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JIT 상황하에서 다품종 조립라인 작업물 투입 순서 결정 방안†

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Sequencing for a Mixed Model Assembly Line in Just-In-Time Production System†

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ABSTRACT

In mixed model assembly lines, products are assembled seqeuntially that have different combination of options specified by customers. In just in time(JIT) environment, production smoothing becomes an important issue for sub-lines which supply the necessary parts to each workstation of the assembly line. Another important issue is to avoid line stopping caused by work overload in workstations.

To find a sequence which minimizes the costs associated with line stoppage and the option parts inventory level, a nonlinear mixed integer programming is formulated. Recognizing the limit of the Branch and Bound technique in large sized problems, a heuristic solution procedure is proposed. The performance of the heuristic is compared with the Branch and Bound technique through randomly generated test problems. The computational results indicate that on the average the heuristic solutions deviate approximately 3.6% from the optimal solutions.

1. INTRODUCTIN

For the assembly of automobile or home appliances, paced assembly lines are commonly used to manufacture different models of the same general product. As manufacturing strategy, manufacturing firms nowadays tend to widen customer specified options which result in a variety of distinct

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products. Each product is characterized by the presence or absence of each of a set of available options or upgrades. For instance, in automobile industry, each product can be identified by the presence or absence of air conditioning(A/CON), anti-break-lock system(ABS), power window (P/WIN) and so forth. In paced assembly lines, jobs arrive at each workstation usually on conveyor at equal intervals of predetermined duration. The worker of a workstation can perform his assigned tasks on the product only when it remains in his working area. If an assembly worker attempts to finish his task crossing the borderline of his workstation, he is likely to end up with interfering a worker in an adjacent station.

Effectiveness of these paced lines dependent on the optimal design of the line(e.g., line balancing, number of workstations, cycle time, length and speed of the line, etc.) and the optimal scheduling of the line(e.g., sequencing, etc.) The sequencing problem has been handled by many research papers with the following objectivs : 1) minimization of the work overload on each workstation(work overload), 2) maintenance of a constant usage rate of every part consumed on the assembly line(production smoothing). Okamura and Yamashina(1979) developed a heuristic to minimize the maximum displacement of operator from starting point of his workstation in order to minimize the risk of stopping the line. Yano and Rachamadugu(1991) provided a sequencing method to minimize the work overload with option products. Burns and Daganzo(1975) emphasized a sequence which balances the setup cost and the work overload cost in non JIT system. Other works with work overload include Kilbridge ar Wester(1963), Thomopoloulos(1970), Macaskill (1972, 1973), and Dar-El and Cother(1975).

In production smoothing, the quantity of each part used by the mixed model assembly line per unit time is to be kept as constant as possible. There shold be very little variability in the usage of each part from one time period to the next, which is also called leveling(Hall, 1983). Monden (1983) discussed the leveling problem and showed how it is being handled at Toyota. Milternburg (1989) presented a smoothing problem whose objective is to find the sequence which minimizes fluctuation of the production quantity at sub-line. In Milternburg and Sinnamon(1989), they extended the earlier research to multi-level situation. The objective of Imman and Bulfin(1992) is to find the sequence which maintains the constant time interval between products which require same option assembly part. Sumichrast(1992) tested various productions smoothing alogrithms by simulation.

The existing works considered these two problems separately even though they are interrelated and equally important to achieve the maximum efficiency of a paced assembly system. In this regard, we formulated a mathematical model whose objective is to determine a sequence which minimizes the sum of two variable cost components ; the line stoppage cost due to work overload schedule and the other from option parts shortage and their inventory holding costs. In the next section, a mathematical model is developed. Section 3 presents a heuristic solution algorithm whose validity is illustrated through computational expriences in section 4. Conclusion appears in section 5.

2. Mathematical Formulation

In each workstation of a mixed model paced assembly line, every product requires ***basic job*** that is common for all products. If a certain product requires optional job, assembly worker must process not only ***basic job*** but also ***optional job***(henceforth called ***option product***). If a product requires only basic job, we call it ***basic product***. To make the problem interesting, we assume that the processing time of basic product is less than unit time and that of option product is longer than unit time. As practiced in JIT system, it is also assumed that each sub−line supplies optional parts to main−line at uniform time intervals. Hereafter, without loss of generality, the length of a cycle time is regarded as one unit time. We will illustrate the nature of our model with a five stage problem.

