⊙ 研究論文

Robust Motion Control of Robotic Manipulators with Nonadaptive Model – based Compensation

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비적응 모델 보상법에 의한 강성로보트의 강인한 동작제어

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Abstract

This article deals with the problem of designing a robust algorithm for the motion control of robot manipulator whose nonlinear dynamics contain various uncertainties. To ensure high performance of control system, a model – based feedforward compensation with continuous robust control has been developed. The control structure based on the deterministic approach consists of two parts : the nominal control law is first introduced to stabilize the system without uncertainties, then a robust nonlinear control law is adopted to compensate for both the resulting errors(or structured uncertainties) and unstructured uncertainties. The uncertainties assumed in this study are bounded by polynomials in the Euclidean norms of system states with known bounding coefficients. The presented control scheme is relatively simple as well as computationally efficient. With a feasible class of desired trajectories, the proposed control law provides sufficient criteria which guarantee that all possible responses of the closed – loop system are uniformly ultimately bounded in the presence of uncertainties. Therefore, the control algorithm proposed is shown to be robust with respect to the involved uncertainties.

1. Introduction

Robotic manipulators con be used in many modern industries to meet various demands for fast motion and high – accuracy operations, such as pick and place, welding, machining, and assembly operations. Furthermore, future practical applications of robots are likely to be extensive. Therefore, the design of high - performance and reliable control algorithm is one of key issues in the current robotics research.

The performance of control system depends largely upon the accuracy of the dynamic model of the system. Unfortunately, many real systems, including robot manipulators, always contain various uncertainties in system modeling and control processes. The uncertainties under consideration include structured uncertainties(e. g., parametric variations, unknown payload, modeling errors) and unstructured uncertainties(disturbances, friction, other unmodelled dynamic effects, etc.). However, a wide range of the current control schemes ignore these uncertainties associated with robot systems because of the complexity of their effects, thus those algorithms sometimes cannot provide general solutions. To achieve satisfactory system performance and to extend the usage of control law, the control strategies should account for possible uncertainties which deteriorate system performances.

Recently, several model - based control schemes have been proposed for motion control of robot manipulators, such as inverse dynamics or computed torque method and passivity - based controllers. Early outstanding results may be found in Ref. [1], among others. Although these control schemes have a certain degree of robustness against uncertainties, they are originally based on exact knowledge of manipulator parameters. Increasing demands on the high - performance system have led to the development of various advanced control strategies. Over the last decade, numerous papers dealing with the control of uncertain dynamical systems including robot manipulators have been published[1~19]. Since the strategies on control of uncertain systems are based on deterministic framework[2~14,19], statistical information(or stochastic approach) about the uncertainties is not required in these researches. In deterministic approach, only possible upper bounds on uncertainties are needed for the control synthesis. One useful design method for controlling uncertain dynamical systems is variable structure-type control scheme[2~13,19]. A recent survey of robust control of robot manipulators is given in Ref.[7]. An alternative approach to solving the problem of such systems is an adaptive control method which has capability of adjusting and tuning time - varying parameters. So far considerable numbers of research have been done in the field of adaptive control[1,7,15~19]. However, most of current approaches have emphasized on estimating the model parameters of system (i. e., centralized adaptive control method). Ortega and Spong[17] presents an overall review of adaptive robot control. Contrary to most existing will algorithms, a decentralized control approach will be introduced in this research. As a result, many advanced control strategies, such as robust controls and adaptive controls, suffer from one or more of the following drawbacks : (i) using discontinuous control laws, (ii) synthesizing computationally inefficient algorithms, and (iii) compensating for small system uncertainties.

The aim of the present paper is to develop a systematic design methodology for an advanced control system that overcomes the defects found in earlier design methods. The proposed control algorithm consists of two parts : the nominal control, utilizing model - based feedforward approach plus proportional - derivatives(PD) compensation, is first introduced to stabilize the system in the absence of uncertainties ; then the robust control law is synthesized to compensate for both the structured and the unstructured uncertainties in the system. Instead of updating model-parameters of robot manipulators as in the centralized adaptive control, the known nominal values of robot parameters con be used in the nominal control law. Then the resulting errors can be handled by a nonlinear robust control law. With a feasible class of desired trajectories which are continuous and bounded desired signals, the proposed control law guarantees that all responses of the closed-loop system are uniformly ultimately bounded(UUB)[2~14,19] under uncertainties.

