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Improving the Efficiency of Marketing Channel between a Wholesaler and a Retailer with Uncertain Characteristics†

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Abstract

The efficiency of marketing channel of distribution between a wholesaler and a retailer with uncertain characteristics can be improved by influencing the retailer's ordering pattern. The wholesaler with large unit inventory holding cost can offer a large quantity discount thanks to the great benefit which comes from the transfer of part of his inventory to retailer. The retailer's increasing average inventory holding cost can be offset by the quantity discount and by savings of the ordering cost. Conditions under which marketing channel improvement can be possible are derived.

1. Introduction

We consider the marketing channel of distribution between a wholesaler and a retailer given a fixed total quantity demanded for a single item. In an earlier paper, Lai and Staelin[4] have taken approach through an integrated inventory model for a marketing channel of distribution where the total inventory related costs are jointly minimized. Such a model is appropriate if collaborative arrangement for the allocation of joint benefit can be enforced by some contractual agreement between a wholesaler and a retailer. When a wholesaler and a retailer are two separate entities, the problem is not that simple. Lee and Rosenblatt[5], Rosenblatt and Lee[9], Eppen and Yehoshua[1] and Monahan[8] have studied the important economic implications of the marketing channel from the supplier's point of view to increase supplier's profitability. Kim and Hwang[3] and Kim[2] analysed the marketing channel between a supplier and multiple retailers with the assumption on the perfect information on the retailer's chracteristics. Lee[6] have analysed the channel for a wholesaler with multiple customers through non-linear pricing.

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In the distribution channel between a manufacturer with several customers, both parties can be beneficial by penalizing the large order quantity[7].

In this paper, we apply the theory of non-linear pricing to improve the efficiency of the marketing channel between a wholesaler and a retailer. The wholesaler is apt to have only the probability density function of the retailer's characteristic rather than the perfect imformation about even one retailer. The behaviors of the wholesaler and the retailer are different from the case of the perfect imformation due to the imperfect imformation.

This non-linear quantity dependent pricing schedule increases the marketing channel efficiency by influencing the retailer's ordering pattern.

2. Model Formulation

Let's consider the situation in which a retailer receives his replenishment stock from a wholesaler. The wholesaler, in return, receives his supply through order from the manufacturer or the other vendor. Assuming that the retailer always responds to the non-linear pricing schedule by the self-selection of optimal order quantity for that schedule, it may be the wholesaler's advantage to offer a quantity discount pricing schedule even though the demand rate is constant. This may affect the expected cost of the wholesaler and result in a higher expected profit.

In this situation, we assume that the ordering cost and demand rate of the retailer are known and the wholesaler knows only the probability density function of the unit inventory holding cost for the retailer. Under some conditions, a non-linear pricing schedule can benefit both the retailer and the wholesaler by transferring part of the inventory holding cost from the wholesaler to the retailer in return for the quantity discount.

As a result, both parties can benefit economically from this use of a nonlinear pricing schedule, although under some conditions, no mutually beneficial pricing schedule exists.

The following assumptions will be used for our analysis.

(1) Demand rate is constant and insensitive to price changes.
(2) No shortages are allowed on either side.
(3) Instantaneous replenishment at both the wholesaler and the retailer.
(4) The wholesaler knows the probability density function of the retailer's unit inventory holding cost.
(5) Demand rate and ordering cost are constant and are known to the wholesaler

The following notations will be used.

$P(q)$ = Total purchasing cost for q units for the retailer from the wholesaler.

$C(q)$ = Total purchasing cost for q units for the wholesaler from the manufacturer or the other vendor.

D = Annual demand rate for the retailer.

K_s = Wholesaler's ordering cost per order.

K_r = Retailer's ordering cost per order.

h_s = Wholesaler's inventory holding cost per unit per period.

h_r = Retailer's inventory holding cost per unit per period.

$f(h_s)$ = Probability density function of retailer's unit inventory holding cost.

p = Present fixed unit price offered to the retailers by the wholesaler.

2.1 Retailer's Behavior

The retailer always responds to the price schedule offered by the wholesaler. The retailer whose unit inventory holding cost is h_r has an optimal order quantity $q^*(h_r)$ that is obtained by minimizing his average total cost, i.e, total of purchasing cost, ordering cost and average inventory holding cost.

$$q^*(h_r) = \arg \mathrm{Min}\ \{P(q(h_r))\frac{D}{q(h_r)} + K_r \frac{D}{q(h_r)} + \frac{q(h_r)}{2} h_r\} \tag{2.1}$$

Thus the first necessary condition for Eq.(2.1), that is, retailer's self-selection condition, can be written as

$$\frac{d}{dq(h_r)} (P(q(h_r))) \frac{D}{q(h_r)} - \{P(q(h_r)) + K_r\} \frac{D}{q(h_r)^2} + \frac{h_r}{2} = 0 \tag{2.2}$$

