connecting condition between each reservoir (N*N matrix); n is the number of reservoirs or hydropower plants, T is the number of total periods for reservoir operation.

And, each element of the system configuration matrix, C of equation (2), depends upon the location of each reservoir, and it accounts for the balance of water from upstream reservoirs. If the value of the element of C Matrix is −1, it represents self release from the reservoir; if the element is 0, the reservoir is connected in parallel; and if it is +1, then it is connected in series.

ε_{tmin} represents the lower limit of the constraint transformed from the multiobjective function which has more than two objectives like hydropower and water supply. A set of nondominated solutions can be generated by parametically varying the bounds of the equation (2), and tradeoff relationships between each objectives can be explicitly obtained. From this tradeoff relationships, the decision maker or the system analyst can select the lower bound of ε_t which is the maximum value of the transformed objective function. This technique which was theoretically established by Haimes et al. (1971) has been suggested as the most desirable multiobjective analysis technique if the multiobjective function is consisted of additive type and Min(Max) or Max(Min) type in reservoir operation (Ko et al., 1992).

The maximum value within the feasible range which is possible to analyze the system even in the low-flow season has to be selected as the minimum release constraint for the Monte Carlo analysis, and ε_t should be pre-determined by the viewpoint of experienced expert after completion of multiobjective analysis according to the historical inflow data.



I : Inflow
V : Storage
D : Diversion
U : Release
E : Evaporation

$$C = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ +1 & +1 & -1 \end{bmatrix}$$

Figure 1 Hypothetical Multiple Reservoir System

3. BASIC CONCEPT OF SIMPLIFICATION OF THE MONTE CARLO TECHNIQUES

The major objective of Monte Carlo analysis of a multiple reservoir system is to identify operation policies or operating rules by analyzing lots of outputs according to the specific season or month, or according to specific reliability or risk levels. The outputs are usually obtained after calculating the reaction upon an optimization model (or simulation model) by using lots of historical or synthetic data generated by a stochastic model based on the observed historical data.

The major procedure of this technique is to compute sets of objective values according to the optimization model of equations from (1) to (4) by using sets of stochastically generated inflow (or demand) data which have certain length of period. The objective values estimated by the optimization model are analyzed to obtain a relationship between the size or variance of inflow for the specific analyzing period or including the relationship between the optimal releases and storage states or inflows in each time period. This process is repeated according to lots of stochastically generated new data in order to include all the possible operational conditions.

Outputs from the system according to the inputs such as inflows or demands which have certain length of time periods can be expressed by the following performance index function.

$$Z_{f(x)} = \zeta_{g(x)} (R_1, R_2, \cdots, R_n : \alpha_1 \alpha_2, \cdots, \alpha_p) \tag{5}$$

Where, $R_1, R_2, \cdots, R_n$ are, as a vector having time function, stochastic input data like inflows into the reservoirs or demands for water supply during the analyzing periods; $\alpha_1, \alpha_2, \cdots, \alpha_p$ are parameters representing a vector for the estimation of physical characteristics of each reservoir, deterministic input data or objective value, Z. ζ is the performance index function represented by equations from (1) to (4). f(x) and g(x) expressed as subscripts are the probability density function (pdf) determined by the magnitude or variance of stochastic input data, and they can be defined as follows.

$$\int_{-\infty}^{\infty} f(x)dx = 1 \tag{6}$$

$$\int_{-\infty}^{\infty} g(x)dx = 1 \tag{7}$$

The original Monte Carlo Techniques require so many computational efforts because the probability density function f(x) and g(x) are not directly estimated. However, computational requirement can be significantly reduced by introducing pre-processing function which estimates f(x) from g(x) according to the objective function.

$$f(x) = h\{g(x)\} | \zeta_{g(x)} \tag{8}$$

Where, function h(.) is a transfer function which is determined by the performance index function, ζ, estimated by the equations from (1) to (4), and it is equivalent to the pre-processor shown in Figure 2.

