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Failure Analysis of the T−S−T Switch Network†

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ABSTRACT

Time−Space−Time(T−S−T) switching network is modeled as a graceful degrading system. Call blocking probability is defined as a measure of performance. Several performance related measures are suggested under the presence of failure. An optimization model is proposed, which determines optimal values of system parameters of the switching network.

1. INTRODUCTIN

Several systems have been modeled as graceful degrading systems[1,2,3]. They are designed to operate at several levels of performance corresponding to the various possible combinations of failures. The T−S−T switching network is another good example for such system. If a single time switch of the switching network system fails, the system may continue to operate without the faulty time switch, but has a lower level of performance until the time switch can be repaired and then reconfigured into the system again. Blocking probability is suggested as a measure of performance, which is defined as the probability that no free path from input channel to output channel is available.

In this paper the following three things are dealt with :

i) propose a model for a T−S−T switching network operating under a graceful degradation policy.

ii) suggest several performance related measures for the combined analysis of reliability and performance.

iii) propose an optimization model, which determines optimal values of system parameters of the switching network.

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2. System Description

2.1 Switching Network[4]

Typical T−S−T switching network switching N channels(time slots) is shown in Figure 1.

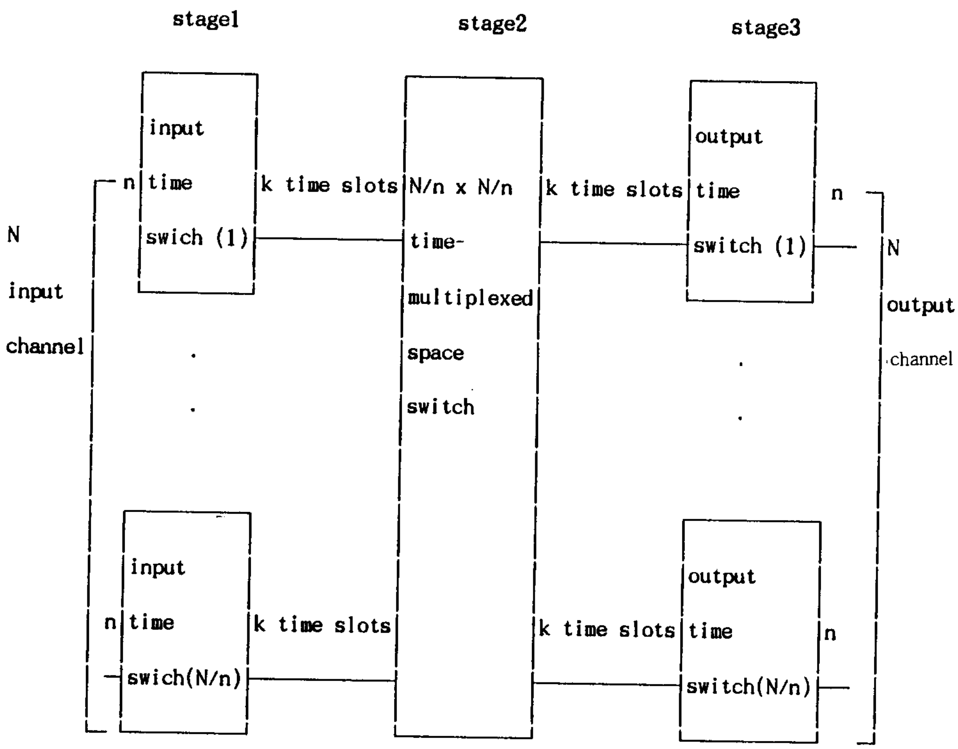

Figure1. T−S−T Switching Network

N channels are grouped into N/n(integer) time switches, each containing n time slots at its input. Let the number of output time slots in a time frame be $k > n$. These N/n time switches are connected to a single $N/n \times N/n$ space switch as shown in Figure 1. The N/n outputs of the space switch are in turn each connected to a time switch in a third stage, with k time slots/frame at its input and n time slots at its output.

For a call to be complete, the connection of the input and the output channels is required through the proper time slots in the input and the output time switches. And a speech path can be established through the space switch only if idle time slots are available in both time switches. Otherwise, the call is said to be blocked. The blocking probability is mainly determined by the traffic load(call attempt rate), and the number of available time slots.

