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Theory of Polarization Grating Formation Induced by the Photosensitivity in Birefringent Optical Fibers*

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The two-photon bleaching model is generalized to derive the dynamical equations which describe the polarization grating formation induced by the photosensitivity in birefringent optical fibers. The frequency response of the induced polarization grating is studied numerically and its relation to the material parameter is investigated.

I. INTRODUTION

Since the discovery of photosensitivity of Ge-doped optical fibers by Hill *et al.*[1] in 1978, there has been considerable efforts both theoretically and experimentally to understand the mechanism of this phenomenon. Although a microscopic description of the formation of the photosensitive grating is still lacking, it was recently shown that the experimentally observed time dependence of the reflectivity can be explained by a local two-photon bleaching model (TPBM),[2-4]

$$\frac{\partial}{\partial t}\epsilon(z,\ t)=AI^2(z,\ t), \qquad (1)$$

where $\epsilon(z,\ t)$ is the effective dielectric constant at a point z along the fiber, $I(z,\ t)$ is the intensity at that point, and A is the constant of proportionality. For a large class of such models, it was demonstrated that there exists a universal evolution parameter which makes it possible to transform the coupled partial differential equations describing the grating growth and state of the electromagnetic field in the fiber into coupled ordinary integro-differential equations; it was then shown that the TPBM leads to a prediction of perfectly phase-matched grating growth, corresponding to a sta-

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Parent *et al.*[5] were the first to observe that the refractive index changes induced in optical fibers by grating formation were anisotropic.[5-7] Hand and Russell demonstrated that the birefringence induced by the visible light in highly birefringent fibers can be used to form polarization couplers.[6] The coupling was shown to be due to a photosensitivity-induced periodic modulation of the birefringence created by the interference of the two forward-propagating polarization eigenmodes with a period equal to the beat length of the fiber (This type of couplers will be called in this paper as the photosensitive polarization gratings to distinguish from the photosensitive Bragg gratings). An and Sipe generalized the TPBM to treat the dynamics of photosensitive polarization grating formation by assuming that the defects which cause the photosensitivity is uniformly distributed along all directions,[8] although later Lauzon *et al.* reported that the distribution of defects could be not isotropic.[9] In this paper the full dynamical equations for the photosensitive polarization grating formation which is written by launching the visible light (at 488 or 514 nm) to the fiber axis (see Fig. 1) are derived in detail. It is shown that under a simple boundary condition, the equations include only one material parameter which characterizes the photosensitivity. The dependence of the frequence response of the induced polarization grating on the material parameters is studied numerically.

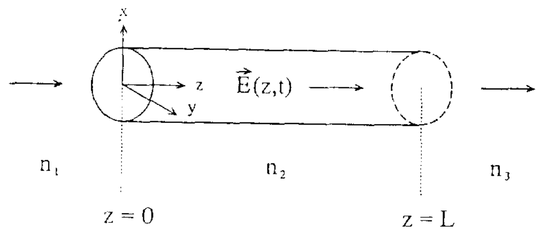

Fig. 1. Schematic of the geometry we consider. We choose x-axis (or y-axis) as the slow (or fast) principal axis of the birefringence fiber. The indices of refraction are given by n_1 for $z<0$, n_2 for $0<z<L$, and n_3 for $z>L$. Note that the light enters from $z=0$ and we neglect the Fresnel reflection from the back surface ($z=L$) of the fiber.

This paper is organized as follows. In Sec. 2 the coupled mode equations which describe the field propagation in a birefringent fiber are derived. In Sec. 3 the TPBM is generalized to derive the equation which could explain the change of dielectric tensor in time due to two-photon absorption. In Sec. 4 the coupled partial differential equations, which was derived in Sec. 2 and 3, are solved numerically and the results are discussed. Conclusions are given in Sec. 5.