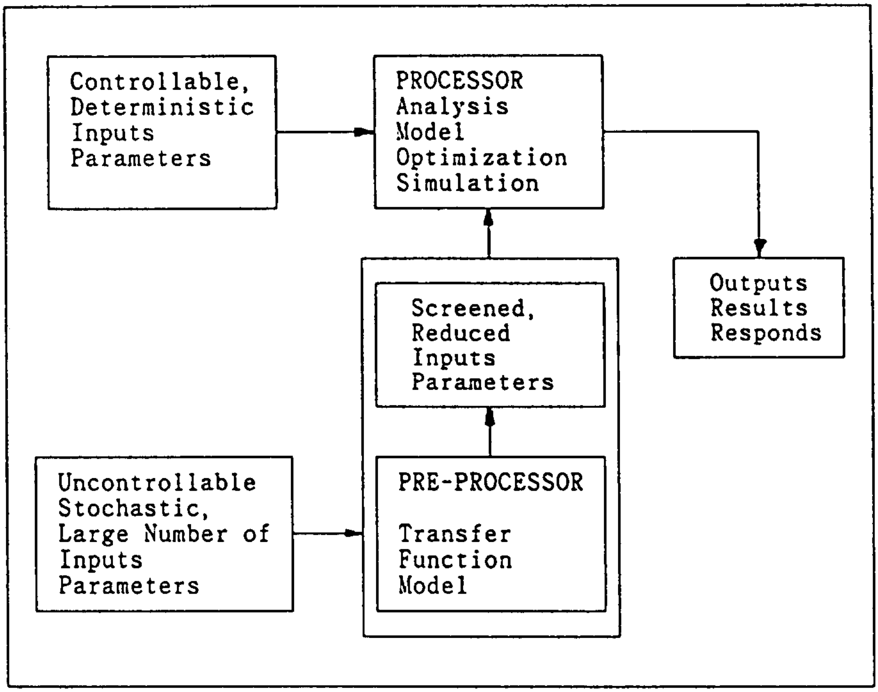


Figure 2 Concept of Screening Function for Monte Carlo Simplification

Therefore, if we can define transfer function model by which objective value can be simply deduced in advance directly with the stochastic input data like inflow or water demand which has certain length of period, only the pre-selected inflow data sets will be analyzed instead of evaluating all the inflow data sets obtained using optimization model in which case needs lots of computational requirement. Especially, the computational burden can be remarkably reduced in case of complex reservoir systems consisted of series and parallel instead of simple reservoir operation problem.

The transfer function of equation (8) can be evaluated not only according to the distribution of stochastic input data in magnitude or variance in time horizon but also according to the physical characteristics of the considering reservoir, operating policy, and criterion of objective function.

4. APPLICATION : ESTIMATION OF HYDROPOWER ENERGY BENEFIT FROM A MULTIPLE RESERVOIRS SYSTEM

4.1 The Basin Outlines and Operation Conditions

In the Han River Basin which supplies various water around the national capital region, there are nine storage reservoirs with hydro-power plants. These reservoirs are the major sources of water supply to the capital city of Seoul including its vicinity areas. Among these reservoirs, the storages of the Chungju and the Soyanggang large-scale multipurpose reservoirs are over 2.7 and 2.9 billion m^3 respectively. These two reservoirs and a flow-through reservoir of Chungju regulation dam have been operated and managed by Korea Water Resources Corporation (KOWACO) after their construction. With another five small-scale flow-through reservoirs, Hwacheon large-scale reservoir is operated and managed by Korea Electric Corporation(KECO). The major purposes of all these reservoirs are water supply, hydro-power generation and flood control including low-flow augmentation, recreation and fishery.

In this study, relatively small reservoirs such as Chuncheon, Euiam, Cheongpyung, Koesan Dams were excluded in the analysis because they can be regarded as flow-through reservoirs. However, Paldang reservoir was included in this analysis since it does a very important role in water supply to the capital region.

Paldang wide region water supply system has been operated in order to supply municipal and industrial water to the capital region including metropolitan city of Seoul since 1977. The equipped capacity at the end of 1992 is 3.93 million m^3 per day, and the capacity will be increased up to 5.46 million m^3 per day by the end of 1995 (Korea Water Resources Corporation, 1992).

Table 1 shows the minimum required flows at each control point in this river basin which are the required absolute minimum amounts for downstream water supply and river maintenance. This table also shows the minimum desired flows which are the preferred flows for river maintenance and water quality improvement in addition to the required amounts. These values were suggested during the feasibility study of the Han River Navigation (Industrial Site & Water Resources Development Corporation, 1988). The amounts of diversion for water supply at the major control points are obtained based on the data from the report of Korea Water Resources Corporation (Industrial Site & Waster Resources Development Corporation, 1986). The rates of return flows were considered as 65 % for municipal and industrial water, and 50 % for irrigation water. Net amounts of diversion water were estimated by deducting the return flows at the return points from total intake amounts.