2.2 Blocking Probability

It can be shown that $k \leq 2n-1$ is sufficient to guarantee an available path through the switching network[4]. By allowing some blocking, however, we can reduce cost and complexity of the switching network. In this study $k<2n-1$ is assumed. The blocking probability(BP) is derived under the following assumptions.

i) incoming traffic is distributed uniformly over the channels

ii) probabilities of finding individual links along a path busy are independent

It can be approximated as [5].

$$BP = [1 - (1 - n \cdot p/k)^2]^k \qquad (1)$$

where p is the probability that a typical input or output channel is busy. Let λ be the constant call offered traffic rate of the switching network. Average call arrival rate of each channel becomes λ/N. Each call is assumed to have exponential holding time of average length $1/\mu$. Then, it can be shown that p can be expressed as $(\lambda/N)/(\lambda/N+\mu)$[6].

2.3 Failure State of the Switching Network

If a time switch of a switching network system fails, the system may continue to operate without the faulty time switch, but has a lower level of performance. That is, call blocking probability is increased.

Under the following assumptions, blocking probability is derived at each failure state.

i) Any failure of time switches causes the offered traffic to be equally distributed to the remaining alive time switches.

ii) Space switch failure does not allow any call connection. That is, all the call is blocked.

iii) If the speech path from subscriber A to B is built through the first input and the second output time switches, the second input and the first output time switches serve as a speech path from subscriber B to A(symmetric path searching). If failure occurs in the second input(or output) time switch, the subscriber B(or A) can't have any speech path. Therefore, any failure of the same numbered input or output time switch does not allow to constitute the speech path using these time switches. Naturally it follows that if a input(or output) time switch failure occurs, any traffic is not offered to the same numbered output(or input) time switch.

Failures in the switching network can be classified into several states depending on the number of failures in the time switch and the space switch. State "i"(i=1,2,. ,N/n) is defined as i operating same numbered pairs of time switches and operating space switch. State "0"is defined as no operating same numbered pairs of time switch or failed space switch. Then, the blocking probability at failure state i, BP(i) can be approximated as

$$BP(i) = (1 - (1 - n \cdot p_i/k)^2)^k, \quad i=1,2,\cdots,N/n \tag{2}$$

where pi is the probability that a typical input or output channel is busy at failure state i. It can be expressed as

$$p_i = (\lambda/i \cdot n)/(\lambda/i \cdot n+\mu)$$

BP(0) is defined as 1.

3. SYSTEM EFFECTIVENESS MEASURES CONSIDERING RELIABILITY AND PERFORMANCE

For the combined analysis of reliability and performance, several performance related measures are defined.

3.1 System without Repair

Assumption

All failure events are statistically independent

Notation

$R_T(t)$: reliability function for time switch

$R_s(t)$: reliability function for space switch

$P_i(t)$: probability that the system is in failure state i at time t

C^* : a specified threshold for blocking probability

$R(C^*,t)$: probability that blocking probability is less than C^* at time t

$MT(C^*)$: mean time for blocking probability to reach C^*

The probability that a pair of time switches is alive is given as $R_T(t)^2$. Under the independence assumption,

$$P_i(t) = \binom{N/n}{i} \{1-R_T(t)^2\}^{N/n-i} \cdot \{R_T(t)^2\}^i \cdot R_s(t), \quad i=1,2,\cdots,N/n \tag{3}$$

Then,

$$R(C^*,t) = \sum_{i\in I} P_i(t), \text{ where } I = \{i \mid BP(i) \le C^*\} \tag{4}$$

And

$$MT(C^*) = \int_0^\infty R(C^*,t)\, dt \tag{5}$$

C^* can be determined in several ways depending upon the types of reliability measures which will be derived. Judging which value is more meaningful as a system reliability measure and to a user requirement, C^* should be chosen. For instance, we can say that the system is in reliable operation mode if the C^* stays below the some given value $C^*=C_1$. Then, $MT(C_1)$ can be used to represent the mean time it takes to come to the 'unreliable operation' mode. Meanwhile, by taking $C^*=1$ MT(1) can be used to represent the mean time to the total failure, i.e., 'no service' mode.

Numerical Example

Input data for the illustrative numerical example are given as follows.