2.2 Wholesaler's Behavior

Suppose that the retailer's economic order quantity is q. Then the demand faced by the supplier is a stream of q units of demand occuring at fixed time interval of $\frac{q}{D}$ time units. Given this stream of demands, q units each, it can be easily shown that the wholesaler's economic order quantity, q_s^*, should be some integer multiple of q. Suppose q_s^* is not an integer multiple of

q. Let $q_s^* = n \cdot q + m$ where n is a positive integer and $0 \le m < q$. For the first $\frac{q}{D}$ time units after the wholesaler's replenishment, his inventory level becomes $(n-1)q+m$. Similary, between $\frac{q}{D}$ and $\frac{2q}{D}$ time units after replenishment, the inventory level is $(n-2)q+m$. Between $\frac{(n-1)q}{D}$ and $\frac{nq}{D}$ time units after replenishment, the inventory level falls to m. Since this inventory level is not large enough to satisfy the next order from the retailer, a replenishment request by the wholesaler is warranted at this time. It can be seen that, by setting $m=0$, the inventory levels during the respective time periods becomes $(n-1)q$, $(n-2)q, \cdots, q$ and 0, and inventory cost are reduced. In view of the above consideration, the wholesaler's order quantity is assumed from now on to be an integer multiple of the retailer's order quantity. We know that average inventory level for the wholesaler is $\frac{q_s - q(h_r)}{2}$. Therefore, if the wholesaler replenishes his economic order quantity from the other vendor just after receiving the retailer's order, which is determined by the pricing schedule given by the wholesaler, the expected annual net profit for the wholesaler is as follows when fixed unit price c is offered to the wholesaler by the other vendor.

$$\begin{aligned}\text{Max } \Pi &= \int_{h_0}^{h_1} \{ P(q(h_r)) \frac{D}{q(h_r)} - cq^*_s \frac{D}{q^*_s} - K \frac{D}{q^*_s} - \frac{q^*_s - q(h_r)}{2} h_s \} f(h_r) dh_r \\ &= \int_{h_0}^{h_1} \{ P(q(h_r)) \frac{D}{q(h_r)} - cD - K_s \frac{D}{n(h_r)q(h_r)} - \frac{n(h_r)\, q(h_r) - q(h_r)}{2} h_s \} f(h_r)\, dh_r \qquad (2.3)\end{aligned}$$

$$h_1 = \text{maximum } h_r \text{ in } f(h_r)$$

$$h_0 = \text{minimum } h_r \text{ in } f(h_r)$$

Finally the wholesaler's problem is to find non-linear pricing schedule $P(q)$ and positive integer value $n(h_r)$ to maximize his expected net profit function Π subject to the optimal retailer's behavior, i.e.

$$\text{Max } \Pi = \int_{h_0}^{h_1} \{ P(q(h_r)) \frac{D}{q(h_r)} + \frac{q(h_r)}{2} h_s - \frac{K_s D}{n(h_r)q(h_r)} - \frac{n(h_r)\, q(h_r)}{2} h_s \} f(h_r)\, dh_r - cD \qquad (2.4)$$

$$s.t. \quad \frac{d}{dq(h_r)} \{ P(q(h_r)) \} \frac{D}{q(h_r)} - \{ P(q(h_r)) + K_r \} \frac{D}{q(h_r)^2} + \frac{h_r}{2} = 0$$

$$n(h_r) = \text{integer} \ge 1$$

This expected net profit of the wholesaler can be increased by offering a quantity discount pricing schedule. The decrease of his gross revenue can be offset by the benefit from the decrease of the inventory hodling cost. Increasing the order quantity of the retailer makes the wholesaler carry a smaller average inventory level. On the retailer side, the benefit from the quantity discounted purchasing cost is offset by the increase of the inventory holding cost.

3. Analysis of Model

We will analyze the model to obtain the pricing policy $P(q)$ and positive integer $n(h_r)$ maximizing the wholesaler's expected net profit and will also investigate how this pricing schedule will benefit both parties, if it is possible to get such a pricing schedule.

We will use a two-step procedure to maximize the wholesaler's expect net profit by maximizing the first part of Eq.(3.1) at the first step and then minimizing the last part of Eq.(3.1) given the information from the first step.

$$\text{Max } \Pi = \int_{h_0}^{h_1} \{ P(q(h_r)) \frac{D}{q(h_r)} + \frac{q(h_r)}{2} h_s \} f(h_r)\, dh_r - cD$$
$$- \int_{h_0}^{h_1} \{ \frac{K_s D}{n(h_r)q(h_r)} + \frac{n(h_r)q(h_r)}{2} h_s \} f(h_r) dh_r \tag{3.1}$$

At first, we can rewrite the wholesaler's problem to maximize the first part of Eq.(3.1) as follows,

$$\text{Max } \Pi' = \int_{h_0}^{h_1} \{ P(q(h_r)) \frac{D}{q(h_r)} + \frac{q(h_r)}{2} h_s \} f(h_r)\, dh_r \tag{3.2}$$

Integrating Eq.(3.2) by parts and substituting in the retailer's self-selection condition by Eq.(2.2) yields

$$\Pi' = P(q_0) \frac{D}{q_0} + \frac{q_0}{2} h_s + \int_{q_0}^{q_1} F(h_r) \{ \frac{K_r D}{q^2} - \frac{h_r}{2} + \frac{h_s}{2} \} dq \tag{3.3}$$

q_0 = minimum $q(h_r)$ = $q(h_1)$

q_1 = maximum $q(h_r)$ = $q(h_0)$

Then, the Euler's first order necessary condition for an interior local maximum is

$$f(h_r) (\frac{K_r D}{q^2} - \frac{h_r}{2} + \frac{h_s}{2}) - \frac{1}{2} F(h_r) = 0 \tag{3.4}$$

By substituting $h_r = h(q)$ from Eq.(3.4) into the retailer's self-selection condition Eq(2.2), we will get

$$\frac{d}{dq} P(q) = \frac{P(q)}{q} + \frac{K_r}{q} - \frac{q}{2D} h(q) \tag{3.5}$$

We find the boundary condition involving q_0 is exactly same as the retailer's self-selection condition Eq(2.2); futhermore, the boundary condition involving q_1 is always true.