II. Coupled mode equations

For the fiber geometry shown in Fig. 1, we write the electric field as $\mathbf{E}(z, t)e^{-i\omega t}+c.c.$, with

$$\mathbf{E}(z, t)=E_1(z, t)\hat{x}+E_2(z, t)\hat{y}.$$

The effective dielectric tensor of the birefringent fiber is then written as

$$\overleftrightarrow{\epsilon}(z, t)=\epsilon_0\overleftrightarrow{\sigma}+\delta\overleftrightarrow{\sigma}_3+\begin{pmatrix}\Delta\epsilon_{11}(z, t) & \Delta\epsilon_{12}(z, t)\\ \Delta\epsilon_{21}(z, t) & \Delta\epsilon_{22}(z, t)\end{pmatrix}, \quad (3)$$

where ϵ_0 describes the background (isotropic) dielectric response, $\Delta\epsilon_{ij}$ describes the change due to two-photon absorption, δ models the original birefringence of the fiber, and $\overleftrightarrow{\sigma}_{0,3}$ are defined by

$$\overleftrightarrow{\sigma}_0\equiv\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},\quad \overleftrightarrow{\sigma}_3\equiv\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \quad (4)$$

To simplify the problem we neglect the Fresnel reflection from the backface (i.e., set $n_2=n_3$ in Fig. 1), and therefore we write

$$\mathbf{E}(z, t)=\vec{\varepsilon}(z, t)e^{ikz}, \quad (5)$$

where $k=\omega\sqrt{\epsilon_0}/c$. Assuming $\epsilon_0\gg\delta, \Delta\epsilon_{ij}$ and substituting Eq. (5) into the wave equation

$$\frac{\partial^2}{\partial z^2}\vec{\varepsilon}(z, t)+\frac{\omega^2}{c^2}\overleftrightarrow{\epsilon}(z, t)\cdot\vec{\varepsilon}(z, t)=0, \quad (6)$$

and applying the slowly varying envelope approximation to $\vec{\varepsilon}(z, t)$, we find the coupled mode equations

$$\frac{1}{k}\frac{\partial\varepsilon_1(z, t)}{\partial z}=\frac{i}{2\epsilon_0}[\delta\varepsilon_1(z, t)+\Delta\epsilon_{11}(z, t)\varepsilon_1(z, t)+\Delta\epsilon_{12}(z, t)\varepsilon_2(z, t)]$$

$$\frac{1}{k}\frac{\partial\varepsilon_2(z, t)}{\partial z}=\frac{i}{2\epsilon_0}[-\delta\varepsilon_2(z, t)+\Delta\epsilon_{12}(z, t)\varepsilon_1(z, t)+\Delta\epsilon_{22}(z, t)\varepsilon_2(z, t)] \quad (7)$$

Now define new envelope functions $U_{1,2}(z, t)$ by

$$\varepsilon_1(z, t)=U_1(z, t)e^{iKz}$$
$$\varepsilon_2(z, t)=U_2(z, t)e^{-iKz}, \quad (8)$$

where $K=(\delta/2\epsilon_0)k$, and put Eq. (8) into Eq. (7). Then the coupled mode equations become

$$\frac{1}{k}\frac{\partial U_1(z, t)}{\partial z}=i[a_1(z, t)U_1(z, t)+b(z, t)U_2(z, t)]$$

$$\frac{1}{k}\frac{\partial U_2(z, t)}{\partial z}=i[b^*(z, t)U_1(z, t)-a_2(z, t)U_2(z, t)], \quad (9)$$

where new grating variables $a_{1,2}(z, t)$ and $b(z, t)$ are defined by

$$\frac{1}{2\epsilon_0}\Delta\epsilon_{ii}(z, t)=a_i(z, t)$$
$$\frac{1}{2\epsilon_0}\Delta\epsilon_{12}(z, t)=b(z, t)e^{2iKz}+c.c., \quad (10)$$

where $i=1$, 2, and $b(z, t)$ is also assumed to be a slowly varying function.

In our assumed geometry, a further simplification follows because isotropic changes in the dielectric constant do not affect the polarization state of the field. Defining

$$\alpha(z, t)\equiv\frac{1}{2}(a_1(z, t)+a_2(z, t))$$

$$\alpha(z,\ t) \equiv \frac{1}{2}(\alpha_1(z,\ t) - \alpha_2(z,\ t)), \tag{11}$$

and putting

$$U_1(z,\ t) = u_1(z,\ t)e^{ik\int_0^z \bar{\alpha}(z',\ t)dz'}$$
$$U_2(z,\ t) = u_2(z,\ t)e^{ik\int_0^z \bar{\alpha}(z',\ t)dz'}, \tag{12}$$

into Eq. (9), we finally have the coupled mode equations which donot involve $\bar{\alpha}(z,\ t)$:

$$\frac{1}{k}\frac{\partial U_1(z,\ t)}{\partial z} = i[\alpha(z,\ t)u_1(z,\ t) + b(z,\ t)u_2(z,\ t)]$$
$$\frac{1}{k}\frac{\partial U_2(z,\ t)}{\partial z} = i[b^*(z,\ t)u_1(z,\ t) - \alpha(z,\ t)u_2(z,\ t)]. \tag{13}$$