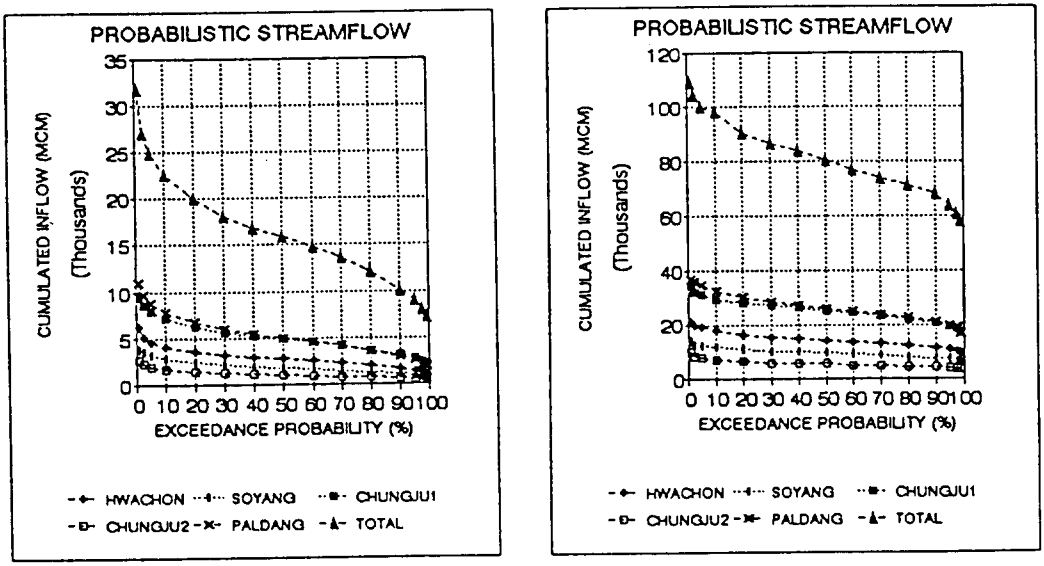
Table 1 Minimum Desired and Required Flows at the Major Control Points

Control Points	Min Required Flow		Min Desired Flow	
	CMS	MCM/M	CMS	MCM/M
North Han				
Hwacheon Dam	7.1	18.7	15.0	39.4
Soyanggang Dam	5.0	13.1	38.3	100.7
South Han				
Chungju Main Dam	10.6	28.0	18.0	47.3
Chungju R/R Dam	13.3	35.0	29.0	76.3
Han River				
Paldang Dam	75.0	197.0	200.0	525.6
Goan Gage	75.0	197.0	200.0	535.6

* MCM/M : million cubic meters per month (10^6m^3/month)
CMS : cubic meters per second (m^3/second)

The equations for hydropower energy production, storage–head, and storage–surface area relationships were derived from the regression analysis and the coefficients of these equations are referred to the proceeding of Korea National Committee on Large Dams (Ko, 1990). For this analysis, monthly inflow data for the considering five points (five reservoirs) from 1917 to 1940 and from 1968 to 1988 were used. The statistic data of this historical data for these five points are also contained in the proceeding above.

The general program of MPAR (1) developed by Ko et al. (1992) was utilized to generate synthetic long–term monthly inflow data for the five control points. The program was based on the multivariate auto–regressive lag–1 model with periodic parameters which was suggested by Salas et al. (1980).



(a) Duration : 1 Year

(b) Duration : 5 Year

Figure 3 Probabilistic Natural Streamflow at Paldang Dam Site

4.2 Approximate Appraisal of a Transfer Function

Monthly inflow series for 1,000 years for the five control points were generated by the stochastic model of MPAR (1). It is not easy to select the real representative inflow series for the considering objective function according to the discrete probabilities since the representation of the objective is decided according to the equations from (1) to (4). However, it is possible to get implicit appraisal of the transfer function from which approximately representative ones can be selected after sorting the generated inflow series according to the assumed criteria.

Equation (8) can be expressed as following when the exceedance probability or reliability levels are considered instead of probability density function.

$$F(x)=H\{G(x)\}|\xi_{g(x)} \tag{9}$$

Where, F(x), G(x) and H(.) are those expressed by the cumulative density function instead of probability density function of f(x), g(x) and h(.) defined by equation (8). In the implicit analysis of transfer function of equation (9) for the simplification of Monte Carlo Techniques in calculating the expected hydropower energy benefit from five reservoirs in the Han River Basin, G(x) was considered as the exceedance probability function on the amount of sum of local inflows into each reservoir during a considering period.

