Derivation of Design Flood by Transformation Method(I)

-On the annual maximum series-

變換法에 의한 設計洪水量의 誘導(I)

-年最高値系列을 中心으로-

Lee, Soon Hyuk · Maeng, Sung Jin
李 淳 赫* · 孟 昇 辰**

摘 要

設計洪水量의 誘道를 위해 一次的으로 極値系列인 錦江, 榮山江 및 蟾津江 水系 3個 流域의 年最高値系列을 中心으로 하여 SMEMAX法, 冪變換(Power Transformation) 및 2段階冪變換(Two Step Power Transformation, TSPT)法에 의해 頻度分布의 正規化를 위한 變換法의 效率性 檢定과 設計洪水量의 誘導 및 比較分析을 수행한 結果 SMEMAX 法과 Power 變換法에서는 正規分布化를 위한 歪曲度係數의 零으로의 接近試圖는 可能하나 尖銳度, 3으로의 條件을 充足시킬 수 없었던 반면 2段階冪變換法에서는 尖銳度, 3에 매우 近接된 結果를 가져오므로서 頻度分布의 正規化를 期할 수 있었고 各各의 變換法에 의해 誘導된 設計洪水量간의 比較分析에서는 SMEMAX 및 Power 變換法에 비해 2段階冪變換法에 의한 設計洪水量이 再現期間 10年 以上에서 實測値에 보다 接近된 좋은 結果를 나타내었다.

I. Introduction

Estimation of design flood has an important role for the rational design of hydraulic structures such as dams, spillways and bridges. Extreme events for getting design flood including annual maximum series can rarely be normally, but is usually distributed asymmetrically. Data from extreme value series form their own distribution. Thus, it is better to find the best fitting distribution for the given data instead of fitting a known dis-

* Professor, Department of Agricultural Engineering, Chungbuk National University
** Research Assistant, Department of Agricultural Engineering, Graduate School of Chungbuk National University

키워드 : SMEMAX 變換, 冪變換, 2段階冪變換, 歪曲度係數, 尖銳度, Weibull 프롯팅, 設計洪水量

tribution to the data. Alternatively, the given data could be reconstituted by some transformations such that transformed series follows a particular distribution.

Transformation methods such as SMEMAX[10]and power transformation[14] have been developed for estimation of design flood by the normalization of skewed distribution. But, those are not truly normalized. Because, it was proved that by these methods the coefficient of skewness can be reduced to a value near to zero, but the kurtosis is not to a value of three.

Gupta et al[5] proposed recently two step power transformation method that satisfy the conditions above mentioned for the normal distribution. Consequently, this study is mainly conducted to estimate rational design flood by comparison of the results of two step power transformation with those computed by SMEMAX and power transformations.

II. Transformation Method

1. Power and Two Step Power Transformation(TSPT)

Box and Cox[3] have suggested the following transformation which is called as power transformation for normalizing the skewed flood series:

$$Y_i = \frac{X_i^{\lambda} - 1}{\lambda}, \text{ in which } \lambda \neq 0 \quad \cdots\cdots(1)$$

where X_i =ith variate of a given series, i. e. original skewed flows, Y_i =ith variate of power transformation series, and λ=parameter of power transformation such that Y_i have zero skewness. Transformation equation (1) hold for $X_i > 0$.

The parameter, λ may be estimated by trial and error method such that the skewness of the transformed series is reduced to zero or nearly zero. The value of λ generally ranges from -1.0 to 1.0. The histogram of the transformed series which the skewness is reduced to zero or nearly zero becomes symmetrical about the vertical axis passing through the mean. But, the transformed distribution can be truly normalized when the kurtosis becomes to three or nearly three. The distribution is said to be leptokurtic or platykurtic when the kurtosis is greater than or less than three respectively.

This trend is much helpful for the estimation of λ. It is generally seemed that the skewness of the transformed annual maximum series becomes zero but the value of kurtosis is not three in the power transformation. Power transformation in this study is required so that the given annual maximum series can be truly normalized with the coefficient of skewness of zero and kurtosis value of three. Therefore, a further transformation procedure of power transformation is needed to get true normalization of skewed data. This double transformation is called as two step power transformation in this study. The correction of kurtosis for making the value of three can be achived through another transformation keeping the coefficient of skewness to be equalled to zero or become nearly to zero in this two step approach. The two limbs of the transformed distribution obtained by power transformation of the first step are equally stretched or reduced by the following transformation:

$$T_i = (\mid Y_i - \overline{Y} \mid)^{\gamma} \quad \cdots\cdots (2)$$

where γ=parameter of TSPT, T_i =ith variate of TSPT series, $\overline{Y}$=mean of power transformed series. The value of the kurtosis to be equalled or approached nearly to three may be acquired by the suitable value of γ.