The layout of the paper is as follows : Preliminaries and system dynamics are presented in Section 2. In Section 3, a robust controller has been proposed for uncertain system dynamics. Section 4 describes the contributions and conclusions of the work.

2. Preliminaries and Uncertain System Dynamics

Unless-mentioned otherwise, the following notations will be used throughout the study. Bold letters represent vectors or matrices and other variables are scalars. R^+ is the set of non-negative real numbers, R^n denotes an n-dimensional vector space over real-valued elements(R), $R^{n\times m}$ is the set of all real-valued $(n\times m)$ matrices, and C^p is the set of p-times continuously differentiable function. The vector norm $\|x\|$ is the Euclidean norm of vector x at time t, i. e., $\|x\| = (x^T x)^{\frac{1}{2}} = [\sum_i^n (x_i)^2]^{\frac{1}{2}}$, $\forall x \in R^n$ where $\|x\| \geq 0$ and $\|x\| = 0$ *iff* $x=0$. The matrix norm is the corresponding induced norm, i. e., for real matrix $A\in R^{n\times n}$,

$$\|A\| = [\sigma_{max}(A^T A)]^{1/2}$$

where $\sigma_{max}(\cdot)(\sigma_{min}(\cdot))$ denotes the maximum (minimum) eigenvalue of the designated matrix $(\cdot)$(if all eigenvalues are real), and $(\cdot)^T$ denotes the transpose of $(\cdot)$.

From the control point of view, the Euler-Lagrangian(EL) formulation of the system dynamics is very useful. By using this approach, an n-DOF rigid robot dynamics with revolute joints con be expressed as follows[1,13~14,18~19] :

$$M(q\,;\Theta)\ddot{q}+C(q,\dot{q}\,;\Theta)q + G(q\,;\Theta)+T_u(q,\dot{q})=T \qquad (1)$$

where q, $\dot{q}$, and $\ddot{q}\in R^n$ are the joint position, velocity, and acceleration vectors, respectively ; -$M(q\,;\Theta)\in R^{n\times n}$ is an inertia matrix ; $C(q,\dot{q}\,;\Theta)\in R^{n\times n}$ is a matrix function represention the centrifugal and Coriolis terms ; $G(q\,;\Theta)\in R^{n\times n}$ is the gravity force/torque vector ; $T_u(q,\dot{q})\in R^n$ represents the unstructured uncertainties, such as friction, unmodelled dynamics, and disturbances ; $T\in R^n$ is the joint torque(or control input) vector supplied by the actuators ; $\Theta\in R^m$ is the parameter vector of bounded system. Note that all quantites in eg. (1) are actually functions of time t. As a matter of fact, the manipulator dynamics (1) is a set of second-order, coupled, and nonlinear differential equations. It is also assumed that the robot moves in a singularity-free region of the workspace with non-redundant mechanism. From now on, some arguments of the dynamics (1) are often omitted for simple notations.

To design advanced control algorithm, one must examine the physical properties of robot model. Although the equation (1) is a complex and highly nonlinear in nature, is has some fundamental properties, which can be summarized in the following[1,6~8,13~14,16~19].

<u>(P1)</u> : $M(q\,;\Theta)$is a symmetric and positive-definite matrix(since all inertial energies are positive), i. e., $M(q\,;\Theta)\ x = M(q\,;\Theta)^T_X > 0$, $\forall$(q, Θ). Furthermore, for finite workspace, $M(q\,;\Theta)$ xand $M(q\,;\Theta)x^1$ are both differentiable matrix functions(C^∞ in q) and uniformly upper and

lower bounded by

$$\eta_l E_n \le M(q) \le \eta_u E_n \text{ and } \frac{E_n}{\eta_u} \le M(q)^{-1} \le \frac{E_n}{\eta_l},$$

for any q and Θ or in the induced matrix norm of the form

$$\delta_l \le \|M(q)\| \le \delta_u \text{ and } \frac{1}{\delta_u} \le \|M(q)^{-1}\| \le \frac{1}{\delta_l}$$

where E_n is an $n \times n$ identity matrix ; η_l, η_u, δ_l, and δ_u are all finite positive constants.

(P2) : The dynamic equation (1) is linear in the system parameters($\Theta \in R^m$) of interests

$$M(q\,;\,\Theta)\dot{x} + C(q,\dot{q}\,;\,\Theta)x + G(q\,;\,\Theta) = R(q,\dot{q},x,\dot{x})\Theta,\ \forall q,\dot{q},x,\dot{x} \in R^n$$

where $R \in R^{n \times m}$ is called the regressor matrix which is consists of known functions of the joint - space variables, and Θ is the unknown robot parameter vector which belongs to a bounded set.