Solving the differential equation Eq(3.5), we will get the $P(q)$.

$$P(q) = -K_r - \frac{q}{2D} \int h(q)dq + C_0 q \tag{3.6}$$

(where C_0 : arbitrary constant number from integration)

Because we assumed that total demand is fixed, a profit maximization criterion will drive up C_0 to infinity, which is clearly unreasonable. We must, therefore, recognize that the fixed demand assumption is reasonable only as long as prices stay within an acceptable range and not attempt to use proft maximization in determining C_0, instead, we will use a satisfying approach and attempt to impose some restrictions on C_0. In particular, we would like to determine C_0 such that

(1) Wholesaler's expected net profit with the new pricing policy is greater than his expected net profit with existing fixed unit price schedule(old price policy).

(2) Retailer's total cost with the new pricing policy is less than his total cost with the existing fixed unit price schedule.

Now we would like to determine the range of C_0 so that the nonlinear price schedule will be beneficial for both parties.

3.1 Wholesaler's Expected Profit

The wholesaler's expected net profit with the old pricing schedule is as follows.

$$\begin{aligned}\Pi_{old} &= \int_{h_0}^{h_1} \{ pq^+ \frac{D}{q^+} - cD - K_s \frac{D}{n(h_r)q^+} - \frac{n(h_r)q^+ - q^+}{2} h_s \} f(h_r)dh_r \\ &= (p-c)D + \int_{h_0}^{h_1} \frac{q^+}{2} h_s f(h_r)dh_r - \int_{h_0}^{h_1} \{ \frac{K_s D}{n(h_r)q^+} + \frac{n(h_r)}{2} q^+ h_s \} f(h_r)\, dh_r \end{aligned} \tag{3.7}$$

q^+ = economic order quantity of the retailer with the fixed price schedule $= (\frac{2K_r D}{h_r})^{\frac{1}{2}}$

$n(h_r)$ = positive integer ≥ 1

To supply the retailer's order quantity without shortage and to minimize the wholesaler's inventory holding cost, $n(h_r)$ should be the positive integer greater than or equal to 1. With a fixed unit pricing schedule, the wholesaler wants to maximize his expected net profit by minimizing the last term of Eq(3.7). Because $n(h_r)$ is the positive integer number greater than or equal to 1, we can find the range of h_r corresponding $n(h_r)$ values minimizing $\frac{K_s D}{n(h_r)q^+} + \frac{n(h_r)}{2} q^+ h_s$ by differentiating with repect to $n(h_r)$ and then choosing the nearest positive integer which makes the smaller value of the minimizing function. For the range of $h_{r(i)} < h_r \leq h_{r(i+1)}$, $n(h_r) = i+1$ is the optimal number(See Appendix). Therefore, the wholesaler's expected net profit with the old pricing policy can be expressed as

$$\Pi_{old} = (p-c)D + \int_{h_0}^{h_1} \frac{q^+}{2} h_s f(h_r)\, dh_r - C_1 \tag{3.8}$$

$$\text{where } C_1 = \text{Min} \int_{h_0}^{h_1} \{ \frac{K_s D}{n(h_r)q^+} + \frac{n(h_r)}{2} q^+ h_s \} f(h_r)\, dh_r$$

$$= \int_{h_0}^{\overset{\circ}{h}_{r(k)}} h_0 \{ \frac{K_s D}{kq^+} + \frac{k}{2} q^+ h_s \} f(h_r)\, dh_r$$

$$+ \sum_{i=k+1}^{l-1} \int_{\overset{\circ}{h}_{r(i-1)}}^{\overset{\circ}{h}_{r(i)}} \{ \frac{K_s D}{iq^+} + \frac{i}{2} q^+ h_s \} f(h_r)\, dh_r$$

$$+ \int_{\overset{\circ}{h}_{r(l-1)}}^{h_1} \{ \frac{K_s D}{lq^+} + \frac{l}{2} q^+ h_s \} f(h_r)\, dh_r \tag{3.9}$$

$\overset{\circ}{h}_{r(k-1)} < h_0 \leq \overset{\circ}{h}_{r(k-1)}$, $\overset{\circ}{h}_{r(l-1)} < h_1 \leq \overset{\circ}{h}_{r(l)}$ (See Appendix for the range of $\overset{\circ}{h}_{r(k)}$)

Now, let's determine the wholesaler's expected net profit with the new pricing policy. The economic order quantity of the retailer with the new pricing policy can be determined from the retailer's self-selection condition. By substituting Eq(3.6) into Eq(2.2), we get

$$q^* = \{ \frac{2 K_r D}{\frac{F(h_r)}{f(h_r)} + h_r - h_s} \}^{\frac{1}{2}} \tag{3.10}$$