Algorithm value of λ and γ, parameters of TSPT should be obtained by iteration such that both the skewness and kurtosis of the transformed series tends to be zero and three respectively. When the normal distribution is obtained by two step power tansformation, flood peaks according to the return period can be calculated by the following equation:

$$T_p = \overline{T} + K_p \cdot S_t \cdots\cdots (3)$$

where T_p =flood peaks according to pth return period in the transformed series, K_p = frequency factor according to the pth return period, and S_t =standard deviation of the transformed series.

X_p, the value corresponding to pth return period in a given series may be calculated by using the following equations:

$$X_p = (\lambda \cdot Y_p + 1)^{1/\lambda} \cdots\cdots (4)$$

where

$$Y_p = \overline{Y} + (T_p)^{1/\gamma} \cdots\cdots (5)$$

where X_p =flood peaks according to pth return period in a given series, Y_p =value corresponding to pth return period in transformed series.

A flow chart and program with functions for two step power transformation are shown in Fig. 1 and Table-1 respectively in this

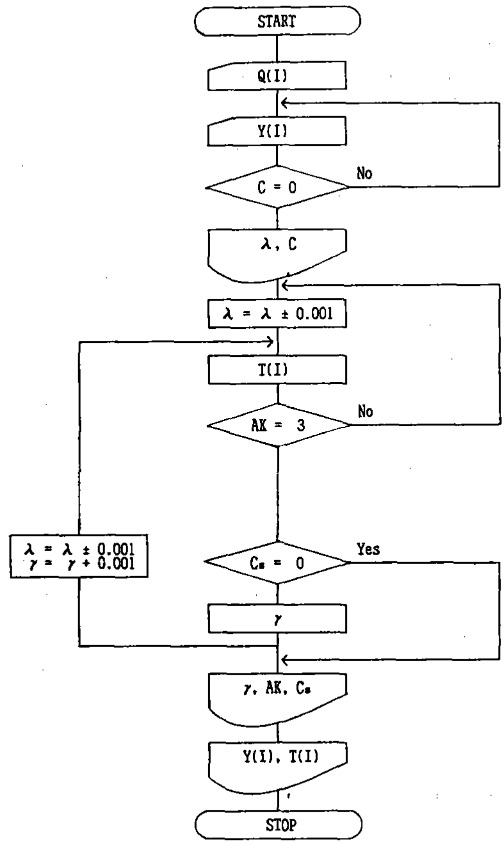

Fig. 1. Flow chart of two step power transformation

Table-1. Program and it's description of variables and parameters for two step power transformation

Item	Description
Q(I)	Flood peaks of given data
Y(I)	Variate of transformed series
λ	Parameter of power transformation
C	Coefficient of skewness for Y(I)
T(I)	Variate of two step power transformed series
γ	Parameter of two step power transformation
AK	Kurtosis for T(I)
C_s	Coefficient of skewness for T(I)

Table-2. Gauging stations and watersheds physical characteristics

River	Station	Area (km^2)	Length of main stream (km)	Average basin width	Shape factor	Observed duration (yrs)	Location
Geum	Yong Dam	989.5	12.0	82.46	6.872	29	Long. 127° 37′ Lati. 35° 58′
Yeong San	Nam Pyeong	576.2	41.1	14.02	0.341	31	Long. 126° 51′ Lati. 35° 03′
Seom Jin	Song Jeong	4255.7	185.7	22.92	0.123	31	Long. 127° 34′ Lati. 35° 11′

Table-3. Basic statistics

River	Station	Mean($\bar{X}$)	Standard deviation(S)	Coefficient of skewness(C_s)	Kurtosis(C_k)
Geum	Yong Dam	682.2	356.1	0.692	3.269
Yeong San	Nam Pyeong	359.6	133.0	0.753	3.514
Seom Jin	Song Jeong	2849.9	1034.6	1.122	4.737

study.

2. SMEMAX transformation[10,13]

SMEMAX transformation transforms a skewed annual maximum series for normalization using the smallest, median and largest value of the given series. SMEMAX is based on the median value, a relatively stable figure that is not affected by chance outliers. But, the resulting series can still have appreciable skewness, even though median value is equidistant from the smallest and the largest value.