- $N = 24$
- $n = 8$
- $k = 10$
- $\lambda = 0.24$ call/sec
- $\mu = 0.02$ call/sec
- $R_T(t) = e^{-10^{-3}t}$, $R_s(t) = e^{-5\cdot10^{-4}t}$

From the equation (2),

$$BP(3) = (1 - (1 - 4/5 \cdot 1/3)^2)^{10} = 0.0004451$$

And

$BP(2) = 0.0035053$

$BP(1) = 0.0427413$

i) $C^* = 0.001$

$I = \{3\}$

$$R(0.001,t) = (R_T(t)^2 2)^3 \cdot R_s(t)$$
$$= e^{-0.0065t}$$

$MT(0.001) = 153.85$

ii) $C^* = 0.01$

$I = \{3,2\}$

$$R(0.01,t) = (R_T(t)^2)^3 \cdot R_s(t) + 3 \cdot (1 - R_T(t)^2) \cdot (R_T(t)^2)^2 \cdot R_s(t)$$
$$= 3 \cdot e^{-0.0045t} - 2e^{-0.0065t}$$

$MT(0.1) = 358.97$

iii) $C^* = 0.1$

$I = \{3,2,1\}$

$$R(0.1,t) = (R_T(t)^2)^3 \cdot R_s(t) + 3 \cdot (1 - R_T(t)^2) \cdot (R_T(t)^2)^2 \cdot R_s(t)$$
$$+ 3 \cdot (1 - R_T(t)^2)^2 \cdot R_T(t)^2 \cdot R_s(t)$$
$$= 3 \cdot e^{-0.0025t} - 2e^{-0.0045t} + e^{-0.0065t}$$

$MT(0.1) = 687.18$

3.2 System with Repair

Assumptions

- All failure events are statistically independent.
- Failures are detected at once during operation(self announcing).
- Transition rates from one state to another are constant.
- Repairs are simultaneous made over all the failures
- In state 0, repair is made to state N/n.

Notation

- α_T : time switch failure rate
- α_s : space switch failure rate
- β_T : time switch repair rate
- β_0 : repair rate from state 0 to state N/n
- P_i : stationary probability that the system is in failure state i
- $A(C^*)$: probability that the switching network is operating with blocking probability less than C^*.

- AVBP : average call blocking probability

Under the assumptions given above and failure state defined in section 2.2, the state transition diagram can be drawn as shown in Figure 2.

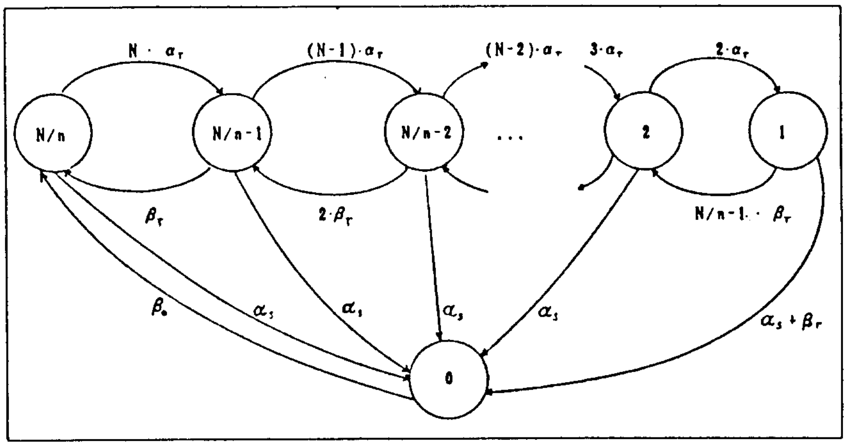

Figure 2. State Transition Diagram of the Switching Network

Based on the above transition diagram of Figure 2, P_i can be obtained by solving a set of following balance equations[6].

$$
\begin{aligned}
&P_{N/n}(N\cdot\alpha_T+\alpha_s) = P_{(N/n-1)}\cdot\beta_T+P_0\cdot\beta_0\\
&P_{(N/n-1)}((N-1)\alpha_T+\beta_T+\alpha_s) = P_{N/n}\cdot N\cdot\alpha_T+P_{N/n-2}\cdot 2\beta_T\\
&\quad\vdots\\
&P_0\cdot\beta_0 = (P_{N/n}+P_{N/n-1}+\cdots+P_2)\cdot\alpha_s+P_1(\alpha_s+\beta_T)
\end{aligned}
\tag{6}
$$