Then,

$$\Pi_{new} = \int_{h_0}^{h_1} [\{-K_r - \frac{q}{2D} \int h(q)dq + C_0 q \} \frac{D}{q} - cD$$

$$- K_s \frac{D}{n(h_r)q(h_r)} - \frac{n(h_r)q(h_r) - q(h_r)}{2} h_s]_{q=q^*} f(h_r) dh_r$$

$$= (C_0 - c)D + \int_{h_0}^{h_1} \left\{ -\frac{K_r D}{q} - \frac{1}{2}\int h(q)dq + \frac{q}{2} h_s \right\}_{q=q^*} f(h_r)dh_r$$

$$- \int_{h_0}^{h_1} \left\{ \frac{K_s D}{n(h_r)q^*} + \frac{n(h_r)}{2} q^* h_s \right\} f(h_r)dh_r \tag{3.11}$$

$n(h_r)$ = positive integer ≥ 1

By the same procedure as in the fixed unit pricing schedule, we get

$$\Pi_{new} = (C_0 - c)D + \int_{h_0}^{h_1} \left\{ -\frac{K_r D}{q} - \frac{1}{2}\int h(q)dq + \frac{q}{2} h_s \right\}_{q=q^*} f(h_r)dh_r - C_2$$

where $C_2 = Min \int_{h_0}^{h_1} \left\{ \frac{K_s D}{n(h_r)q^*} + \frac{n(h_r)}{2} q^* h_s \right\} f(h_r)dh_r$

$$= \int_{h_0}^{h^n_{r(k)}} \left\{ \frac{K_s D}{kq^*} + \frac{k}{2} q^* h_s \right\} f(h_r)dh_r$$

$$+ \sum_{i=k+1}^{l-1} \int_{h^n_{r(i-1)}}^{h^n_{r(i)}} \left\{ \frac{K_s D}{iq^*} + \frac{i}{2} q^* h_s \right\} f(h_r)dh_r$$

$$+ \int_{h^n_{r(l-1)}}^{h_1} \left\{ \frac{K_s D}{lq^*} + \frac{l}{2} q^* h_s \right\} f(h_r)dh_r \tag{3.12}$$

$h^n_{r(k-1)} < h_0 \leq h^n_{r(k)}$, $h^n_{r(l-1)} < h_1 \leq h^n_{r(l)}$ (See Appendix for the the range of $h_{rk}{}^n$)

Thus, the wholesaler's expected net profit condition $\Pi_{new} \geq \Pi_{old}$ implies

$$C_0 \geq p + \frac{1}{D}\int_{h_0}^{h_1} \frac{q^+}{2} h_s f(h_r)dh_r$$

$$+ \frac{1}{D}\int_{h_0}^{h_1} \left\{ \frac{K_r D}{q} + \frac{1}{2}\int h(q)dq - \frac{q}{2} h_s \right\}_{q=q^*} f(h_r)dh_r - \frac{C_1 - C_2}{D} \tag{3.13}$$

For the convenience of notation, let $\int h(q)dq = H(q)$. Then we get the following necessary condition for C_0 which assures that the wholesaler will choose the nonlinear price schedule over a fixed unit price.

$$C_0 \geq p + \int_{h_0}^{h_1} \left\{ \frac{q^+}{2D} h_s + \frac{K_r}{q^+} + \frac{1}{2D} H(q^*) - \frac{q^*}{2D} h_s \right\} f(h_r)\, dh_r - \frac{C_1 - C_2}{D} = C_{min} \tag{3.14}$$

3.2 Retailer's Total Cost

Retailer's total cost condition $TC_{new} \leq TC_{old}$ implies

$$C_0 \leq p + \left(\frac{2K_r h_r}{D}\right)^{\frac{1}{2}} + \frac{1}{2D} H(q^*) - \frac{q^*}{2D} h_r = C_{max}(h_r) \tag{3.15}$$

If it was possible to get C_0 satisfying Eqs.(3.11) and (3.15) for all values of h_r, we would obtain a non-linear pricing schedule that is beneficial for both parties.

3.3 Existence of a Pareto Superior Pricing Schedule

The behavior of $C_{max}(h_r)$ is not only dependent on the size of the retailer's economic order quantity with the old pricing schedule but also that of the new pricing schedule. For the whole range of the retailer's holding cost satisfying $q^+ > q^*$, $C_{max}(h_r)$ is increasing function of h_r and for the whole range of the retailer's holding cost satisfying $q^+ < q^*$, $C_{max}(h_r)$, is decreasing function of h_r. If $\frac{F(h_r)}{f(h_r)}$ is not a monotone increasing function of h_r, it makes this analysis much more complicated. To simplify our analysis, let's restrict $\frac{F(h_r)}{f(h_r)}$ to the monotone increasing function of h_r. This restriction is not severe since it is met by many common distributions such as the uniform, exponential, and normal distribution.

There are three possibilities which must be considered.

Possibility 1. C_0 exists for the entire range of h_r.