Note that for a given manipulator, the dimension of Θ or the degree of m is usually greater than n. Therefore, the dimension of the parameter vector is not unique and can vary in different applications.

(P3) : $x^T(^1/_2\dot{M} - C)x = 0,\ \forall\ x \in R^n$ with $\|x\| < \infty$, that is, $(^1/_2\dot{M} - C)$ is a skew - symmetric matrix (or $\dot{M} = C + C^T$) provided that C is properly defined. Then, it can be expressed as

$$C(q,\dot{q}\,;\,\Theta) = [\dot{q}^T C_k(q\,;\,\Theta)]_{k=1,\cdots,n}$$

where $C_k \in R^{n \times n}$ is symmetric and bounded matrix for all $q \in R^n$ and defined as

$$C_k(q\,;\Theta) = \frac{1}{2}\left(\frac{\partial m_k}{\partial q} + \frac{\partial m_k}{\partial q}^T - \frac{\partial M}{\partial q_k}\right)$$

In this formulation, m_k denotes the k th column(or row) or M.

(P4) : $C(q,x)y = C(q,y)x,\ \forall (q,x,y)$. Furthermore, it is known that the norm of C satisfies $\|C\| \le \alpha_1 \|\dot{q}\|$ for any $(q, \dot{q})$, where α_1 is a positive constant.

(P5) : There exists a finite constant α_2 such that for any q, we have $\|G(q)\| \le \alpha_2$.

Note that the physical properties stated above can be easily justified for a large class of manipulators. For example, the skew - symmetric property$(^1/_2\ \dot{M} - C)$ could be obtained from the fact that the mapping from joint torgue T to joint velocity $\dot{q}$ is passive.

Now, the following assumptions are made for the system formulation.

(A1) : Each degree of freedom of the robot manipulator is powered by an independent control input or actuator.

(A2) : The nature of unstructured uncertainties(T_u) strongly influences system performance.

(A3) : The system state vectors$(q, \dot{q})$ are measurable for all $t \ge 0$; however, the information about the joint acceleration($\ddot{q}$) is not necessary.

(A4) : The model parameter vector $\Theta = [\theta_1, \cdots, \theta_m]^T$ is assumed to be unknown, but the variation of parameter θ_i is within the range $\Psi_i := [\underline{\theta}_i, \bar{\theta}_i]$, $\forall_i \in [1, m]$, where $\underline{\theta}_i$ and $\bar{\theta}_i$ are known positive constants. Therefore, $\Psi := \Psi_1 \times \Psi_2 \times \cdots \times \Psi_m$ and $\Theta \in \Psi \subset R^m$.

A class of allowable desired trajectories can be described as follows.

(A5) : For sufficiently smooth trajectories, the desired trajectory($q_d \in C^2$) and its time derivatives(assuming that inverse kinematics problem has been solve) are all continuous and uniformly bounded as :

$$d_i = \sup_{t \in R^+} \left\| d^{(i)} q_d \big/ dt^i \right\| < \infty,\ \forall i = 0, 1, 2$$

where d_0, d_1, and d_2 are some positive constants.

It should be mentioned that the above assumptions are neither restrictive nor unrealistic in robot dynamics.

Definition 1 : Let B represent the closed ball in Rn of radius $\zeta > 0$ centered at $x = 0$:

$$B_\zeta(x) = (x \in R^n : \|x\| \leq \zeta)$$

Before going further, a number of tracking error vectors are defined. $e \in R^n$ is the vector of the position tracking error which is given as $e = q - q_d$, where $q_d \in R^n$ is the desired joint position vector. Then the reference tracking error $e_r \in Rn$ is defined by $\dot{e}_r = \dot{q}_d - \wedge e$, where $\wedge \in R^{n \times n}$ is a positive-definite gain matrix chosen by the designer, $\wedge = diag(\mu)$, with $\mu > 0$.

Now define the sliding surface vector($e_s \in R^n$) as $e_s = \dot{q} - \dot{e}_r = \dot{e} + \wedge e$.

