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Calculation of Gaussian-Wavefront Variations Induced by a Photorefractive Grating with non-90 Phase Shift

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Variations in the amplitude and phase distributions of an incident Gaussian beam are derived analytically when it propagates through a photorefractive crystal in the presence of another strong beam and resulting photorefractive grating that is phase shifted by other than 90° relative to the intensity grating. In the presence of the photorefractive grating the beam is shown to propagate along a straight line in a new direction with an increased wave number.

I. INTRODUTION

When two coherent optical beams interact in a photorefractive crystal and result in a photorefractive grating that is phase shifted by 90° with respect to the incident intensity grating, energy of one beam is transferred to the other beam while there is no change in the phase fronts of the beams.[1-3] In this case the propagation of the beams in the crystal can be well explained with Maxwell's equations by assuming of plane waves and slowly varying envelopes. When,however,the intensity grating results in a photorefractive grating with non-90° phase shift,because of photovotaic effect,applied electric field,[5-6] moving grating[7-8] and so on,the phase fronts of the beams should be changed during propagation through the crystal. Recently, it has been observed that an absorption grating can arise in a photorefractive crystal due to difference in absorption cross sections of the full and empty traps,which is in phase with the intensity grating,and therefore change the phase fronts of the beams.[9-10] In most cases,however,it has been just described qualitatively as they cause the phase fronts of the incident beams to change and the resulting photorefractive grating to tilt and bend in the crystal.[11]

In this paper,the variations in the amplitude and phase distributions of an incident Gaussian beam are derived analytically,for the forst time to our knowledge, at a steady state by assuming a photorefractive grating with non-90° phase shift in a photorefractive crystal. In the calculation a weak "signal" is assumed to have a Gaussian intensity profile and another strong "reference" to have a planar phase front. In the presence of the photorefractive grating it is assumed that the amplitude and phase distributions of the weak signal are modified by a multiplication of slow exponential functions in the transverse plane, while the strong reference remains unchanged in the crystal. In this case, the interference fringe is first obtained from the assumed expressions of the beams. Next, the standard band conduction model[12] is solved for space-charge electric field by using the intensity distribution and by assuming no movement of charge carriers along the bisector of the beams. Then, the exact amplitude and phase distributions of the weak signal is derived by forcing the beam to satisfy Maxwell's equation in the presence of the resulting photorefractive grating with non-90° phase shift in the crystal.

II. THEORY

Fig. 1 shows geometry of a photorefractive crystal and two incident beams used in our calculations. The weak signal is assumed to have a Gaussian intensity profile only in one direction parallel to xz-plane and, therefore,no variation in its amplitude and phase along y-axis,for simplicity in calculations. The strong reference is assumed to have a planar phase front for the

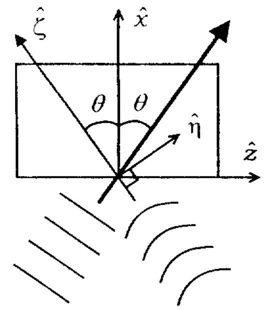

그림 1. A weak signal with a Gaussian profile interacts with a strong reference with a planar wave front in a photorefractive crystal. xz-coordinates are to represent the crystal and the strong reference while $\xi\eta$-coordinates to represent the weak signal in the crystal.

simplicity as before. The beams are assumed to cross each other such that the bisector is parallel to the entrance surface normal, or x-axis in Fig. 1. In addition to xyz-coordinate system,another coordinate system is defined such that ξ-axis is parallel to the propagation direction of the weak signal and η-axis is perpendicular to ξ-axis and parallel to xz-plane. Since a function represented in one coordinate system can be easily transformed into the other at later times,the weak signal can be represented by ξ and η axes as $\boldsymbol{e}_1(\boldsymbol{r}, t)=\frac{1}{2}\times E_1\exp\left[\frac{1}{2}iQ\eta^2+iP+ik_1\xi-iwt\right]+c.c$, when there is no photorefractive grating in the crystal,while the strong reference by x and z axes as $\boldsymbol{e}_2(\boldsymbol{r}, t)=\frac{1}{2}\boldsymbol{E}_2\exp[i\,\boldsymbol{k}_2\cdot\boldsymbol{r}-iwt]+c.c.$ Here Q and P are Gaussian beam parameters, k_1 and k_2 are wave number of the weak signal and wave vector of the strong refernce, respectively, $\boldsymbol{r}$ is position vector represented in xyz-coordinates, ω is angular frequency, and $c.c$ stands for complex conjugate. It is noted from the expression of the weak gignal that the point, $\xi=\eta=0$, corresponds to the center of beam waist of the weak signal. If there is no photorefractive grating in the crystal,the Gaussian beam parameters can be obtained from Maxwell's equations as $Q=k_1/(\xi-i\xi_0)$ and $P=\frac{1}{2}i ln(1+i\xi/\xi_0)$, where ξ_0 is Rayleigh range and a factor of $\frac{1}{2}$ in P results from our assuming of no variation along y-axis.[13]