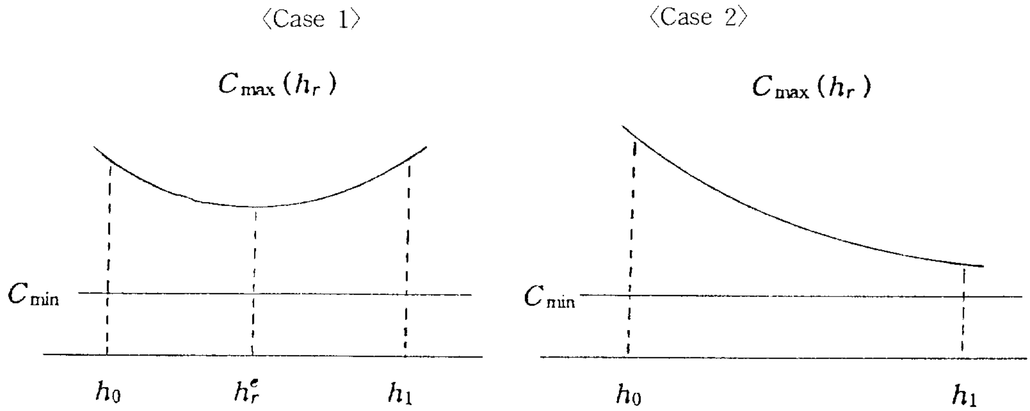

Fig.3.1 Range of C_0 that yields a Pareto superior nonlinear price schedlule

We can summarize the result for both parties as follows.

Table 1. Range of C_0 which is beneficial to both parties

Range of C_0		Result	
Case 1	Case 2	Retailer	Wholesaler
$C_{min} < C_0 < C_{min}(h_r^e)$	$C_{min} < C_0 < C_{min}(h_1)$	Better	Better
$C_0 = C_{min}$	$C_0 = C_{min}$	Better	Same*
$C_0 = C_{min}(h_r^e)$	$C_0 = C_{min}(h_1)$	Same*	Better

* 'Same' means expected net profit of the wholesaler or the retailer's total cost is the same as in the fixed unit pricing schedule

Table 2. Benefit of Wholesaler and Retailer

Wholesaler	Retailer
$\Pi_{new} - \Pi_{old} = (C_{mix} - C_{min})D$	$TC_{new} - TC_{old} = (C_{mix}(h_r) - C_{min})D$

* C_{max} is the fixed $C_{max}(h_r^e)$ or $C_{max}(h_1)$

Possibility 2. C_0 doesn't exist for the entire range of h_r

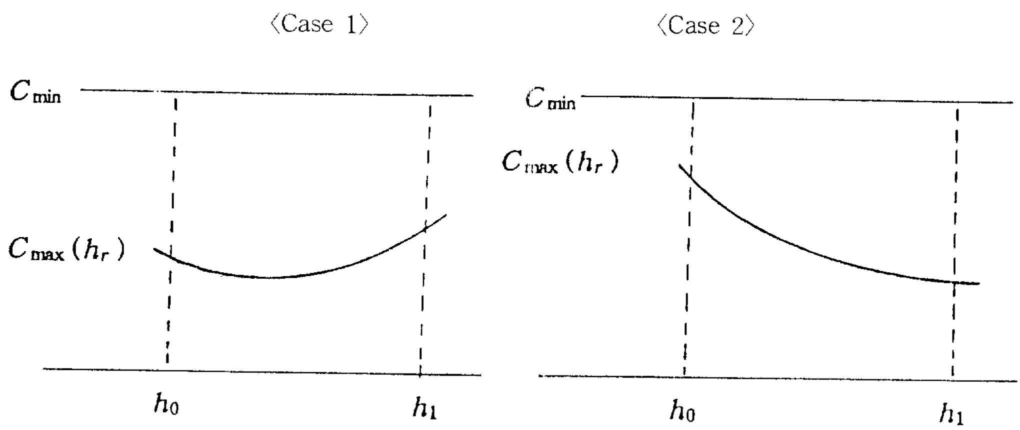

Fig.3.2 Range of C_{min} and $C_{max}(h_r)$

If C_{min} is determined to be beneficial for one party, the other party is always worse off. So the wholesaler should stick with the old fixed unit pricing policy rather than non-linear pricing policy.

Possibility 3. C_{min} can be determined such that the wholesaler will benefit but the retailer might be worse off with certain probability ($TC_{new} \geq TC_{old}$)

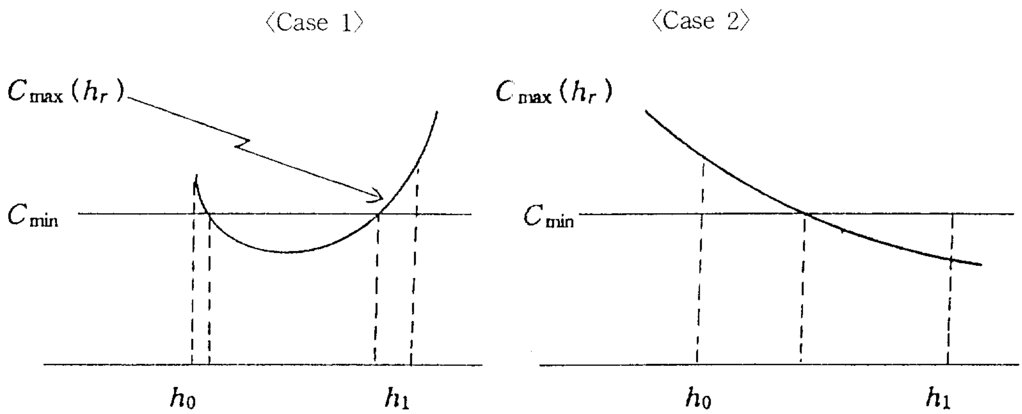

Fig.3.3 Range of retailer's holding cost that leads to $TC_{new} \geq TC_{old}$

There is a trade-off between the probability that the retailer will be worse off and the wholesaler's expected net profit. If the retailer is not allowed to be worse off than he was under the old fixed unit pricing policy, behavior of both parties will be the same as Possibility 2. If the retailer is allowed to be worse off with certain probability α (and better off with probability $1-\alpha$), then the wholesaler can increase his expected net profit until α is reached. The behavior of both parties will be the same as Possibility 1 until the mentioned probability falls below α. If this probability is greater than α, the behavior of both parties will be the same as Possibility 2. This process can be applied to Possibility 1 to increase the wholesaler's expected net profit until the appropriate probability is less than or equal to α.