Now

$$A(C^*) = \sum_{i\in I} P_i,\ I=\{i\,|\,BP(i)\le C^*\} \tag{7}$$

The average call blocking probability of the switching network is determined as the sum of all its "operating state probability" times the "corresponding call blocking probability".

$$AVBP = \sum_{i=0}^{N/n} P_i\cdot BP(i) \tag{8}$$

Numerical Example

For the illustrative numerical example, following input data are used.

- $N = 24$
- $n = 8$
- $k = 10$
- $\lambda = 0.24$ call/sec.
- $\mu = 0.02$ call/sec.
- $\alpha_T = 10^{-3}$/hour, $\alpha_s = 5 \cdot 10^{-4}$/hour, $\beta_T = 0.5$, $\beta_0 = 0.2$

The state transition diagram can be drawn as follows.

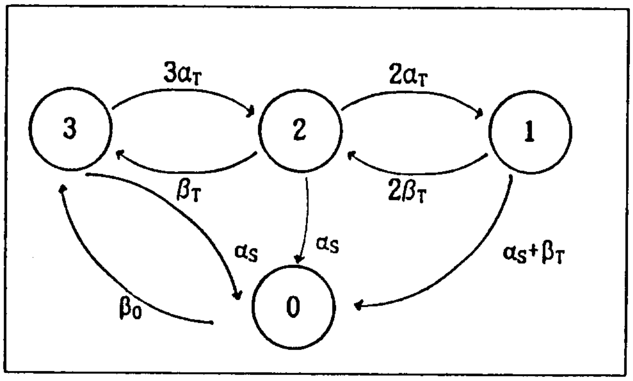

Figure 3. state transition diagram with N/ n=3

The stationary state probability P_i is obtained as

$P_3 = 0.99155$

$P_2 = 0.00594$

$P_1 = 0.00001$

$P_0 = 0.00250$

From the equation (7), A(C*) can be obtained as

i) $C^* = 0.001$

$I = \{3\}$

$A(0.001) = P_3 = 0.99155$

ii) $C^* = 0.01$

$I = \{3,2\}$

$A(0.01) = P_3 + P_2 = 0.99749$

iii) $C^* = 0.1$

$I = \{3,2,1\}$

$A(0.1) = P_3 + P_2 + P_1 = 0.9975$

And AVBP is calculated from the equation (8)

$AVBP = 0.0029625$

4. OPTIMIZATION MODEL

Given N input channel, the cost of the TST switching network in Figure 1 can be expressed as a function of the switching network parameters n(number of input time slots in time switch) and k(number of output time slots in time switch). It can be assumed to have the functional form c(n, k). And the switching network can have the different values of blocking probability corresponding to the various possible combinations of n and k.

In this section, an optimization model is just presented for a. repairable case, which determines the optimal values of n and k. Under the constraints of $A(C^*) \le A_1$ (for instance, $A(0.01) \le 0.95$) and cost, it minimizes AVBP over n and k.

- objective function : $\underset{(n,k)}{\text{minimizes}}\ AVBP$
- constraints :
  - i) $A(C^*) \le A_1$
  - ii) $C(n,k) \le COST$
  - iii) $n \le k < 2n-1,\ n > 0$
  - iv) N/n should be integer

5. SUMMARY

For the system which is designed to operate at several levels of performance corresponding to the various possible combinations of failures, it is very unreasonable to define system state in binary way, i.e., "up" state and "down" state. Several different reliability measures of the switching network can be obtained considering its performance, the call blocking probability. Judging which value is more meaningful as a system reliability measure and to a user requirement, C^* should be chosen.

Under the assumption of no failure in the switching network, the blocking probability is calculated as 0.0004451. Under the presence of failure, the average blocking probability becomes

AVBP = 0.0029625.

The difference between these values explains why the switching network is modeled as a graceful degrading system.

An optimization model is just suggested without solving, which determines optimal values of system parameters of the switching network given number of input channels and C*.

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