In the crystal, a photorefractive grating with non-90° phase shift can decompose into a 90°-phase-shifted component and an in-phase component. The 90°-phase-shifted component changes the envelope of the weak signal as a function of the distance from the entrance surface of the crystal, which is what the conventional slowly varying envelope approximation (SVEA) is based on. In a similar way one may assume that the in-phase component changes the phase of the weak signal as another function of the distance from the entrance surface. Since the conventional SVEA predicts the envelope of the beam to vary exponentially along the entrance surface normal, one can thus assume that the photorefractive grating changes both amplitude and phase of the weak signal exponentially in the transverse plane of the beam,starting from the point where the plane meets the entrance surface of the crystal. In addition,we assume that the photorefractive grating makes the weak signal displace in its transverse plane. Therefore,in the presence of the photorefractive grating with non-90° phase shift and at a steady state,we assume the electric fields of the weak signal and the strong reference as[14]

$$\boldsymbol{e}_1(\boldsymbol{r}, t)=\frac{1}{2}\boldsymbol{E}_1\exp\left[\frac{1}{2}iQ\eta^2+iP+ik_1-iwt\right]$$

$$\exp[(S+iT)\{\eta+(\xi-\xi_1)\cot\theta\}+iU+c.c] \quad (1)$$

$$\boldsymbol{e}_2(\boldsymbol{r}, t)=\frac{1}{2}\boldsymbol{E}_2\exp[i\,\boldsymbol{k}_2\cdot\boldsymbol{r}-iwt]+c.c. \quad (2)$$

Here S represents a photorefractive gain resulting from the 90°-phase-shifted component and T a phase front change resulting from the in-phase component. ξ_1 is the value of ξ-axis at the entrance surface and, therefore, $\{\eta+(\xi-\xi_1)\cot\theta\}$ represents the distance of a point (ξ, η), measured in the transverse plane, from the entrance surfance of the crystal. U represents the transverse displacement of the beam, which depends only on ξ. Then we obtain the resulting interference fringe as

$$I(x, z)=I_0+\frac{1}{2}mI_0\exp[-iKz]\exp\left[\frac{1}{2}iQ^2+iP\right]\times$$

$$\exp[(S+iT)\{\eta+(\xi-\xi_1)\cot\theta\}+iU_\eta]+c.c, \tag{3}$$

where K is the magnitude of the grating wave vector defined by $\boldsymbol{K}\equiv\boldsymbol{k}_1-\boldsymbol{k}_2$. When the reference is much stronger than the signal, $E_2|\gg E_1$, the modulation index can be assumed to be $m\approx 2E_1E_2/|E_2|^2$ and the average light intensity I_0 to be constant in the crystal.

When the light intensity distribution is given in the crystal as in (3), the band conduction equations can be solved for space-chage electric field as follows. To ignore higher-order spatial harmonics the intensity modulation m is assumed much smaller than unity.[15] Then, the excited charge carrier density, n, empty trap density, N^i_D, and space-charge electric field, e_{sc}, can be assumed to have the same spatial variation as the intensity distribution. By inserting the expressions for n, N^i_D and e_{sc} into the band conduction model and by assuming a steady state one may obtain the space-charge electric field. Since, in this case,the band conduction equations become highly nonlinear, a numerical method is required for the exact solution. However, an approximate analytic solution can be obtained by assuming the gradient of the intensity distribution along x-axis to be negligible compared with that along z-axis. This approximation can be written as

$$w/\xi_0\ll\sin\theta, \tag{4}$$

where w is beam width measured in the crystal, ξ_0 is Rayleigh range, and θ is an angle between the weak signal and the entrance surface normal. With the condition of (4) the space-charge electric field can be obtained in the same way as for two incident plane waves.

$$e_{sc}=\frac{1}{2}m\,E_{sc}\exp[-iKz]\exp\left[\frac{1}{2}iQ^2+iP\right]$$
$$\exp[(S+iT)\{\eta+(\xi-\xi_1)\cot\theta\}+iU\eta]+c.c, \tag{5}$$

In the presence of an applied field E_0, for example,the complex amplitude E_{sc} can be obtained,in a photorefractive crystal where hole is the dominant charge carrier, as

$$E_{sc}=\frac{-E_0-iKK_BT/q}{1+(K^2/K_0^2)(1-iE_0q/KK_BT)}\hat{z}, \tag{6}$$

where K_B is Boltzmann constant, T is temperature, q is absolute value of electron charge, K_0 is Debye wave number, and $\hat{z}$ is a unit vector in z-axis.