Notice, from the definitions (5), (8), and (12), $u_{1,2}(z,\ t)$ satisfy the same boundary conditions as $E(z,\ t)$ at $z=0$, i.e.,

$$u_1(0,\ t) = E_1(0,\ t)$$
$$u_2(0,\ t) = E_2(0,\ t) \tag{14}$$

III. Generalization of the TPBM

In order to explain the change of dielectric tensor in time due to two-photon absorption, we generalize the simplest TPBM[2-4] tothe geometry shown in Fig. 1;

$$\frac{\partial}{\partial t}\epsilon_{ij}(z,\ t) = \sum_{k,l,m,n=1}^{2} A_{ijklmn}E_k(z,\ t)E_l(z,\ t)E_m^*(z,\ t)E_n^*(z,\ t) \tag{15}$$

which in general involves 64 ($=2^6$) independent complex constants A_{ijklmn}. However, since the medium is assumed isotropic, all that survives is $J=0$ representation of that 6-th rank Cartesian tensor, which involves 15 independent components, and each component may be taken as the products of 3 Kronecker deltas. Most generally we can write

$$\begin{aligned} A_{ijklmn} = {} & a_1\delta_{ij}\delta_{kl}\delta_{mn} + a_2\delta_{ij}\delta_{km}\delta_{ln} + a_3\delta_{ij}\delta_{kn}\delta_{lm} \\ & + a_4\delta_{ik}\delta_{jl}\delta_{mn} + a_5\delta_{ik}\delta_{jm}\delta_{ln} + a_6\delta_{ik}\delta_{jn}\delta_{lm} \\ & + a_7\delta_{il}\delta_{jk}\delta_{mn} + a_8\delta_{il}\delta_{jm}\delta_{kn} + a_9\delta_{il}\delta_{jn}\delta_{km} \\ & + a_{10}\delta_{im}\delta_{jk}\delta_{ln} + a_{11}\delta_{im}\delta_{jl}\delta_{kn} + a_{12}\delta_{im}\delta_{jn}\delta_{kl} \\ & + a_{13}\delta_{in}\delta_{jk}\delta_{ln} + a_{14}\delta_{in}\delta_{jl}\delta_{kn} + a_{15}\delta_{in}\delta_{jn}\delta_{kl}. \end{aligned} \tag{16}$$

Now since ϵ_{ij} is symmetric tensor, however, we can require A_{ijklmn} to be symmetric in the first two indices, (ij). Similarly, since all $\boldsymbol{E}(z,\ t)$ and $\boldsymbol{E}^*(z,\ t)$ fields are the same, we also require A_{ijklmn} to be symmetric in (kl) and (mn). So instead of A_{ijklmn} we can work with $\bar{A}_{ijklmn}$ which is symmetric in (ij), (kl), and (mn), i.e.,

$$\begin{aligned} \bar{A}_{ijklmn} \equiv {} & \frac{1}{8}(A_{ijklmn} + A_{ijlkmn} + A_{ijklnm} + A_{ijlknm} + A_{jiklmn} \\ & + A_{jilkmn} + A_{jiklnm} + A_{jilknm}) \\ = {} & b_1\delta_{ij}\delta_{kl}\delta_{mn} + b_2[\delta_{ij}\delta_{km}\delta_{ln} + \delta_{ij}\delta_{lm}\delta_{kn}] \\ & + b_3[\delta_{ik}\delta_{jl}\delta_{mn} + \delta_{il}\delta_{jk}\delta_{mn}] \\ & + b_4[\delta_{im}\delta_{jn}\delta_{kl} + \delta_{in}\delta_{jm}\delta_{kl}] \\ & + b_5[\delta_{ik}\delta_{jn}\delta_{lm} + \delta_{ik}\delta_{jm}\delta_{ln} + \delta_{il}\delta_{jm}\delta_{kn} + \delta_{il}\delta_{jn}\delta_{km} \\ & + \delta_{im}\delta_{jk}\delta_{ln} + \delta_{im}\delta_{jl}\delta_{kn} + \delta_{in}\delta_{jk}\delta_{lm} + \delta_{in}\delta_{jl}\delta_{km} \end{aligned} \tag{17}$$