<u>Lemma 1</u> : If $\| e_s(t) \| \leq \gamma < \infty$ is satisfied for any $t \in [t_0, \infty]$ with a scalar constant γ and some t_0, then

$$\|e(t)\| \leq \exp[-\mu(t-t_0)]\left\{\|e(t_0)\| - \frac{\gamma}{\mu}\right\} + \frac{\gamma}{\mu}$$

and

$$\| \dot{e}(t) \| \leq \gamma + \mu \| e(t) \|$$

Proof : See Refs[13~14,19] for the proof.

In fact, this lemma shows that the ultimate boundednesses of $e(t)$ and $\dot{e}(t)$ can be obtained from that of e_s, that is,

$\lim_{t \to \infty} \|e(t)\| \leq \frac{\gamma}{\mu}$ and $\lim_{t \to \infty} \|\dot{e}(t)\| \leq 2\gamma$.

Moreover, if $\gamma = 0$, then

$\lim_{t \to \infty} \|e(t)\| = 0$ and $\lim_{t \to \infty} \|\dot{e}(t)\| = 0$.

With these definitions, the next step is to formulate the problem of this study. This research presents a design methodology for robot controller that guarantees the following problem statement, provided that some system states(q, $\dot{q}$) are available and also that the desired trajectories(q_d, $\dot{q}_d$, $\ddot{q}_d$) are all continuous and bounded.

<u>*Problem Statement*</u> : For the given uncertain system dynamics (1), derive realizable control law $T = h(t, q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d ; \Theta, \Theta_0)$ such that despite uncertainties, every signal in the resulting closed-loop system remains within the desired degree of accuracy in some sense, i.e., guaranteeing uniform ultimate boundednesses of tracking errors, where Θ_0 is the nominal values of the true parameter values Θ, and $h(\cdot) \in R^n$ is smooth and continuous nonlinear function.

As indicated above, the design objective is the formulate a set of control inputs so that the actual system responses track the desired quantities as closely and rapidly as possible irrespective of uncertainties. In this study, the general control structure takes the following form,

$$T = T^n + T^r, t \geq 0 \tag{2}$$

where

$T^n = M_0(q_d ; \Theta_0)\ddot{e}_r + C_0(q_d, \dot{q}_d ; \Theta_0)\dot{e}_r + G_0(q_d ; \Theta_0) - K_a e_s$; M_0, C_0, and G_0 denote the estimatesor available values of the true values M, C, and G via modeling, respectiveiy ; the gain matrices $k_a = k_a E_n$ are chosen by the designer($k_a > 0$) ; $T^r = -f(e, \dot{e}, e_s)$, where $f(\cdot)$ are some continuous and smooth nonlinear feedback functions on(e, $\dot{e}$, e_s),over vector fields in R^n. Thus, the control algorithm consists of the nominal control vector, T^n, which is a modelbased feedforward scheme plus proportional-derivative (PD) control law to stabilize the nominal system in the absence of the uncertainties, and the robust control law, T^r, which is used to compensate for both the compensation error and unstructured uncertainties. This twostage control scheme is intended to achieve better robustness and tracking performance to signifincant uncertainties. The torque computation in the model-based portion can be performed off-line since the desired trajectories (q_d, $\dot{q}_d$, $\ddot{q}_d$) and the nominal values of system parameters Θ_0 are known in advance, while many other methods rely heavily on the on-line computations. Note that the quantities M_0, C_0, and G_0 are not updated on-line in this algorithm. The nonlinear structure of robust control T^r will be specified later. In the case of $M_0=C_0=G_0=[0]$ ineg. (2), the control structure is simply reduced to $T=-k_a e_s+T^r$. Therefore the control scheme presented here is very general form and can be applied to a wide variety of important systems for trajectory tracking purposes.

Remark : Even if the true values($\Theta \in R^m$) are unavailable, the possible ranges of parameter variations may be given. For example, the nominal value may be selected as $\theta_{0i}={}^1/_2(\underline{\theta}_i+\bar{\theta}_i)$, i. e., the mean value of admissible range of θ_i, for designer's convenience. That is, instead of using the true parameter values which are unknown, the proposed control law(feedforward term) is based on the nominal values Θ_0 which are known fixed values.