The space-charge electric field modulates the electric susceptibility in the crystal via the linear electro-optic effect. Then,the nonlinear polarization for the weak signal results in the crystal as

$$\Delta P_1(r,\ t)=-\varepsilon_0[\dot{\varepsilon}_\omega\cdot(\ddot{R}\cdot e_{sc})]\frac{1}{2}E_2\exp[ik_2\cdot r-i\omega t]+\text{c.c},$$

where ε_0 is vacuum permittivity, $\dot{\varepsilon}_\omega$ is the second-rank dielectric tensor, $\ddot{R}$ is the third-rank electro-optic tensor. In the presence of the nonlinear polarization the weak signal should satisfy the wave equation, $\nabla^2e_1+k_1^2e_1+(\omega/c\varepsilon_0)^2\Delta P_1=0$, where c is speed of light in vacuum. By assuming S and T to be much smaller than $k_1\tan\theta$ and magnitude of U, $dQ/d\xi$, $dP/d\xi$, and $dU/d\xi$ to be much smaller than k_1 we can reduce the wave equation to

$$\eta^2\left[Q^2+k_1\frac{\partial Q}{\partial\xi}\right]+\eta\left[iQ(S+iT)-QU-k_1\frac{\partial U}{\partial\xi}\right]+$$
$$\left[iQ-2k_1\frac{\partial P}{\partial\xi}+i2k_1(S+iT)\cot\theta-k_1\left(\frac{\omega}{c}\right)n_1^3r_{eff}E_{sc}\right]=0 \tag{7}$$

where n_1 is refractive index of the weak signal and r_{eff} is effective electro-optic coeffient given by $r_{eff}=\ddot{R}\cdot\hat{z}$. Since the equation should be satisfied regardless of a value of η, each bracket should be zero in the crystal. Using the equation $Q=k_1/(\xi-i\xi_0)$ in the second bracket we obtain

$$U=\frac{i\xi(S+iT)+C}{\xi-i\xi_0}, \tag{8}$$

where C is a constant to be determined later. If the Gaussian beam parameters Q and P are assumed to follow their equations of motion even in the presence of the photorefractive grating, the first bracket becomes zero automatically and the third bracket gives expressions for S and T as

$$S=(\omega/2c)n_1^3r_{eff}I_m[E_{sc}]\tan\theta. \tag{9}$$

$$T=-(\omega/2c)n_1^3r_{eff}R_e[E_{sc}]\tan\theta, \tag{10}$$

where I_m and R_e stand for imaginary and real parts,respectively.

We next obtain propagation directions of the energy and phase of the weak signal as follows. Eqs. (1) and (8) give the transverse amplitude distribution of the weak signal as $\exp\left[-\frac{1}{2}Q_i(\eta-\Delta\eta)^2+\frac{1}{2}Q_i(\Delta\eta)^2\right]$, where Q_i represents the imaginary part of Q and $\Delta\eta$ a transverse displacement of the amplitue distribution given by $\Delta\eta=(T\xi+S\xi_0-C)/k_1$. Then, at the center of the amplitude distribution,or at a point (ξ, $\eta=\Delta\eta$), we can obtain the propagation direction of the phase front by taking the gradient of the phase, $\psi=ik_1\xi+\frac{1}{2}iQ_r\eta^2+iP_r+iT\{\eta+(\xi-\xi_1)\cot\theta\}+iU_r\eta$, as $\mathrm{grad}(\psi)=k_1\hat{\xi}+T\hat{\eta}$. In the equations Q_r, P_r and U_r represent real parts of the respective functions and $\hat{\xi}$ and $\hat{\eta}$ are unit vectors in ξ and η directions, respectively. In the calculation of the gradient, S and T are ignored whenever they compete with k_1. We note in the equations of $\Delta\eta$ and $\mathrm{grad}(\psi)$ that the propagation directions of the energy and phase of the weak signal are the same regardless of a value of C. We obtain the value of C from the boundary condition that the transverse displacement of the weak signal should be zero at the entrance surface of the crystal as

$$C=S\xi_0+T\xi_1 \tag{11}$$

This gives the transverse displacement of the beams as $\Delta\eta=T(\xi-\xi_1)/k_1$. By inserting (8) and (11) into (1) we obtain

$$e_1(r,\ t)=\frac{1}{2}E_1\exp\left[\frac{1}{2}iQ\{\eta^2+2(T/k_1)(\xi_1-i\xi_0)\eta\}+iP+(S+iT)(\xi-\xi_1)\cot\theta\right]\exp[ik_1\xi-i\omega t]+c.c, \tag{12}$$

which is the electric field of the weak signal in the presence of the photorefractive grating with non-90° phase shift at a steady state.

III. DISCUSSION

In conclusion,we show an analytic expression of an incident Gaussian beam, for the first time to our knowledge,in the presence of a photorefractive grating with non-90° phase shift in a photorefractive crystal. Our calculation shows that the beam propagates in a new direction, $k_1\hat{\xi}+T\hat{\eta}$, with an increased wave number of $k_1\sqrt{1+(T/k_1)^2}$ and that the center of the beam displaces by $\Delta\eta=T(\xi-\xi_1)/k_1$ in its transverse plane. When the incident plane formed by two incident beams is parallel to xz-plane of a $BaTiO_3$ crystal, for example, with an applied electric field along +C axis,our calculation shows that the beam always tilts towards the entrance surface normal, or x-axis in Fig. 1, regardless of the orientation of the +C axis and of the type of the dominant charge carrier. Our method may be also used for calculation of the electric field of the weak signal in the presence of a moving grating or an absorption grating.

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