where the cosfficients b_1, b_2, b_3, b_4, and b_5, which are linear combinations of the cosfficients a_i's in Eq. (16), are in general complex numbers. Now if we assume that ϵ_{ij} remains real (i.e., that no significant linear real absorption is induced), then

$$\bar{A}_{ijklmn} = \bar{A}^*_{ijklmn}, \tag{18}$$

and this gives the conditions

$$\begin{aligned} b_1 &= b_1^* \equiv r_1 \\ b_2 &= b_2^* \equiv \frac{1}{2}r_2 \\ b_3 &= b_4^* \equiv \frac{1}{2}c \\ b_5 &= b_5^* \equiv \frac{1}{4}r_3 \end{aligned} \tag{19}$$

where r_1, r_2, r_3 are real numbers, and c is a complex number.Substituting Eqs. (17) and (19) into Eq. (15) with A_{ijklmn} replaced by $\bar{A}_{ijklmn}$, we obtain the two-photon equations which only have 1 complex and 3 real parameters:

$$\begin{aligned} \frac{\partial \epsilon_{ij}}{\partial t} = {} & [r_1(\boldsymbol{E}\cdot\boldsymbol{E})(\boldsymbol{E}^*\cdot\boldsymbol{E}^*) + r_2(\boldsymbol{E}\cdot\boldsymbol{E}^*)^2]\delta_{ij} \\ & + r_3(E_iE_j^* + E_i^*E_j)(\boldsymbol{E}\cdot\boldsymbol{E}^*) \\ & + cE_iE_j(\boldsymbol{E}^*\cdot\boldsymbol{E}^*) + c^*E_i^*E_j^*(\boldsymbol{E}\cdot\boldsymbol{E}), \end{aligned} \tag{20}$$

where $i(j)=1,\ 2$.

Putting Eqs. (5), (8), and (10) into Eq. (20), and keeping only the phase-matched terms, we have

$$\frac{\partial \alpha_1(z,\ t)}{\partial t} = 2B_s|U_1(z,\ t)|^4 + 2B_o|U_2(z,\ t)|^4 + 2B_m|U_1(z,\ t)|^2|U_2(z,\ t)|^2$$

$$\frac{\partial \alpha_2(z,\ t)}{\partial t} = 2B_s|U_2(z,\ t)|^4 + 2B_o|U_1(z,\ t)|^4$$

$$+2B_m|U_1(z,\ t)|^2|U_2(z,\ t)|^2$$

$$\frac{\partial b(z,\ t)}{\partial t}=(B_s-B_o)[|U_1(z,\ t)|^2+|U_2(z,\ t)|^2]U_1(z,\ t)U_2^*(z,\ t)$$
$$+iB_x[|U_1(z,\ t)|^2-|U_2(z,\ t)|^2]U_1(z,\ t)U_2^*(z,\ t), \tag{21}$$

where the coefficients B_s, B_o, B_m, and B_x are defined by

$$B_s=\frac{1}{4\epsilon_0}[r_1+r_2+2r_3+2Re(c)]$$
$$B_o=\frac{1}{4\epsilon_0}[r_1+r_2]$$
$$B_m=\frac{1}{2\epsilon_0}[r_2+r_3]$$
$$B_x=\frac{1}{2\epsilon_0}Im\ (c) \tag{22}$$

Finally, using the definitions (10) and (11) and removing theequation for $\partial a(z,\ t)//\partial t$, we have

$$\frac{1}{B}\frac{\partial a(z,\ t)}{\partial t}=(|u_1(z,\ t)|^4-|u_2(z,\ t)|^4)\cos\theta$$
$$\frac{1}{B}\frac{\partial b(z,\ t)}{\partial t}=(|u_1(z,\ t)|^2+|u_2(z,\ t)|^2)u_1(z,\ t)u_2^*(z,\ t)\cos\theta$$
$$+i(|u_1(z,\ t)|^2-|u_2(z,\ t)|^2)u_1(z,\ t)u_2^*(z,\ t)\sin\theta \tag{23}$$

where two real constants B and θ are defined by

$$B=\sqrt{(B_s-B_o)^2+B_x^2}$$
$$\tan\theta=(B_s-B_o)/B,$$

and Eq. (23) are subjected to the initial conditions

$$a(z,\ 0)=b(z,\ 0)=0. \tag{25}$$

Thus if we absorb the overall constant B in Eq. (23) into the time variable t to define a new time unit, the aspect of the photosensitive response of a birefringent optical fiber that is important for polarization grating formation is specified by one real parameter θ.