Figure 1 shows the system configuration of the problem whose data given in Table 1. In the table, the option(basis) product is indicated by binary value one(zero) with its processing times in parenthesis. The base products are launched at the beginning of the main conveyor line at equal time intervals. Each assembly worker does basic or optional job riding on the conveyor which moves in one direction. When he completes the assigned job of the current stage, he walks upstream with constant speed to catch the product of the succeeding stage. In case the product does not yet enter his workstation, he becomes idle at the starting point of his workstation until it arrives. The line may be stopped if required option part is not readily available from sub−line or job cannot be completed within his workstations.

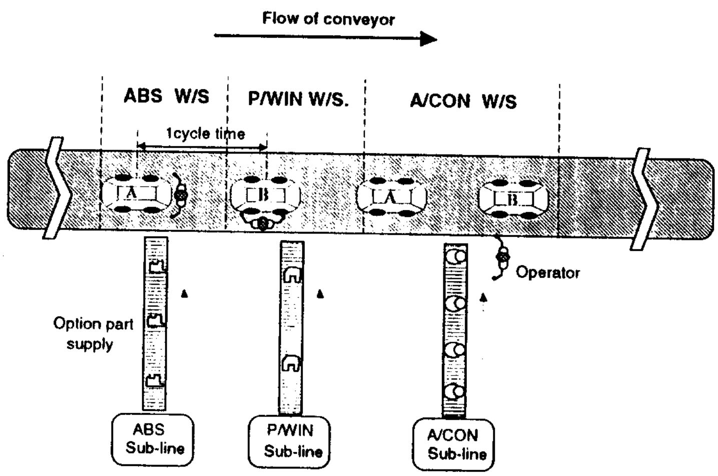

Figure 1. Configuration of the paced assembly line

Table 1. Product specification matrix

options / product type	ABS	Pow Win	Aircon	Demand
A	1(1.5)	0(0.8)	1(1.2)	3 units
B	0(0.5)	1(1.2)	1(1.3)	2 units

Figure 2 shows the loci of products flows and workers movements. Vertical axis and horizontal axis represent elapsed time and the length of each workstation expressed in terms of unit time, repectively. In the figure, each worker can move upstream 0.1 unit time by foot for the next job coming toward him on the conveyor. For an example, we consider ABS assembly workstation with its sub-line supplying ABS parts at 2 unit time intervals.