After substituting eg. (2) into eg. (1) and subtracting $M\ddot{e}_r+C\dot{e}_r+G$ on both sides of the resulting equation, the error dynamics under the control law (2) can be expressed in the general form as

$$M(q\ ;\ \Theta)\dot{e}_s=-C(q,\dot{q}\ ;\ \Theta)e_s-(T_u+\Delta R_r)-k_a e_s+T^r, \quad (3)$$

where ΔR_r, is such that

$$\Delta R_r=[M(q\ ;\ \Theta)-M_0(q_d\ ;\ \Theta_0)]\ddot{e}_r+[C(q,\dot{q}\ ;\ \Theta)-C_0(q_d,\dot{q}_d\ ;\ \Theta_0)]\dot{e}_r+[G(q\ ;\ \Theta)-G_0(q_d\ ;\ \Theta_0)] \quad (4)$$

In this formulation, ΔR_r represents the structured uncertainties due to compensation eror and may be zero for the complete model-following of the system. However, in most real applications, the estimated parameters always differ from the actual ones, in other words, $\Delta R_r \neq 0$.

The first step in robust controller design is to examine the possible bounds of ΔR_r and T_u. As shown later, both ΔR_r and T_u are assumed to be bounded in magnitude, usually in their Euclidean norms. In order to develop the bound of ΔR_r, we first introduce the following assumptions.

<u>(A6)</u> : There exist nonnegative constants ρ_{11}, ρ_{12}, ρ_{13}, and ρ_{14} such that

(i) $\rho_{11}:=\sup_{\Theta\in\Psi}\ \sup_{(q,q_d)\in R^n\times R^n}\|M(q;\Theta)-M_0(q_d;\Theta_0)\|$

(ii) $\|C(q,\dot{q};\Theta)-C_0(q_d,\dot{q}_d;\Theta_0)\|\le\rho_{12}\|\dot{q}\|+\rho_{13}\|\dot{q}_d\|$

in which

$$\rho_{12}:=\sup_{\Theta\in\Psi}\ \sup_{q\in R^n}\sum_{k=1}^{n}\|C_k(q;\Theta)\|$$

and $\rho_{13}:=\sup_{\Theta_0\in\Psi}\ \sup_{q_d\in R^n}\sum_{k=1}^{n}\|C_{0k}(q_d;\Theta_0)\|$

(iii) $\rho_{14}:=\sup_{\Theta\in\Psi}\ \sup_{(q,q_d)\in R^n\times R^n}\|G(q;\Theta)-G_0(q_d;\Theta_0)\|$.

Based on the above observations, the following lemma provides the possible upper bound on ΔR_r.

<u>Lemma 2</u> : The structured uncertainies ΔR_r is bounded in the form

$$\|\Delta R_r\|\le c_0+c_1\|e\|+c_2\|\dot{e}\|+c_3\|e\|\|\dot{e}\|,$$

where $c_i(i=0,1,2,3)$ are known finite constants that depend on the size of parametric variations as well as the upper bounds of the desired trajectories.

Proof : The proof of this lemma can be found in Appendix.

Now, the following assumption is made on the uncertainties(T_u).

<u>(A7)</u> : The unknown function $T_u(t, e, \dot{e}) : R^1\times R^n\times R^n \to R^n$ is bounded by

$$\|T_u\|\le d_0+d_1\|e\|+d_2\|\dot{e}\|+d_3\|e\|\|\dot{e}\|,$$

where $d_i(i=0,1,2,3)\in R^+$ are known constants.

As stated before, the robust control(T^r) is intended to cope with the total uncertainties(T_u

3. Synthesis of Robust Control Law

Given the general structure of control corresponding to equation(2), a methodology for the design of a control law T^r will be presented in this section.

In order to fulfill the requirements of designing a robust controller, *a priori* knowledge on possible bounds(in Euclidean norm) of the uncertainties is required.

Now, the control law T^r takes the following polynomial - type form

$$T^r = -k_c \| e \|^2 e_s \tag{5}$$

Then, the complete control law can be expressed as

$$T = M_0(q_d\,;\,\Theta_0)\ddot{e}_r + C_0(q_d, \dot{q}_d\,;\,\Theta_0)\dot{e}_r + G_0(q_d\,;\,\Theta_0) - k_a e_s - k_c \| e \|^2 e_s \tag{6}$$

where the control gain $k_c \in R^{n \times n}$ may be selected as diagonal matrix, that is, $k_c = k_c E_n$, with $k_c > 0$.

The stability and the tracking properties of the closed - loop system(3) with the control law (5) are analyzed in the following theorem.