The coupled partial differential equations (13) and (23), together with initial conditions (25) and the specification of the incident field at $z=0^+$, define our dynamical problem. We take that field to be

$$\varepsilon_1(0,\ t)=U_1(0,\ t)=\cos\psi$$
$$\varepsilon_2(0,\ t)=U_2(0,\ t)=e^{i\chi}\sin\psi \tag{26}$$

where ψ and χ are taken to be constant, and any overall amplitude factor we take to be absorbed (see Eq. (23)) in the time unit.

The physical significance of θ can be elucidated by returning to Eq. (20) and considering various local polarizations and the birefringences they induce. For example, for the polarization state given by Eq. (26) at $z=0$, which represent an elliptical polarization, the induced dielectric tensor after an infinitesimal time Δt is given by

$$\Delta\vec{\epsilon}\propto\begin{pmatrix}\cos(2\psi)\cos\theta & \sin(2\psi)\Theta(\psi,\ \chi,\ \theta)\\ \sin(2\psi)\Theta(\psi,\ \chi,\ \theta) & -\cos(2\psi)\cos\theta\end{pmatrix}, \tag{27}$$

where $\Theta(\psi,\ \chi,\ \theta)\equiv\cos\chi\cos\theta+\cos(\psi)\sin\chi\sin\theta$, and we neglected the induced isotropic change of the dielectric tensor.

By diagonalizing the tensor given by Eq. (27) we can study how the polarization state of the field at an instant time induce the birefringence in the fiber for a given material parameter θ. For the special case of linear polarization, the principal axes of the induced birefringence are those of the polarization direction and that direction perpendicular to it; for $-\pi/2<\theta<\pi/2$ (or $\pi/2<\theta<3\pi/2$) the induced change in dielectric constant is greater (or lesser) along the polarization axis. More generally, for elliptically polarized light, if $\theta=0$ or π the principal axes of the induced birefringence are those defined by the major and minor axes of the ellipse, while for $\theta=\pm\pi/2$ the principalaxes are at $\pm 45°$ to the major axes (see Fig. 2); the other directions correspond to the other values of θ. For circularlypolarized light only isotropic changes are induced, as expe-

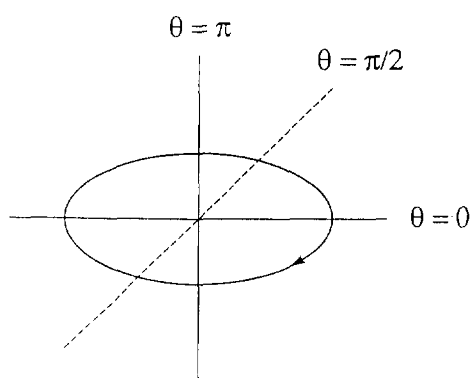

Fig. 2. Polarization ellipse of a light and the principal axes of the induced birefringence for different θs.

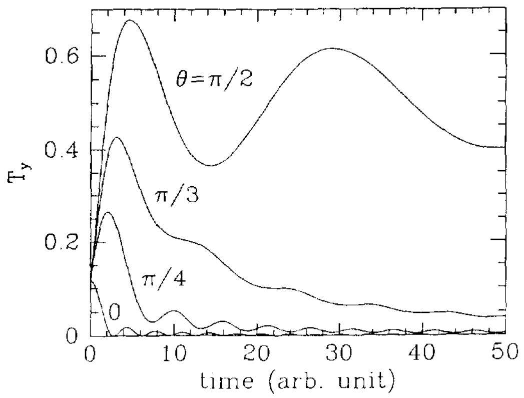

Fig. 3. Plot of T_y versus t for $\chi=0$, $\psi=20°$, and $\theta=0$, $\pi/4$, $\pi/3$, and $\pi/2$. Note that as $t\to\infty$, $T_y\to 0$ for $\theta\neq\pi/2$, and $T_y\to\frac{1}{2}$ for $\theta=\pi/2$.