And the function, H(.) was approximately estimated through trial and error method by combining one or several elements among various factors of the system. The factors considered in this study are geometric average of monthly inflow in a year, effective storage volume of each reservoir, installed capacity of hydropower plant, standard deviation of monthly inflow, and amount of inflow during flood season in addition to the total amount of inflow considered in G(x).

Figure 3 shows the sorted result by the Weibull's Plotting Position method according to the exceedance probability of the function, G(x). In this function, G(x) is based on the total amount of inflow during a certain period of time from the generated 1,000 years' monthly inflow data for the five control points by the adopted MPAR (1).

4.3 Analysis of the Application Results

Figure 4 shows the annual energy outputs from the Han River reservoirs system according to the exceedance probability of the transfer function of G(x) and duration of analyzing period. In this case the function considers only the cumulated amount of inflow during specified periods as shown in the Figure 3. Energy outputs were computed by the optimization model of equations from (1) to (4), and the incremental dynamic programming technique was utilized to solve this optimization of five reservoirs operation problem.

The values of minimum desired flows shown in the Table 1 were applied for the values of ε_t in equation (4) that considers transfer objective function in the ε-constraint multiobjective optimization problem.

Table 2 Criteria for Preprocessing and Regression Coefficients

Gases	Criteria for Exceedance Probability	Regression, R
1	Cumulative Inflow (Total Inflow)	0.8424
2	Geometric Average of Annual Monthly Inflow	0.8663
3	Total Inflow + Effective Storage Capacity	0.8747
4	Total Inflow + Power Facility Capacity	0.8635
5	Total Inflow − Standard Deviation of Inflow	0.8644
6	Total Inflow − Flood Season Inflow	0.9172
7	Total Inflow + Effective Storage Capacity-(minus) The Standard Deviation of Inflow	0.9078

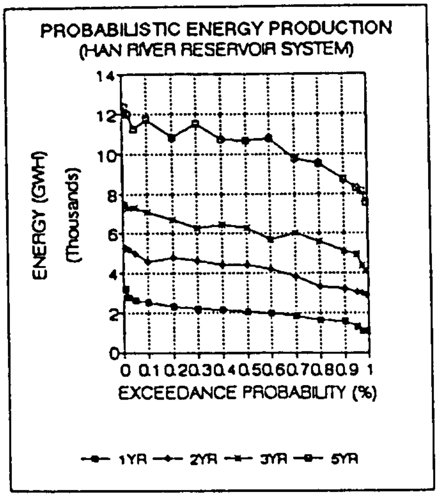


Figure 4 Probabilistic Energy Output According to the Preprocessed Inflow Series

Curves in Figure 4 are not smoothly distributed compared to the curves in the Figure 3. This means that even though the expected energy outputs according to the discretized exceedance probability levels are not very closely related with the cumulated amount of inflows during specified periods, this transfer function can be considered and has the room of improvement.

Table 2 shows alternative cases of the transfer function of G(x) which considers geometric average of inflow, effective storage capacity, power facility capacity, standard deviation of inflow, or flood season inflow in addition to the cumulated amount of inflow. Figure 5 shows the results which were computed based on the alternative cases described in Table 2. From this figure and regression coefficients shown in Table 2, it can be found that the similarities of distribution were much increased.