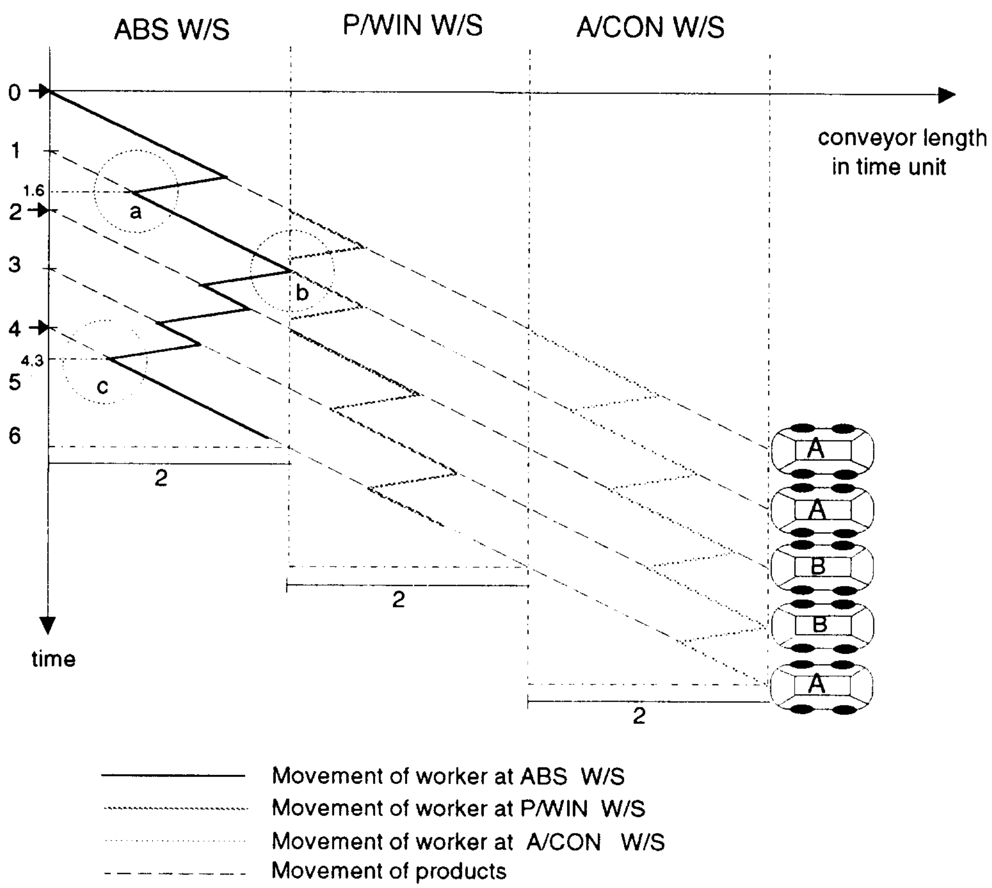

Figure 2. Loci of operators and products' movements

Let s(i) denote the type of the base product which is to be assembled at stage i and S be the sequence vector where K is the number of stages. S=(s(1), s(2), ⋯, s(K)).

With S=(A,A,B,B,A), three kinds of variable costs can occur in ABS work station as follows : (i) The worker begins his work on the product of the second stage at the time 1.6 while its option part is available at the time 2.0, which results in a potential line stoppage for the duration of 0.4 unit time due to the shortage(indicated by circle a). Instead of stopping line, our model will assume that additional work force is utilized in the sub−line to expedite the production of the required part for timely delivery and imposes the penalty cost which is proportional to the additional labor cost. (ii) For the product of the second stage, only 1.4 unit time is available for the worker whereas 1.5 unit time is needed to complete his task(indicated by circle b). In this case, we will impose another penalty cost, instead of stopping the line, assuming that the remaining work is done at a separate area. (iii) The earliest time the assembly work can start on the product of the third stage is 4.3, which implies that the corresponding ABS part has to wait for 0.3 unit time as inventory(indicated by circle c).

Based on the above procedures, a mathematical model is developed whose objective is to find a product sequence which minimizes the sum of the additional labor cost at sub−lines, the work overload cost and the inventory holding cost at the assembly line. For the model, the following notations are adopted :

C_1 = inventory holding cost per unit time.

C_2 = additional labor cost per unit time at sub−line required to prevent part shortages at main line.

C_3 = additional labor cost per unit time due to work overload.

j = workstation /option index j=1,2,⋯,J.

h = product type index h = 1,2,⋯,H.

$$t_{jh} = \begin{cases} 1, & \text{if option } j \text{ is required for product type } h,\ j=1,2,,\cdots,J\ \ h=1,2,\cdots,H \\ 0, & \text{otherwise} \end{cases}$$

Q = demand vector = (q_1, q_2, ⋯, q_H) where q_h is the total production requirement for product type h during production horizon.

$K = \sum_h^H q_h$ = total production requirement during production horizon.

k = stage index representing the location in S, k=1,2,⋯,K.

$d_j = \sum_h^H t_{jh}\, q_h$ = total demand of option j.

u_{hk} = decision variables $= \begin{cases} 1, \text{ if product type } h \text{ is scheduled at stage } k. \\ 0, \text{ otherwise.} \end{cases}$

$x_{jk} = \sum_{k}^{H} t_{jh}\, u_{hk} = \begin{cases} 1, \text{ if the product type of stage k is option product at station } j. \\ 0, \text{ otherwise.} \end{cases}$

$y_{ik} = \sum_{l}^{k} x_{jk}.$

wo_{jk} = the amount of work overload occurred at workstation j at stage k.

P_{hj} = processing time for product type h at station j.

L_j = length of workstation j in time unit.

s_{jk} = time for the worker at station j to begin the assembly task at stage k.

f_{jk} = time for the worker at station j to complete the assembly task at stage k.

$W = \dfrac{V_C}{V_C+V_O}$ = length of the time for the worker to walk upstream for the next job where

V_C = conveyor speed, V_C = operator speed.

For the convenience of analysis, without loss of generality s_{j1} will be set zero for every j throughout this paper. It is assumed that the replenishment rate of option parts from sub−line to the corresponding work station j is uniform with its interval r_j where

r_j = (length of the producton horizon of work station j) / (number of option parts supplied during production horizon) $= (K-1+L_j)/d_j$.