3.4 Discussion and Modification

Discussion

We want to make several additional remarks with respect to this model. Generally, we can assume that $P(0) = 0$ and $P'(0) \geq 0$ in a general non-linear quantity discount pricing schedule. But our pricing policy has a little different shape from the general case. From the Fig. 3.4, we note that our schedule has negative value on the $P(q)$ axis while the pricing policy is effective only on the range of $q_0 \leq q \leq q_1$. The fact that $P(0) < 0$ is the result of the extension of our effective pricing schedule to the $P(q)$ axis. We want to find the non-linear quantity discount pricing schedule to increase the wholesaler's or manufacturer's expected net profit as compared with the old fixed unit pricing schedule. If the supplier can benefit from this non-linear pricing schedule, we don't need to worry about the special feature of this shape. If he cannot benefit from this new pricing schedule, he should stick with the old pricing schedule. In general case, the pricing schedule is positive in the effective zone due to the fact that the unit inventory holding cost is less than the average unit purchasing price of the product.

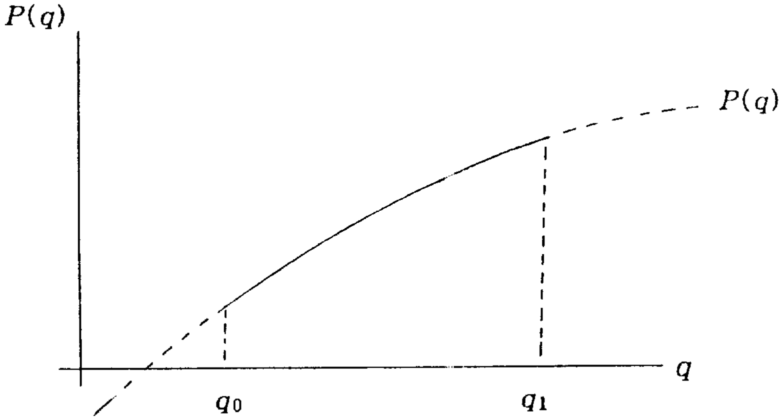

Fig.3.4 Effective pricing schedule zone

Modification

Our pricing schedule Eq.(3.6) implicitly assumes that $\frac{F(h_r)}{f(h_r)} + h_r - h_s > 0$ for the entire range of h_r to get the retailer's economic order quantity with the new pricing schedule. This means that if $\frac{F(h_r)}{f(h_r)} + h_r - h_s \leq 0$, then the retailer's economic order quantity should be infinitely large. Futhermore, even in the fixed unit price schedule, the retailer's economic order quantity grows very large for the very small inventory holding cost.

To keep a very large inventory only for the reason of inventory holding cost and ordering cost is impractical due to the deterioration of the goods, storage space limitation, technological improvement of the product, etc. Let's define Q to be the largest possible retailer's order quantity, $h_{r\,old}^*$ to be the retailer's holding cost, which gives the largest possible retailer's order quantity in a fixed unit pricing schedule and $h_{r\,new}^*$ to be the retailer's holding cost, which gives the largest possible retailer's economic order quantity in the new pricing schedule. Then $h_{r\,old}^*$ and $h_{r\,new}^*$ can be expressed as below.

$$h_{r_{old}}^* = \frac{2K_rD}{Q^2}$$

$$\frac{F(h_{r\,new}^*)}{f(h_{r\,new}^*)} + h_{r\,new}^* - h_s = \frac{2K_rD}{Q^2} \tag{3.16}$$

Accordingly, we can easily modify our pricing policy by thye following steps.

Step 1. Change C_{min} by recalculating C_1 and C_2

Step 2. Change C_{max} (h_r) by recalculating TC_{old} and TC_{new}

Step 3. Check the existence of C_0 by the same method as in section (3.3)

The old pricing schedule is no longer a tangent line on the new pricing schedule, however, the new pricing schedule crosses the old pricing schedule from below at the maximum quantity of Q as illustrated. The average price charge is less than the reference fixed unit price and the new profit is lower.

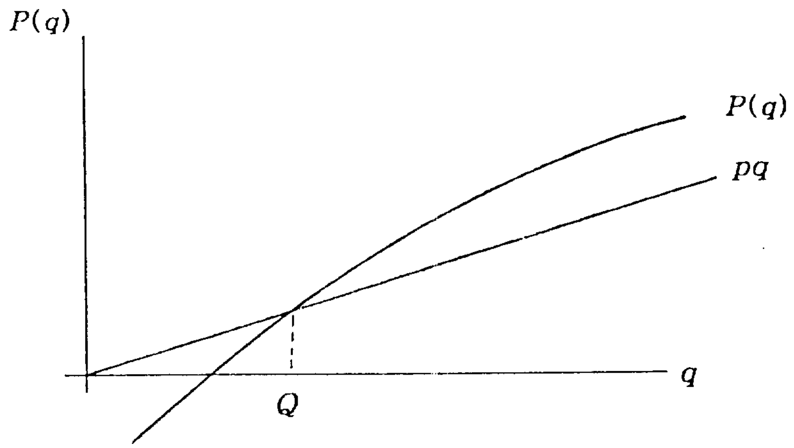

Fig. 3.5 Shape of old and new pricing schedule

4. Numerical Example

As we take the wholesaler's point of view to find a quantity dependent pricing schedule that will be benificial to the wholesaler and the retailer, we concentrate only on the highest pricing schedule in the ranges of the Pareto superior schedules.