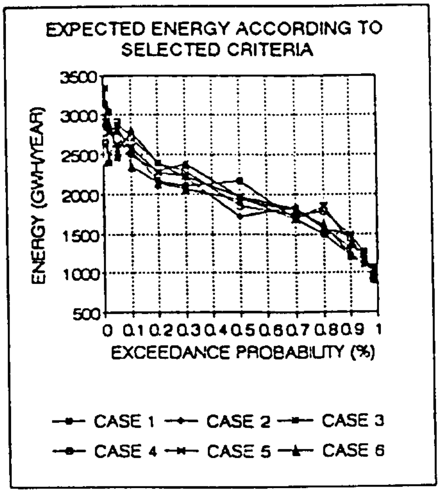
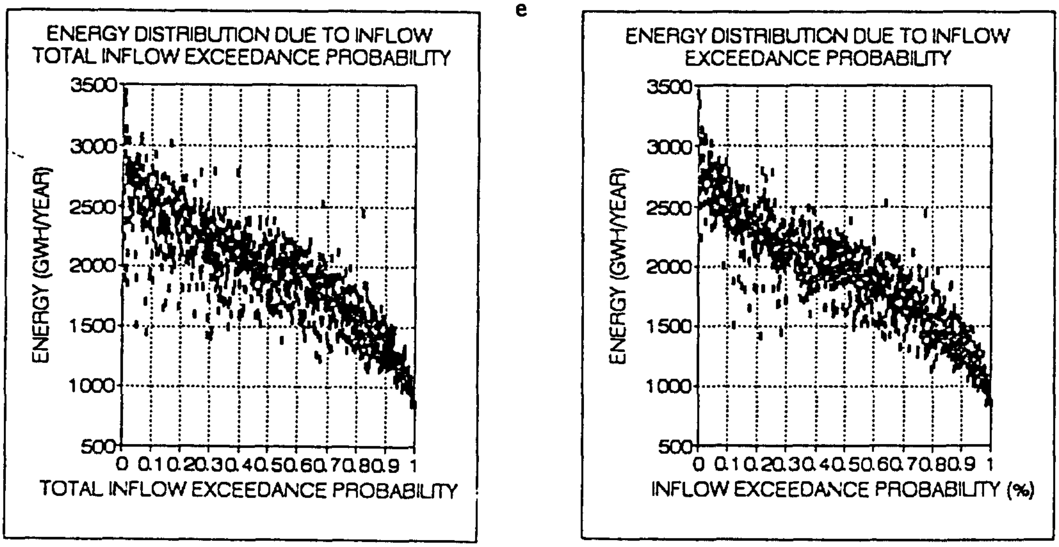


Figure 5 Distribution of Expected Benefit According to the Selected Criteria

Figure 6 shows the results of the application of the original Monte Carlo Techniques which maximize the hydropower energy benefits according to the reliability levels with satisfying the water demands from the system. These results were based on all the stochastically generated inflow series of 1,000 years. Figure 6 (a) shows the distribution of expected energy output according to the exceedance probability which is based on the Case-1 of Table 2, and Figure 6 (b) displays the distribution based on the alternative Case-6. It is noted that the distribution of Figure (b) is less scattered than that of Figure (a).

For the estimation of expected benefits from a multiple reservoir system by considering the stochastic property of inflow, the Monte Carlo Techniques can be simplified through pre-processing the stochastic input data. In the estimation of the power benefit by applying the simplified technique from the Han River reservoirs system, more reliable analysis was possible by considering the physical property or operation purpose of the system to the pre-processing procedure in addition to the stochastic inflow itself. Moreover, computational requirements are significantly reduced by analyzing only the representative data which are selected through the pre-processing procedure instead of analyzing all the data that are stochastically generated.



(1) CASE 1 (2) CASE 2

Figure 6 Energy Distribution According to the Monte Carlo Technique

5. CONCLUSIONS

For the long or mid-term operation planning of a reservoir system, the hydrologic uncertainty such as the reservoir inflow has to be especially considered. As a commonly using stochastic analysis technique, the Monte Carlo Techniques can directly consider the risks or reliabilities incorporated with this uncertainty. However, the techniques require lots of computational burden. Moreover, due to the non-linearity and non-convexity of the problem, the burden is severely increased if the problem includes optimization of hydropower energy from multiple reservoir system like the reservoirs in the Han River.

Therefore, application of the conventional Monte Carlo Techniques for the estimation of power benefit by considering reliability levels from a multiple reservoirs system has required tremendous computational efforts. The Monte Carlo Techniques can be significantly simplified by adding a pre-processing procedure of the stochastic input data like reservoir inflow with regard to the property of the objective function or the physical conditions. Then only the representative input data are analyzed according to the discretized reliability levels.

Hydro-power energy benefits according to the discrete reliability levels from the five reservoirs in the Han River Basin were estimated by using the simplified technique which adopts a pre-processing procedure of the input data. As for the criteria on the pre-processing of the stochastically generated long term inflows, not only total amount of inflow within the basin during an operation period but also the distribution status of the inflows during the period, and the effective storage and power capacities of the reservoirs were considered. The application result of the pre-processing shows that the reliability level of the distribution of the energy output was improved compared with the case of considering the amount of inflow only.

Especially, if the amount of inflow during flood season is considered in addition to the total amount of inflow, the distribution of power energy benefit in each reliability level was much smoother. Moreover, if all or some of these factors are considered at a same time, the reliability of the distribution can be improved according to the selection of the weighting factors to combine these criteria.

Considering the complexity of the stochastic analysis techniques like Monte Carlo Techniques or reliability programming, the simplified technique can be utilized for the estimation of expected benefit according to the assessed reliability levels with significantly less computational effort in the long or mid-term operational planning of single or multiple reservoirs.

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