Note that the part arrival time at station j from sub−line which is required for the product of stage k is $(\sum_{l}^{k} x_{jl} - 1)r_j$. Let $G = s_{jk} - (\sum_{l}^{k} x_{jl} - 1)r_j$. If G is negative, then the shortage cost $-C_2\, x_{jk}\, G$ occurs and if G is positive, the inventory cost $C_1\, x_{jk}\, G$ occurs. Since $(k-1+L_j)$ denote the time the product of stage k leaves workstation j, work overload occurs whenever the required processing time exceeds the available time, i.e., $\sum_{h}^{H} u_{hk}\, P_{hj} > k-1+L_j-s_{jk}$. Therefore, the amount of work overload, wo_{jk}, which may occur in stage k at workstation j becomes

$$wo_{jk} = \max(0,\ s_{jk} + \sum_{h}^{H} u_{hk}\, P_{hj} - (k-1+L_j)). \tag{1}$$

The earliest time the worker at station j can begin his task is the larger of $k-1$ and $f_{j,k-1}$+W. The time he can complete his task is the smaller of $(k-1+L_j)$ and $s_{jk} + \sum_{h}^{H} u_{hk}\, P_{hj}$.

Therefore, we have

$$s_{jk} = \max(k-1,\ f_{j,k-1} + W), \tag{2}$$

$$f_{jk} = \min(s_{jk} + \sum_{h}^{H} u_{hk} P_{hj},\ (k-1+L_j)). \tag{3}$$

Based on the above findings, our problem can be formulated as follows :

(P1)

$$\min \quad C \sum_{j}^{J} \sum_{k}^{K} x_{jk} \mid s_{jk} - (\sum_{l}^{k} x_{jl} - 1) r_j \mid + C_3 \sum_{j}^{J} \sum_{k}^{K} wo_{jk} \tag{4}$$

$$\text{s.t.} \quad wo_{jk} = \max(0,\ s_{jk} + \sum_{h}^{H} u_{hk} P_{hj} - (k-1+L_j)) \qquad \forall\ j, k \tag{5}$$

$$s_{jk} = \max(k-1,\ f_{j,k-1}+W) \quad k=2,\cdots,K \qquad \forall\ j \tag{6}$$

$$s_{j1} = 0 \qquad \forall\ j \tag{7}$$

$$f_{jk} = \min(s_{jk} + \sum_{h}^{H} u_{hk} P_{hj},\ (k-1+L_j)) \qquad \forall\ j, k \tag{8}$$

$$x_{jk} = \sum_{h}^{H} t_{jh} u_{hk} \qquad \forall\ j, k \tag{9}$$

$$C = \begin{cases} C_1, & \text{if } s_{jk} - (\sum_{l}^{k} x_{jl} - 1) r_j \geq 0 \\ C_2, & \text{otherwise} \end{cases} \qquad \forall\ j, k \tag{10}$$

$$\sum_{h}^{H} u_{hk} = 1 \qquad \forall\ k \tag{11}$$

$$\sum_{k}^{K} u_{hk} = q_h \qquad \forall\ h \tag{12}$$

$$u_{hk} = 0, \text{ or } 1 \qquad \forall\ h, k \tag{13}$$

To eliminate the avsolute value terms in equation (4), we utilize $z_{jk}^{+} - z_{jk}^{-} = s_{jk} - (\sum_{l}^{k} x_{jl} - 1) r_j$ with $z_{jk}^{+} \cdot z_{jk}^{-} = 0$. The equality constraints containing terms with min and max operators can be easily transformed into the corresponding inequality constraints.

With the above measures, (P1) can be transformed into (P2) as follows :

(P2)

$$\min \quad C_1 \sum_{j}^{J} \sum_{k}^{K} x_{jk} z_{jk}^{+} + C_2 \sum_{j}^{J} \sum_{k}^{K} x_{jk} z_{jk}^{-} + C_3 \sum_{j}^{J} \sum_{k}^{K} wo_{jk} \tag{14}$$

$$\text{s.t.} \quad z_{jk}^{+} - z_{jk}^{-} = s_{jk} - (\sum_{l}^{k} x_{jl} - 1) r_j \qquad \forall\ j, k \tag{15}$$

$$z_{jk}^{+} \cdot z_{jk}^{-} = 0 \qquad \forall\ j, k \tag{16}$$

$$wo_{jk} \geq s_{jk} + \sum_{h}^{H} u_{hk} P_{hj} - (k-1+L_j) \qquad \forall\ j, k \tag{17}$$

$$wo_{jk} \geq 0 \qquad \forall\ j, k \tag{18}$$

$s_{jk} \geq k-1 \qquad k=2,\cdots,K \qquad \forall j \qquad (19)$

$s_{jk} \geq f_{j,k-1}+W \qquad k=2,\cdots,K \qquad \forall j \qquad (20)$

$f_{jk} \geq s_{jk}+\sum_{h}^{H} u_{hk} P_{hj}-My_{jk} \qquad \forall j, k \qquad (21)$