Three common probability distribution functions whose $\frac{F(h_r)}{f(h_r)}$ are monotone increasing function of h_r are chosen for the retailer's holding cost : *Uniform*, *Truncated Exponential*, *Truncated Normal*.

We assume the following parameter values.

Table 3. Sample Data

K_s	K_r	h_s	h_{min}	h_{max}	h_{mean}	$\sigma^2{}_{hr}$	D	p	Q
100	50	3.0	4.0	6.0	5.0	0.5	150	10	150

※ ($\sigma^2{}_{hr}$ is given for the truncated normal distribution only.)

The retailer's economic order quantity depends on the value of h_r^e. If the retailer's unit inventory holding cost lies between h_{min}, and h_r^e, he will increase his ordering quantity. If it lies between and h_r^e, and h_{min}, he will reduce his ordering quantity. To reduce his expected inventory level, the wholesaler offers a pricing schedule that induces the retailer to increase his order quantity. The following table gives the ranges of the retailer's economic order quantities for the sample data.

Table 4. Ranges of the Retailer's EOQ.

EOQ	Distribution Function		
	Uniform	Exponential	Normal
q_0^* q_1^*	54.8 122.5	52.4 122.5	46.1 122.5
q_0^+ q_1^+	50.5 61.2		

The behavior of the Pareto superior pricing schedule and its range is closely related to the behavior of the retailers' economic order quantities, which, in turn, are dependent on the probability distribution, range of the holding cost, the value of h_r^e, and so on. The possibility of the two cases already discussed further complicates the analysis of the pricing schedule behavior. After examining C_{min}, $C_{max}(h_r)$ and $P(q)$, it is very difficult to obtain the general pattern of the pricing schedule and its sensitivity to the changes in the model parameters.

Nevertheless, we can conclude that a non-linear pricing schedule can benefit both the retailers and the supplier by transferring part of the inventory holding cost from the wholesaler to the retailers in return for the reduced gross revenue.

Table 5. Range of C0 and wholesaler's benefit

Distribution	h_s	q^*_0	q^*	C_{max}	C_{min}	Expected benefit
Uniform	2.0	50.0	86.6	11.00	10.99	1.5
	3.0	54.8	122.5	11.09	11.05	6.0
Exponential	2.0	48.2	86.6	10.97	10.76	31.5
	3.0	52.4	122.5	10.95	10.50	67.5
Normal	2.0	43.1	86.6	10.97	10.75	33.0
	3.0	46.1	122.5	10.99	10.52	70.5