Theorem 1 : Consider the closed loop system dynamics(3) with known coefficients on the uncertainty bounds. Then the solutions $(e(t), e_s(t))$ under the control law(5) requiring only position and velocity measurements are uniformly ultimately bounded. That is, there sxists the compact set Ω_f such that for all$(e(0), e_s(0)) \in \bar{\Omega}$, the system responses globally converge to the following compact(residual) set :

$$\Omega_f = \{(t, e, e_s) \in R^+ \times R^n \times R^n : V(t, e, e_s) \le V_f\}$$

with its ultimate bound $V_f = \frac{\gamma_3}{\gamma_0}$, where the set Ω_f is a subset of $\bar{\Omega}$(i.e., $\Omega_f \subset \bar{\Omega}$).

Proof : Consider a Lyapunov function candidate(a C^1 function), $V(\bullet) : R^+ \times R^n \times R^n \to R^+$, as

$$V = \frac{1}{2} e_s^T M(q) e_s + \frac{1}{2} e^T F e \tag{7}$$

where $F = \varepsilon E_n$, $\varepsilon > 0$. For a real, symmetric, and positive - definite matrix $A \in R^{n \times n}$, the following inequality can be established by using Rayleigh's Principle

$$\sigma_{min}(A) \| x \|^2 \le x^T A x \le \sigma_{max}(A) \| x \|^2, \ \forall x \in R^n$$

Based on the above relation, it is clear that the bounds of the Lyapunov function(7) can be estimated as

$$\frac{1}{2}\min\{\sigma_{min}(M), \varepsilon\}[\|e_s\|^2 + \|e\|^2] \le \frac{1}{2}\sigma_{min}(M)\|e_s\|^2 + \frac{1}{2}\varepsilon\|e\|^2 \le V \le \frac{1}{2}\sigma_{max}(M)\|e_s\|^2 + \frac{1}{2}\varepsilon\|e\|^2 \le \frac{1}{2}\max\{\sigma_{max}(M), \varepsilon\}[\|e_s\|^2 + \|e\|^2]$$

which implies that V is clearly a legitimate Lyapunov function candidate(since it is positive definite) and further radially unbounded scalar function, i.e., $V \to \infty$ as $\| e_s \| \to \infty$ and $\| e \| \to \infty$.

The time derivative of V along the equation (3) is given by

$$\begin{aligned} \dot{V} &= e_s^T M \dot{e}_s + \frac{1}{2} e_s^T \dot{M} e_s + e^T F \dot{e} \\ &= e_s^T\{-C(q,\dot{q})e_s - (T_u + \Delta R) - [k_a + k_c\|e\|^2]e_s\} \\ &\quad + \frac{1}{2} e_s^T \dot{M} e_s + e^T F \dot{e} \end{aligned} \tag{8}$$

By utilizing(P3), we obtain the upper bound on the time derivative of the Lyapunov function as

$$\begin{aligned} \dot{V} &\le \|e_s\|\{(c_0 + d_0) + (c_1 + d_1)\|e\| + (c_2 + d_2)\|\dot{e}\| \\ &\quad + (c_3 + d_3)\|e\|\|\dot{e}\|\} - \{k_a + k_c\|e\|^2\}\|e_s\|^2 \\ &\quad + e^T F(e_s - \wedge e) \end{aligned}$$

which leads to

$$\begin{aligned} \dot{V} &\le (c_0 + d_0)\|e_s\| + (c_1 + d_1)\|e\|\|e_s\| + (c_2 + d_2)\|e_s\|\|\dot{e}\| \\ &\quad + (c_3 + d_3)\|e\|\|e_s\|\|\dot{e}\| - k_a\|e_s\|^2 - k_c\|e\|^2\|e_s\|^2 \\ &\quad + \varepsilon\|e\|\|e_s\| - \varepsilon\mu\|e\|^2. \end{aligned} \tag{9}$$

For futher simplification, use inequality $\| \dot{e} \| \le \| e_s \| + \mu \| e \|$, then eg. (9) satisfies

$$\dot{V} \le (c_0+d_0)\|e_s\| + (c_1+d_1)\|e\|\|e_s\| + (c_2+d_2)\|e_s\| \{\|e_s\|+\mu\|e\|\} + (c_3+d_3)\|e\|\|e_s\|\{\|e_s\|+\mu\|e\|\} - k_a\|e_s\|^2 - k_c\|e\|^2\|e_s\|^2 + \varepsilon\|e\|\|e_s\| - \varepsilon\mu\|e\|^2$$

Further manipulation yields

$$\dot{V} \le (c_0+d_0)\|e_s\| + (c_1+d_1)\|e_s\| + (c_1+d_1)\|e\|\|e_s\| + (c_2+d_2)\|e_s\|^2 + (c_2+d_2)\mu\|e\|\|e_s\| - k_a\|e_s\|^2 - k_c\|e\|^2\|e_s\|^2 + \varepsilon\|e\|\|e_s\| - \varepsilon\mu\|e\|^2 \quad (10)$$