III. Data used for application

Three stations selected for application are Yong Dam, Nam Pyeong and Song Jeong of Geum, Yeong San and Seom Jin rivers respectively in Korea. The data used are the annual maximum series at three selected stations. Physical characteristics of the given basins are shown on Table-2.

IV. Rusults of Analysis

1. Basic statistics

Basic statistics of observed annual maximum series from applied stations are shown in Table-3.

Those were within the range of 133.0 to 1034.6, 0.692 to 1.122 and 3.269 to 4.737 for standard deviation, coefficient of skewness and kurtosis respectively.

2. Efficiency of transformation

Efficiency of transformation for the normalization of frequency distribution can be judged by checking whether the coefficient of skewness and kurtosis tend to zero and three respectively. It can be seen from Table-4 that SMEMAX transformation reduces the skewness and kurtosis in comparison with those of original data but still cannot be considered as a normally distributed series and in power transformation the skewness could be bro-

Table-4. Effect of different transformations on coefficient of skewness and kurtosis

River	Station	Observed duration (yrs)	Original data		SMEMAX transformed data		Power transformed data			TSPT transformed data			
			Skewness	Kurtosis	Skewness	Kurtosis	Lambda (λ)	Skewness	Kurtosis	Lambda (λ)	Gamma (γ)	Skewness	Kurtosis
Geum	Yong Dam	29	0.692	3.269	0.0988	2.4376	0.558	-0.000479	2.6128	0.315	0.4728	0.0097599	3.0199
Yeong San	Nam Pyeong	31	0.753	3.514	0.1225	3.2341	0.479	0.000016	3.4152	0.489	0.1993	0.0098407	3.0331
Seom Jin	Song Jeong	31	1.122	4.737	0.1191	3.2218	0.234	0.000377	3.3973	-0.720	0.3909	0.0098139	3.0003

ught to closer zero to a satisfactory degree, but the kurtosis are increased to some degree than those of SMEMAX. Thus, the power transformed series also cannot be considered as really normalized. Two step power transformation of original data series was attempted for both the skewness and the kurtosis could be brought as close to zero and three respectively in this study. In this method the value of λ obtained in power transformation was used as initial trial value and thereafter it was subsequently modified with determination of γ iteratively. Consequently, it is proved that two step power transformation can transform an annual maximum series with any skewness and kurtosis into a really normal distribution one.

3. Flood flows prediction by SMEMAX

1) Basic statistics

Basic statistics calculated by SMEMAX transformation are within the range of 233.3 to 1775.5, 94.59 to 786.95, 0.0988 to 0.1225 and 2.4376 to 3.2341 for the mean, standard deviation, coefficient of skewness and kurtosis respectively as shown in Table-5.

2) Flood flows prediction

The flood flows of different return periods were estimated by SMEMAX transformation for applied stations as shown in Table-6.

Table-5. Basic statistics calculated by SMEMAX transformed data

River	Station	Mean	Standard deviation	Coefficient of skewness	Kurtosis
Geum	Yong Dam	526.4	270.51	0.0988	2.4376
Yeong San	Nam Pyeong	233.3	94.59	0.1225	3.2341
Seom Jin	Song Jeong	1775.5	786.95	0.1191	3.2218

Table-6. Flood flows prediction calculated by SMEMAX transformation

River	Station	Return Period(yrs)					
		5	10	20	50	100	200
Geum	Yong Dam	975	1179	1348	1537	1664	1781
Yeong San	Nam Pyeong	471	542	600	666	710	750
Seom Jin	Song Jeong	3718	4329	4833	5400	5778	6130

4. Flood flows prediction by power transformation

1) Basic statistics

Basic statistics calculated from transformed data by power transformation are within the range of 22.91 to 64.16, 2.29 to 20.36, -0.000479 to 0.000377 and 2.6128 to 3.4152 for the mean, standard deviation, coefficient of skewness and kurtosis respectively as shown in Table-7.

2) Flood flows prediction

The flood flows of different return periods were estimated by power transformation for applied stations as shown in Table-8.