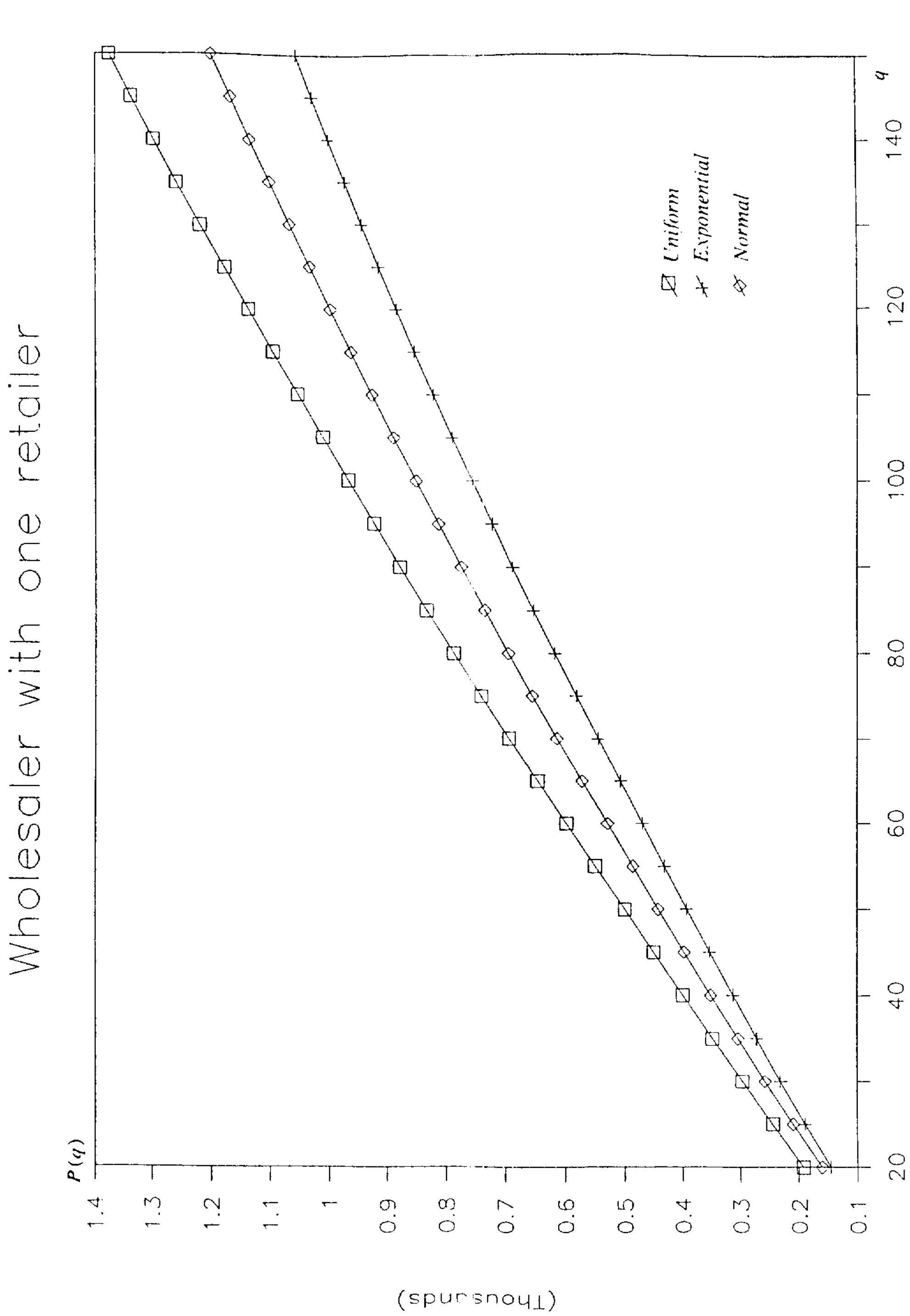

Fig.4.1 Pareto Superior Pricing Schedule for Various Probability Distribution

5. Conclusion

The wholesaler with large unit inventory holding cost can offer a large quantity discount thanks to the great benefit which comes from the transfer of part of his inventory to the retailer. The retailer's increasing average inventory holding cost can be offset by the quantity discount and by the savings of the ordering cost. When the wholesaler's unit inventory holding cost is not large enough to bring considerable benefit, there is no reason for the wholesaler to offer the non-linear pricing schedule instead of uniform fixed pricing schedule. Consideration of an upper limit on the retailer's order quantity affects the existence of a Pareto superior pricing schedule as well as the specific structure of the pricing schedule which maximizes the wholesaler's profit.

APPENDIX. RANGE OF h_r FOR INTEGER VALUE OF $n(h_r)$

A.1. Old pricing schedule.

Let's assume $n(h_r)$ is a continuous variable. Then, by solving the following equation

$$\frac{d}{dn(h_r)}\left\{\frac{KsD}{n(h_r)q^+}+\frac{n(h_r)}{2}q^+ h_s\right\}=0 \text{ where } q^+=\left(\frac{2K_rD}{h_r}\right)^{\frac{1}{2}}$$

we will get

$$n(h_r)^2=\frac{K_sh_r}{h_sK_r}$$

Therefore,

$0<n(h_r)<1$ if $0<h_r\leq K_rh_s/K_s$

$1<n(h_r)<2$ if $K_rh_s/K_s\leq h_r\leq 4K_rh_s/K_s$

$2<n(h_r)<3$ if $4K_rh_s/K_s\leq h_r\leq 9K_rh_s/K_s$

$k<n(h_r)<k+1$ if $k^2K_rh_s/K_s<h_r\leq(k+1)^2K_rh_s/K_s$

From the condition that $n(h_r)$ should be an integer greater than or equal to 1, $n(h_r)=1$ for $0<n(h_r)\leq 1$. For $k<n(h_r)\leq k+1$, $n(h_r)$ can be either k or $k+1$.

We can find the range of h_r, whose optimum is $n(h_r) = k$ or $k+1$, by equating

$$\frac{K_sD}{kq^+} + \frac{k}{2}q^+ h_s = \frac{K_sD}{(k+1)q^+} + \frac{(k+1)}{2}q^+ h_s$$

Finally, we have the range of h_r for the integer value of $n(h_r)$

$n(h_r) = 1$ if $0<h_r\leq 2K_rh_s/K_s = h_{r(1)}^0$

$n(h_r) = 2$ if $2K_rh_s/K_s<h_r\leq 6K_rh_s/K_s = h_{r(2)}^0$

$n(h_r) = 3$ if $6K_rh_s/K_s<h_r\leq 12K_rh_s/K_s = h_{r(3)}^0$

$n(h_r) = k$ if $(k-1)kK_rh_s/K_s<h_r\leq k(k+1)k_rh_s/K_s = h_{r(k)}^0$

It can be immediately seen that the situation of lot-for-lot arises if $0<h_r\leq\frac{2K_rh_s}{K_s}$

A.2. New pricing schedule

By the same procedure, we can get the range of h_r under the new pricing schedule.

$n(h_r) = 1$ if $0<h_r\leq h_{r(1)}^n$

$n(h_r) = 2$ if $h_{r(1)}^n<h_r\leq h_{r(2)}^n$

$n(h_r) = 3$ if $h_{r(2)}^n<h_r\leq h_{r(3)}^n$

$n(h_r) = k$ if $h_{r(k-1)}^n<h_r\leq h_{r(k)}^n$

(where $h_{r(k)}^n$ is the h_r satisfying $\frac{F(h_r)}{f(h_r)} + h_r - h_s = \frac{k(k+1)K_rh_s}{K_s}$

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