After grouping terms, the inequality eg.(10) can be rewritten as

$$\dot{V} \le \{-k_a + (c_2+d_2)\}\|e_s\|^2 - k_c\|e\|^2\|e_s\|^2 + (c_2+d_2)^2\|e\|\|e_s\|^2 + (c_3+d_3)\mu\|e\|^2\|e_s\| + \{(c_1+d_1) + (c_2+d_2)\mu + \varepsilon\}\|e\|\|e_s\| - \varepsilon\mu\|e\|^2 + (c_0+d_0)\|e_s\| \quad (11)$$

Completing squares and regrouping terms by using inequality $abc \le \frac{1}{4}b^2 + a^2c^2 (a, b, c \in R^+)$ gives

$$\dot{V} \le -\left\{\frac{k_a}{2} - (c_2+d_2) - \frac{(c_3+d_3)^2}{2k_c} - [(c_1+d_1) + (c_2+d_2)\mu+\varepsilon]^2\right\}\|e_s\|^2 - \left\{\varepsilon\mu - \frac{\mu^2(c_3+d_3)^2}{2k_c} - \frac{1}{4}\right\} \|e\|^2 + \frac{(c_0+d_0)^2}{2k_a} - \frac{k_c}{2}\|e_s\|^2\left\{\|e\| - \frac{(c_3+d_3)}{k_c}\right\}^2 \frac{k_c}{2}\|e\|^2\left\{\|e_s\| - \frac{(c_3+d_3)\mu}{k_c}\right\}^2 - \frac{k_a}{2}\left\{\|e_s\| - \frac{(c_0+d_0)}{k_a}\right\}^2 \quad (12)$$

Dropping the last three negative terms in eg. (12) yields

$$\dot{V} \le -\left\{\frac{k_a}{2} - (c_2+d_2) - \frac{(c_3+d_3)^2}{2k_c} - [(c_1+d_1) + (c_2+d_2)\mu+\varepsilon]^2\right\}\|e_s\|^2 - \left\{\varepsilon\mu - \frac{(c_3+d_3)^2\mu^2}{2k_c} - \frac{1}{4}\right\} \|e\|^2 + \frac{(c_0+d_0)^2}{2k_a} \quad (13)$$

Letting

$$\gamma_1 = \frac{k_a}{2} - (c_2+d_2) - \frac{(c_3+d_3)^2}{2k_c} - [(c_1+d_1) + (c_2+d_2)\mu+\varepsilon]^2$$

$$\gamma_2 = \varepsilon\mu - \frac{(c_3+d_3)^2\mu^2}{2k_c} - \frac{1}{4}$$

$$\gamma_3 = \frac{(c_0+d_0)^2}{2k_a}$$

where γ_1, γ_2, and γ_3 can be positive constants with proper choices of design parameters(k_a, k_c, and μ). Then, the differential inequality eg. (13) can be expresed in more compact form as

$$\dot{V} \le -\gamma_1\|e_s\|^2 - \gamma_2\|e\|^2 + \gamma_3 \quad (14)$$

Let $\gamma_0 = \min\left\{\frac{2\gamma_1}{\sigma_{\max}(M)}, \frac{2\gamma_2}{\varepsilon}\right\}$, then

$$\dot{V} \le -\gamma_0 V + \gamma_3 \quad (15)$$

Therefore, for $V_f = \frac{\gamma_3}{\gamma_0} \ge 0$, one has $V<0$ if $V>V_f$(or $V \subset \Omega_f^c$), where Ω_f^c denotes the complement of Ω_f, i. e., $\lim_{t\to\infty}\int_0^t V(\tau)d\tau = V_f - V_0$ and $|V_f - V_0| < \infty$ with $V_0 = V(0, e(0), e_s(0))$.