Table-7. Basic statistics calculated by power transformed data

River	Station	Mean	Standard deviation	Coefficient of skewness	Kurtosis
Geum	Yong Dam	64.16	20.36	-0.000479	2.6128
Yeong San	Nam Pyeong	32.31	6.23	0.000016	3.4152
Seom Jin	Song Jeong	22.91	2.29	0.000377	3.3973

Table-8. Flood flows prediction calculated by power transformation

River	Station	Return Period(yrs)					
		5	10	20	50	100	200
Geum	Yong Dam	968	1163	1336	1543	1688	1829
Yeong San	Nam Pyeong	467	537	598	671	723	772
Seom Jin	Song Jeong	3644	4215	4737	5381	5847	6307

5. Flood flows prediction by two step power transformation

1) Basic statistics

Basic statistics calculated from two step power transformed data are within the range of 0.444 to 1.693, 0.162 to 0.631, -0.720 to 0.489, 0.1993 to 0.4728, 0.0097599 to 0.0098407 and 3.0003 to 3.0331 for the mean, standard deviation, coefficient of transformation λ, coefficient of transformation γ, coefficient of skewness and kurtosis respectively as shown in Table-9.

Table-9. Basic statistics calculated by two step power transformed data

River	Station	Mean	Standard deviation	Coefficient of transformation (λ)	Coefficient of transformation (γ)	Coefficient of skewness	Kurtosis
Geum	Yong Dam	1.693	0.631	0.315	0.4728	0.0097599	3.0199
Yeong San	Nam Pyeong	1.328	0.206	0.489	0.1993	0.0098407	3.0331
Seom Jin	Song Jeong	0.444	0.162	-0.720	0.3909	0.0098139	3.0003

Table-10. Flood flows prediction calculated by two step power transformation

River	Station	Ruturn Period(yrs)					
		5	10	20	50	100	200
Geum	Yong Dam	1175	1380	1591	1882	2114	2361
Yeong San	Nam Pyeong	517	583	654	758	845	941
Seom Jin	Song Jeong	4161	4868	5673	6929	8057	9398

Table-11. Comparison of design flood calculated by SMEMAX, Power and Two step power transformation methods

River	Station	Method	Return Period(yrs)					
			5	10	20	50	100	200
Geum	Yong Dam	SMEMAX	975	1179	1348	1537	1664	1781
		Power	968	1163	1336	1543	1688	1829
		TSPT	1175	1380	1591	1882	2114	2361
Yeong San	Nam Pyeong	SMEMAX	471	542	600	666	710	750
		Power	467	537	598	671	723	772
		TSPT	517	583	654	758	845	941
Seom Jin	Song Jeong	SMEMAX	3718	4329	4833	5400	5778	6130
		Power	3644	4215	4737	5381	5847	6307
		TSPT	4161	4868	5673	6929	8057	9398

2) Flood flows prediction

The flood flows of different return periods were calculated by two step power transformation method for applied stations as shown in Table-10.

6. Comparison of design floods calculated by different transformation methods

Design floods of different return periods were calculated by SMEMAX, Power and Two step power transformation methods as shown in Table-11.

Comparing the relative suitabilities of the transformation methods including SMEMAX, Power and Two step power transformations, these results are plotted along with the observed data on a normal probability paper as shown in Fig. 2 to Fig. 4.

The plotting position is based on the weibull formula in which the probability of nonexceedance is calculated as $p=1-m/n+1$, in which n is the sample size and m is the rank commencing with the largest value. It can be seen from Fig. 2 to Fig. 4 that the design floods based on two step power transformation are much closer to the observed data in comparison with the other methods at the return period of more than ten years.

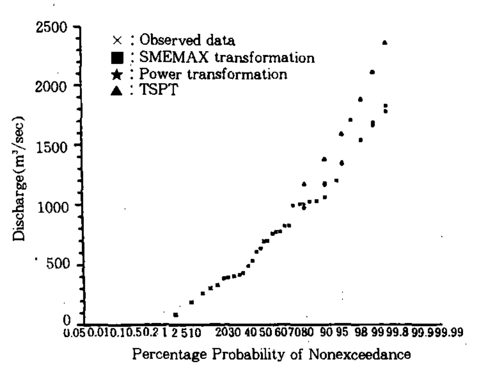

Fig. 2. Comparison of design floods flows at Yong Dam watershed in the Geum river, Korea

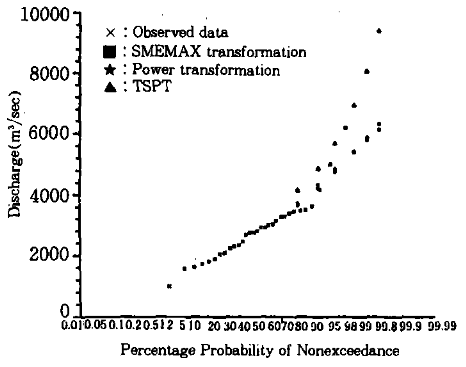

Fig. 3. Comparison of design floods at Nam Pyeong watershed in the Yeong San river, Korea