$f_{jk} \geq k-1+L_j-M(1-y_{jk}) \qquad \forall j, k \qquad (22)$

$y_{jk} = 0, \text{ or } 1 \qquad \forall j, k \qquad (23)$

(7), (9), (11), (12) and (13)

where M is large number.

It can be noted that (P2) is a nonlinear mixed integer programming whose solution cannot be obtained within reasonable comutation time as the size of problem increases.

3. Development of heuristic algorithm

We note that an optimal solution of (P2) can be found with a branch and bound procedure. Since the procedure becomes computationally infeasible if K, the number of states of sequence, is much more than 15, a heuristic alogrithm is developed in this section. The heuristic consists of two phases. Based on a construction scheme, the first phase develops an initial sequence which is improved by the pairwise interchange technique in the second phase. Let S_k be the partial sequence where the sequence for the first k stages are specified. Also, let T_{jk} denote the time interval between the job starting times on two successive products of stages k and $k+1$ at station j. $T_{jk} = [s_{jk}, f_{jk}+W]$ or $[s_{jk}, k]$ depending on the existence of idle time. Let $[\alpha]$ denote the largest integer equal to or less than α. For instance, $[3.4]=3$.

The following properties are utilized for the development of an initial sequence.

Property 1

With the partial sequence S_{k-1} given, the minimum work overload $mw_{j,k-1}$ expected to occur at station j during stages $k, k+1,\cdots,K$ is given by

$$mw_{j,k-1} = \max(0, \sum_{i \in \bar{S}_{k-1}} P_{ij} +(K-k)W-(K-1+L_j-\max(k-1, f_{j,k-1}+W))).$$

The corresponding minimum of the total work overload tw_{k-1} of the system is

$$tw_{k-1} = \sum_{j=1}^{J} mw_{j,k-1}.$$

(proof)

At station j, the time required for the worker to complete his task on the product i and to be ready for the next to job is at least $P_{ij}+W$. Therefore, the total time required at the station for the jobs of stages $k, k+1,\cdots,K$ equals or larger than $\sum_{i\in \bar{S}_{k-1}} P_{ij} + (K-k)W$ where $\bar{S}_k$ stands for the partial sequence such that $S_k \cup \bar{S}_k = S$. And the total available time for the station is simply $(K-1+L_j-\max(k-1, f_{j,k-1}+W))$. Hence, $mw_{j,k-1} = \max$ [0, (the total required time)−(the total available time)] $= \max [0, \sum_{i\in \bar{S}_{k-1}} P_{ij} + (K-k)W-(K-1+L_j-\max(k-1, f_{j,k-1}+W))]$.

Proterty 2 (proof omitted)

Suppose S_{k-1} is known and the product at stage k is option product at station j.

(i) The number IOE_{jk} of the option parts available at the station at the end of T_{jk} is

$$IOE_{jk} = \max \{0, [\frac{f_{jk}+W-(y_{jk})-1)r_j}{r_j}]\}.$$

(ii) The number IOB_{jk} of the option parts available at the station at the beginning of T_{jk} is

$$IOB_{jk} = \max \{0, [\frac{s_{jk}-(y_{jk})-1)r_j}{r_j}]\}.$$

Property 3 (proof omitted)

Suppose S_{k-1} is known and the product at stage k is basic product at station j. Let IBB_{jk} be the number of option parts availabel at the station at the beginning of T_{jk}.

Then

$$IBB_{jk} = [\frac{s_{jk}-y_{jk}r_j}{r_j}] + 1.$$

Property 4 (proof omitted)

Suppose S_{k-1} is known and the product at stage k is basic product at station j. Let IBE_{jk} and $IBEI_{jk}$ be the number of option parts availabel at the beginning of T_{jk} for $T_{jk} = [s_{jk}, f_{jk} + W]$ and $T_{jk} = [s_{jk}, k]$, respectively.