A detailed solution of eg. (15) for all $t \ge 0$ can be expressed as

$$V \le \exp(-\gamma_0 t)\left\{V_0 - \frac{\gamma_3}{\gamma_0}\right\} + \frac{\gamma_3}{\gamma_0}, \ t \ge 0 \quad (16)$$

where the function $V(t, e, e_s)$ decreases monotonically at rate of $\exp(-\gamma_0 t)$ until the solution reaches the target ball(or the residual set) Ω_f in some finite time. Consequently, V is uniformly bounded and its ultimate bound is given as $0 \le \lim_{t\to\infty} V = \inf_t V = V_f \le V_0 < \infty$. In other words, the system state variables are also uniformly bonuded for all $t \ge 0$. Moreover, the norm bounds of tracking errors(e and e_s) given by

$$\|e_s\| \le \frac{1}{\sqrt{\sigma_{\min}(M)}}\left\{\exp(-\gamma_0 t)\left[V_0 - \frac{\gamma_3}{\gamma_0}\right] + \frac{\gamma_3}{\gamma_0}\right\}^{\frac{1}{2}} \quad (17)$$

$$\|e\| \le \frac{1}{\sqrt{\varepsilon}}\left\{\exp(-\gamma_0 t)\left[V_0 - \frac{\gamma_3}{\gamma_0}\right] + \frac{\gamma_3}{\gamma_0}\right\}^{\frac{1}{2}} \quad (18)$$

The above results also imply that the system responeses converge th the closed ball as $t \to \infty$:

$$B(e)=\left\{e\in R^n:\|e\|\le\frac{1}{\sqrt{\varepsilon}}\left(\frac{\gamma_3}{\gamma_0}\right)^{\frac{1}{2}}\right\}$$

and $$B(e_s)=\left\{e_s\in R^n:\|e_s\|\le\frac{1}{\sqrt{\sigma_{\min}(M)}}\left(\frac{\gamma_3}{\gamma_0}\right)^{\frac{1}{2}}\right\}$$

As a consquence, the global uniform ultimate boundedness results of all signals are established with respect to V_f in this design. In other words, the Euclidean norms of tracking errors never leave the closed ball after a finite time. One can manipulate the design parameters to determine the size of the residual set. From the practical point of view, the system designer should determine the trade - off between the minimization of size of the residual set(or better system performances) and practical control gains(or large control energy).

Remark : From Lemma 1, the global boundedness of the sliding surface vector(e_s) guarantees that of $\dot{e}$.

4. Conclusions

This paper has presented dynamic compensation methodology for motion control of uncertain robot system. A model - based feedforward scheme with continuous nonlinear robust controller has been proposed to meet design objective. Furthermore, the controller can be implemented in a decentralized manner. Stability and robustness issues have been investigated extensively and rigorously by the Lyapunov theory. The outstanding contributions of the proposed control algorithm are summarized as follows : (1) the presented control law do not require the exact information about the system parameters and dynamics ; (2) the joint acceleration is not required in the control law ; (3) the torque computations in the model - based portion can be calculated off - line if the desired trajectories and the nominal values of dynamic parameters are known in advance. This implies high promises for real - time control applications ; (4) the proposed control law can guarantee the global uniform ultimate boundedness results for the system responses under uncertainties.

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Proof of Lemma 2 :

The upper bound of the modeling error ΔR_r, can be estimated as follows.

Taking the norms on both sides of (4) gives

$$\| \Delta R_r \| \leq \| M - M_0 \| \| \dot{e}_r \| + \| C - C_0 \| \| e^r \| + \| G - G_0 \| .$$

Now, with (A5) and (A6), the following inequality is obtained

$$\| \Delta R_r \| \leq \rho_{11} \| \dot{e}_r \| + (\rho_{12} \| q \| + \rho_{13} \| q_d \|) \| e_r \| + \rho_{14} \leq \rho_{11}(\| q_d \| + \mu \| e \|) + [\rho_{12}(\| q_d \| + \| e \|) + \rho_{13} \| q_d \|] (\| q_d \| + \mu \| e \|) \rho_{14} \leq [\rho_{11} d_3 + (\rho_{12} + r_{13}) d_2^2 + \rho_{14}] + (\rho_{12} + r13)md2 \| e \| + (\rho_{11}\mu + \rho_{12} d_2) \| e \| \mu\rho_{12} \| e \| \| e \|$$

Then, it follows that

$$\| \Delta R_r \| \leq c_0 + c_1 \| e \| + c_2 \| e \| + c_3 \| e \| \| e \| ,$$

where

$c_0 = \rho_{11} d_3 + (\rho_{12} + \rho_{13}) d_2^2 + \rho_{14}$, $c_1 = (\rho_{12} + \rho_{13}) \mu d_2$

$c_2 = \rho_{11}\mu + \rho_{12} d_2$, $c_3 = \mu\rho_{12}$

The proof is completed.