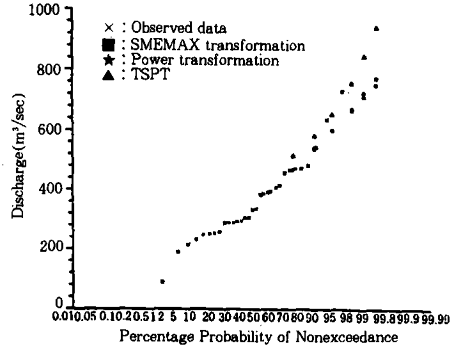

Fig. 4. Comparison of design floods at Song Jeong watershed in the Seom Jin river, Korea

V. Conclusions

It was attempted to get suitable design floods of different return periods can be estimated by means of two step power transformation which is more efficient for the normalization of frequency distribution than any other transformation methods in making the coefficient of skewness and the kurtosis nearer to zero and three respectively and to compare with the results computed by SMEMAX and power transformation methods with the annual maximum series of three stations along Geum, Yeong San and Seom Jin rivers in Korea. The results were analyzed and summarized as follows.

1. Two step power transformation has found to be the best one in comparison with SMEMAX and Power transformation for bring the skewness and the kurtosis into closer zero and three respectively as a means of getting the normalization of frequency distribution.

2. Design floods were derived by SMEMAX, Power and Two step power transformation methods for the applied watersheds.

3. Judging by the relative suitabilities of the three transformation methods, it was confirmed that the design floods calculated using Two step power transformation are nearer to the observed data as compared with other methods at the return period of more than ten years.

Acknowledgement

"This papar was supported by RESEARCH FUND for Institute attached to University, Korea Research Foundation, 1993."

References

1. Benjamin, J. R. and C. A. Cornell(1970), Probability, Statistics, and Decision for civil Engineers, McGraw-Hill, New York, pp. 370-500, English.
2. Bethlahmy, N. (1977). Flood Analysis by SMEMAX Transformation, Journal of the Hydraulic Division, ASCE, Vol. 103, No. HY1. Proc. Paper 12701 Jan., pp. 69-

80, English.

3. Box, G. E. P. and D. R. Cox(1964), An analysis of transformation, Journal of the Royal Statistical Society, Vol. B 26, pp. 211-252, English.
4. Emir Zelenhasic(1970), Theoretical probability distribution for flood peaks, Hyd. Papers, No. 42, C. S. U., pp. 1-21, English.
5. Gupta, D. K., B. N. Asthana and A. N. Bhargawa(1987), Application of Frequency and Risk in Water Resources, D. Reidel publishing Co., pp. 101-111, English.
6. Ministry of Construction (1962-1978), Hydrological Investigation of Korean Watersheds (Rainfall and Water Stage), Korean.
7. Ministry of Construction(1964), An Annual Report of Hydrological Investigation in Korea, Korean.
8. Ministry of Construction(1974), Watershed Investigation Report in Korea, Korean.
9. Ministry of Construction(1979-1982), An Annual Report of Hydrological Investigation in Korea, Korean.
10. Lee, S. H. and M. K. Park(1985), Flood frequency analysis by SMEMAX transformation Journal of Chung Buk National University, Vol. 30, pp. 165-175, Korean.
11. Lee, S. H. and M. K. Park(1985), Hydrology studies on the design flood and risk of failure of the Hydraulic structure (I) (on the annual maximum series), Journal of the Korean Society of Agricultural Engineers, Vol. 27(2), pp. 23-37, Korean.
12. Lee, S. H., S. P. Hong and M. K. Park (1988), A study on the flood frequency analyzed in consideration of low outliers, Journal of the Korean Society of Agricultural Engineers, Vol. 30(4), pp. 62-70, Korean.
13. Rasheed, H. R., M. V. Ramamoorthy and A. S. Aldabbagh(1982), Modified SMEMAX transformation for frequency analysis, W. R. P., Vol. 18, No. 3, pp. 509-511, English.
14. Subbash Chander, S. K. Spolia, and Arun Kumar(1978), Flood frequency Analysis by power transformation, Journal of the Hydraulic Division, Vol. Hy 11, pp. 1495-1503, English.
15. Yevjevich, V. (1972), Probability and Statistics in Hydrology, W. R. P. Colorado, pp. 118-167, English.