Then

$$IBE_{jk} = [\frac{f_{jk}+W-y_{jk}r_j}{r_j}] + 1 \text{ and } IBEI_{jk} = [\frac{k-y_{jk}r_j}{r_j}] + 1.$$

Additional notations used in developing the algorithm are listed in the following ;

NOP_{jk} = the number of option parts arrived at station j from sub−line during T_{jk}.

$QU_k = (qu_{k1}, qu_{k2}, \cdots, qu_{kH})$ where qu_{kh} is the quantity of product type h which is to be scheduled for stages $k, k+1, \cdots, K$.

Given S_{k-1}, suppose product type i is scheduled for stage k. We want to determine the length of inventory carrying period (z_{jk}^{+}) as well as shortage occuring period (z_{jk}^{-}) which may occur during T_{jk} at station j. z_{jk}^{+} and z_{jk}^{-} depend on the number of conditions such as (i) the product type at stage k (option or not) (ii) the number of option parts available at the begining of T_{jk} (iii) the number of option parts arrived during T_{jk} (iv) the existence of worker's idle time in T_{jk}.

Table 2 lists 10 cases we have to consider, i.e., 4 cases with the product at stage k being option and 6 cases being basic. Note that for the case 1 in the table, worker idle time never appears in T_{jk} since the processing time of option product is not smaller than the cycle time. For each case, z_{jk}^{+} and z_{jk}^{-} are determined utilizing properties 2 to 4 presented in Table 3. For instance, consider case 1.2(refer to Figure 3) in which the number of option part is not available at time s_{jk} and at least one option part arrives from the sub-line during T_{jk}. It implies $s_{jk} \leq (y_{jk}-1)r_j \leq f_{jk}+W$. Then the length of time shortage occurs becomes $z_{jk}^{-} = (y_{jk}-1)r_j - s_{jk}$ and the length of time during which option parts are stored as inventory is $z_{jk}^{+} = \sum_{i=1}^{IOB_{jk}} (f_{jk}+W-(y_{ik}+i-1)r_j)$. Figure 3 illustrates the situation with $NOP_{jk} = 2$ during $T_{jk} = [s_{jk}, f_{jk}+W]$. The arrival times of the option parts are indicated with arrow signs. The first arrived parts is for the product of stage k and the other is kept as inventory until the time $f_{jk}+W$.

Table 2. Classfication of cases

	Case 1 : The product at stage k is option product.
1.1	$IOB_{jk} = 0$ and $NOP_{jk} = 0$
1.2	$IOB_{jk} = 0$ and $NOP_{jk} \geq 0$
1.3	$IOB_{jk} \geq 1$ and $NOP_{jk} = 0$
1.4	$IOB_{jk} \geq 1$ and $NOP_{jk} \geq 0$
	Case 2 : The product at stage k basic product.
2.1	$IBB_{jk} = 0$, $NOP_{jk} > 1$, and $T_{jk} = [s_{jk}, f_{jk} + W]$
2.2	$IBB_{jk} = 0$, $NOP_{jk} = 1$, and $T_{jk} = [s_{jk}, k]$
2.3	$IBB_{jk} \geq 1$, $NOP_{jk} = 0$, and $T_{jk} = [s_{jk}, f_{jk} + W]$
2.4	$IBB_{jk} \geq 1$, $NOP_{jk} \geq 1$, and $T_{jk} = [s_{jk}, f_{jk} + W]$
2.5	$IBB_{jk} \geq 1$, $NOP_{jk} = 0$, and $T_{jk} = [s_{jk}, k]$
2.6	$IBB_{jk} \geq 1$, $NOP_{jk} \geq 1$, and $T_{jk} = [s_{jk}, k]$

Table 3. Length of inventory carrying period (z_{jk}^{+}) and shortage occurring period(z_{jk}^{-})