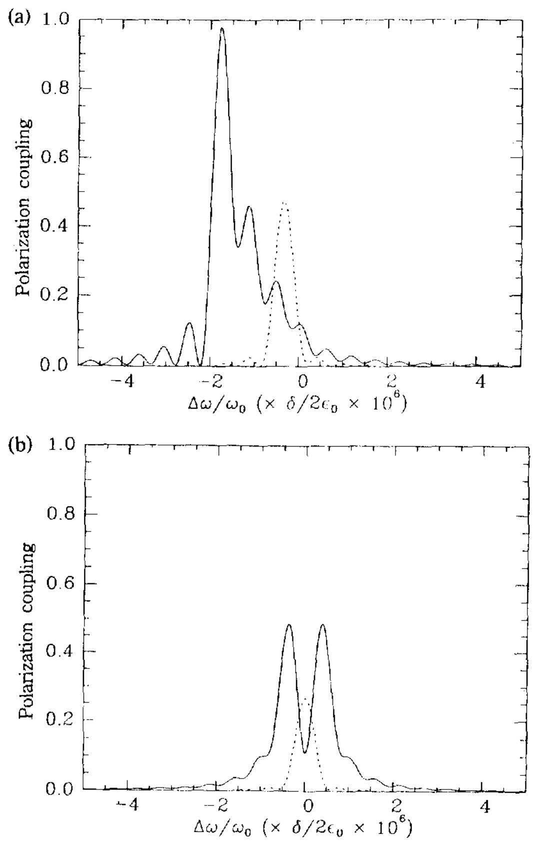

Fig. 4. Polarization coupling as a function of detuning frequency $\Delta\omega$ ($\equiv\omega-\omega_0$) for the grating which was written by the light of frequency ω_0, for $\chi=0$, $\psi=20°$, and (a) $\theta=0$ or (b) $\theta=\pi/2$. $t=50$ (dotted line) and $t=200$ (solid line).

cted from the assumption that the medium is isotropic.

IV. Numerical results and discussions

We use a fourth-order Runge-Kutta method to solve numerically the coupled partial differential equations defined in Sec. 2 and 3. In Fig. 3 we plot the intensity of light transmitted through a y-axis polarizer at the output end of the fiber, T_y, versus time for various values of θ, when $\chi=0$ and $\psi=20°$. The shapes of the curves for different θ are quite different, which of course could provide one possible way of experimentally determining the magnitude of material parameter θ

In Fig. 4 we plot the frequency responses of the grating which was written by the light of frequency ω_0, for $\chi=0$, $\psi=20°$, and (a) $\theta=0$ or (b) $\theta=\pi/2$. We stop growing the grating numerically, at $t=50$ or $t=100$, and put a probe beam of frequency ω into the front surface of the fiber to study the polarization coupling as a function of frequency for each time and θ. For small time (e.g., at $t=50$) we see that the shapes of coupling curves as function of frequency are similar between $\theta=0$ and $\theta=\pi/2$, except that the maximum coupling of $\theta=0$ case are much larger than that of $\theta=\pi/2$ case and that the center of the peak for $\theta=0$ is shifted from $\Delta\omega=0$. As time increases (e.g., $t=200$), we see that the shapes of coupling curves become totally different between $\theta=0$ and $\theta=\pi/2$. For $\theta=0$ the coupling curve has one central peak with many sidelobes and the shape of the curve is asymmetric. For $\theta=0$, on the other hand, the coupling curve has two peaks and it is symmetric. We also notice that the center of the central peak for $\theta=0$ is shifting away from $\Delta\omega=0$ as time increases. Therefore we expect that this kind of experiment could provide another possible way of determining the magnitude of material parameter θ.

V. Conclusions

The two-photon bleaching model was generalized to describe the photosensitive polarization grating formation in birefringent optical fibers. The assumption of the isotropic symmetry of the medium and the simplification of the fiber geometry were used to derive the dynamical equations of the system. These equations were used to study numerically the frequency response of the induced polarization grating and it was shown that for a mature grating different types of coupling curves are obtained for different values of the material parameter.

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