Case	z_{jk}^{+}	z_{jk}^{-}
1.1	0	$(y_{jk}-1)r_j-s_{jk}$
1.2	$\sum_{i=1}^{IOB_{jk}}(f_{jk}+W-(y_{jk}+i\ -1)r_j)$	$(y_{jk}-1)r_j-s_{jk}$
1.3	$(f_{jk}+W-s_{jk})IOB_{jk}$	0
1.4	$(f_{jk}+W-s_{jk})IOB_{jk}+\sum_{i=1}^{IOB_{jk}-IOB_{jk}}(f_{jk}+W-(y_{jk}+IOE_{jk}-i)r_j)$	0
2.1	$\sum_{i=0}^{IBB_{jk}-1}(f_{jk}+W-(y_{jk}+i\ )r_j)$	0
2.2	$k-y_{jk}\ r_j$	0
2.3	$(f_{jk}+W-s_{jk})IBB_{jk}$	0
2.4	$(f_{jk}+W-s_{jk})IBB_{jk}+\sum_{i=1}^{IBB_{jk}-IBB_{jk}}(f_{jk}+W-(y_{jk}+IBB_{jk}+i-1)r_j)$	0
2.5	$(k-s_{jk})\ \mathrm{IBB}_{jk}$	0
2.6	$(k-s_{jk})\ \mathrm{IBB}_{jk}+\sum_{i=1}^{IBB_{jk}-IBB_{jk}}(k-(y_{jk}+IBB_{jk}+i-1)r_j)$	0

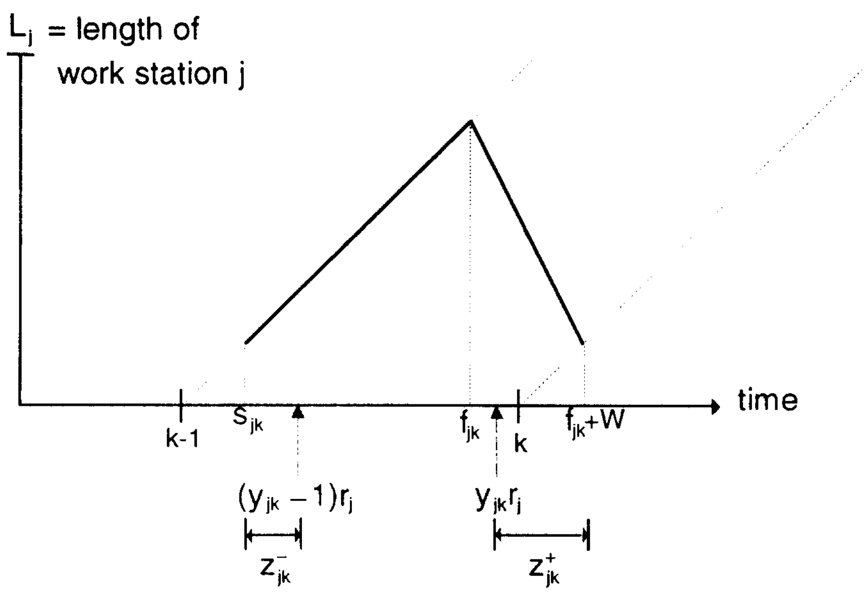

Figure 3. Graphic representation of case 1.2

Now, we are going to propose a way to determine the product type for stage k, given S_{k-1}. Suppose product type i is scheduled for $s(k)$. Property 1 enables us to find the minimum amount of work overload expected at stages $k+1$, $k+2,\cdots,K$. Also from table 2, 3 and equation (1), the amount of work overload, the inventory carrying cost and the shortage cost can be determined. Let $f_k(i)$ be the cost function which is the sum of four kinds of the cost elements.

$$f_k(i) = \text{ the cost associated with } s(k) = i, \text{ given } S_{k-1}$$
$$= C_3\, tw_k + C_3 \sum_j^J wo_{jk} + C_1 \sum_j^J z_{jk}^+ + C_2 \sum_j^J z_{jk}^-$$

Suppose $\min_i f_k(i) = f_k(i^0)$ then we will set $s(k) = i^0$.

Summarizing the above procedures, the following alogrithm is presented.

Algorithm

(Phase 1)

Setp 1 : Find d_j and r_j from the given product specification matrix and demand vector Q.
Set $k=1$, $s_{j1}=0$, $S_0=\phi$, $QU_1=Q$.

Step 2 : Set $k=k+1$
Determine $f_k(i)$ for each of i which satisfies $qu_{ki} > 0$.

Step 3 : Find $\min_i f_k(i) = F_k(\mathrm{i}^0)$.

Step 4 : $qu_{k\rho} = qu_{k-1,\rho} - 1$ and $s(k) = i^0$.

Step 5 : If $k < K$, go to Step 2
Otherwise, $S = (s(1), s(2), \cdots, s(K))$.

(Phase 2)

An ordinary pairwise interchange method is employed with the maximum number of trials N_{mas} specified by the user.

4. Computational Experience

To examine the heuristic proposed, the problem with the following parameters is solved ; $H=5$, $J=7$, $Q=(1,2,3,2,2)$, $W=0.1$, $L_i=20\ \forall i$

The effects are also examined of the cost ratio, the variations in processing times and the frequencies option products appeared in production specification matrix. Table 4 shows the structure of the test problems.

Table 4. Factors tested

cost ratio	frequencies options are required	variations of processing times
1:1:1 1:1:3 1:1:7	Sparse(7) Medium(14) Dense(21)	Small Large

In the table, 'Sparse(7)' implies that among 5×7 entries in the product specification matrix, only 7 entries have binary variable one. 'Medium(14)' and 'Dense(21)' can be interpreted in similar ways. The uniform distributions are utilized to generate processing times for each problem category as follows :

(i) for option product with 'Small' variations : $U[1.0,\ 1.5]$

(ii) for basic product with 'Small' variations : $U[0.5,\ 1.0]$

(iii) for option product with 'Large' variations : $U[1.0,\ 2.0]$

(iv) for basic product with 'Large' variations : $U[0,\ 1.0]$

For each combination of three factors, twenty problems are randomly generated and altogether 3×3×2×20 problems become the subjects of the sensitivity analysis. We represent each problem category with three letters, each standing for the corresponding level of factor. For instance, 3DS implies the problem with $C_1 : C_2 : C_3$ = 1:1:3, Dense (21) and Small. Table 5 shows the average percent deviation of the heuristic solutions obtained with N_{max} = 40 from those by the branch and bound technique (B&B). For instance, for the problems idenified by 3DS, on the average the deviations of 0.42% are resulted with the average computation time of 0.5 sec. for heuristic and 47.16 sec. for B&B.

In general, the heuristic solutions deviate approximately 3.6% from the B&B solutions with the range of 0.42%~7.74%. It is observed that slightly poor performances are observed for the problems with large variations in the processing times.

For our interest, 20 problems with the population size of K=20 are randomly generated whose optimal solutions are impossible to obtain by B&B. The solution of the heuristic is compared with the best of 10,000 random solutions of each problem and the results are 18 out of 20 in favor of the heuristic.

Table 5. Test results for K=10

problem	% deviation form opt.	run time(sec.) (PC 386 DX)	
		B&B	heuristic
1SS	1.21	14.19	0.31
1SL	1.84	3.87	0.25
1MS	4.28	25.88	0.37
1ML	5.58	10.53	0.31
1DS	1.35	34.45	0.34
1DL	7.29	17.44	0.40
3SS	1.15	16.76	0.40
3SL	4.75	5.27	0.31
3MS	2.17	35.07	0.37
3ML	6.58	10.25	0.33
3DS	0.42	47.16	0.50
3DL	4.63	20.08	0.28
7SS	5.04	11.94	0.29
7SL	6.95	5.53	0.21
7MS	1.19	31.67	0.34
7ML	7.74	6.73	0.28
7DS	0.47	64.35	0.35
7DL	2.36	13.86	0.33
average	3.61	16.47	0.30

5. Conclusion

This paper considers the sequencing problem of a mixed model assembly line in JIT environment. In the JIT production system, an important issue is how to keep a constant rate of parts usage at main−line in order to smooth workloads at sub−lines. An ill−scheduled sequence in terms of smoothing will cause option parts inventories at assembly line or line stoppings due to part shortages. Another equally important issue is to avoid line stoppings resulted by work overloads in workstaions.

Recognizing that these two problems have been treated independently in the previous research works, this study preents a nonlinear mixed integer programming which consider these two issues simultaneously. The objective is to find a sequence which minimizes the total variable costs associated with production smoothing and work overload. Two−phased heuristic is proposed whose validity is examined with test problems. Also sensitivity analysis is carried out to examined the performance of the heuristic. Computational experiences with $K=10$ show that the heuristic generate relatively efficient solutions which deviate approximately 3.6% from the optimal solutions.

Since an optimal solution is almost impossible to obtain for a real world problem due to its problem size, we hope that the heuristic proposed in this study could contribute in enhancing the systms efficiency of the industries which produce their products on mixed model assembly lines under JIT environment.

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