The Journal of Engineering Geology, Vol.4, No.3, December., 1994, pp.311-319.

Hydraulic Parameter Evaluation by Sensitivity Analysis of Constant and Variable Rate Pump Test in Leaky Fractal Aquifer
누수성 프락탈 대수층내의 일정 또는 다단계 양수시험의 민감성 분석에 의한 수리상수 결정

함 세 영 (Hamm, Se-Yeong) 한국자원연구소

Abstract/요약

This paper presents a sensitivity analysis to obtain best fit of hydraulic parameters of leaky fractal aquifer. The sensitivity analysis uses the least squares method. The hydraulic parameters (generalized transmissivity and generalized storage coefficient) can be easily determined by the sensitivity analysis for various flow dimensions and different values of the leakage factor. Furthermore, the sensitivity analysis was applied to variable-rate pump tast at several abstraction wells.

A computer program was developed to evaluate the hydraulic parameters by the sensitivity analysis.

본 논문은 누수를 포함하는 프락탈 대수층의 수리상수의 최적값을 구하기 위한 민감성 분석에 대한 것이다. 민감성 분석은 최소자승법을 이용한다. 민감성 분석에 의하여 수리상수(일반화 투수량계수와 일반화 저유계수)는 여러가지 흐름의 차원과 여러 값의 누수계수에 대해서 쉽게 결정될 수 있다. 아울러, 민감성 분석은 다수의 양수정의 다단계 양수에도 적용되었다.

민감성 분석을 이용한 프락탈 대수층의 수리상수 산출을 위하여 컴퓨터 프로그램이 개발되었다.

INTRODUCTION

The characteristics of ground water flow in the fractured media have been studied since several tens years (Barenblatt et al., 1960 ; Warren and Root, 1963 ; Streltsova, 1976 ; Boulton and Streltsova, 1977 ; Duguid and Lee, 1977 ; Gringarten and Witherspoon, 1872 ; Gringarten and Ramey, 1974 ; Gringarten, Ramey and Raghaven, 1974, 1975 ; Ferris et al., 1962 ; Jenkins and Prentice, 1982 ; Cinco-Ley and Samaniego, 1981). Unfortunately, all these methods can not treat correctly physical properties of flow in fractured media.

In recent years, ground water flow in fractured rocks has been analyzed by a fractal theory (Barker, 1988). Bangoy et al. (1992) applied the fractal theory to the observed data of two experimental sites and proved that the fractal model agreed with the data very well. The fractal model can explain some characteristics of groundwater flow in fractal aquifers which are composed of the fractal network of fractures. An important parameter of the fractal model is the non-integer flow dimension. However, it will be necessary to use another fractal model for simulating the fractal aquifer with leakage from aquitard. A fractal model of groundwater flow with leakage from aquitard was proposed by Hamm and Bideaux (1994). It comprises the storage capacity and the well loss effects at the production well.

Since many years, a number of methods of pumping test analyses using computers have been developed to calculate the hydraulic parameters in two dimensional radial aquifers (Saleem, 1970 ; McElwee, 1980a ; Rayer, 1980 ; Grimestad, 1981 ; Das Gupta and Joshi, 1984 ; Hamm, 1987 ; Smith, 1987 ; Yeh, 1988). However, until now, no paper on computer analysis of pump test in fractal aquifers has been published. Type curves for different flow dimensions with different values of the leakage factor were made for the abstraction well and the observation well (Hamm and Bideaux, 1994). However, Type-curve matching is often troublesome to use because it is necessary many type-curves for different dimensions and for different parameters.

MATHEMATICAL BACKGROUND

Let us consider a borehole to which the flow converse in a fractal aquifer. In the fractal aquifer, the section of flow, A is proportional to a non-integral power of r (distance from the production well) :

$$A(r)=b^{3-n}r^{n-1}(2\pi^{n/2}/\Gamma(n/2)) \cdots\cdots\cdots (1)$$

where b is the transverse extension [L] of the aquifer to the flow path, and r is the radial distance [L] from the production well, along the flow line.

For instance,

– for linear flow(n=1), A=2we

— for cylindrical flow($n=2$), $A=2\pi er$

— for spherical flow($n=3$), $A=4\pi r^2$

where e is the thickness [L] of a cylindrical symmetry aquifer, and w is the width [L] of the flow front in linear geometry.

The diffusivity equation of flow in a leaky fractal aquifer without storage in the confining layer(Hamm and Bidaux, 1994) can be expressed as :

$$\frac{K_f}{r^{n-1}}\frac{\partial}{\partial r}\left(r^{n-1}\frac{\partial s}{\partial r}\right)=S_{sf}\frac{\partial s}{\partial t}+K_f\frac{s}{B^2} \quad \cdots\cdots (2)$$

K_f : hydraulic conductivity of the aquifer $[LT^{-1}]$

S_{sf} : specific storage of the aquifer $[L^{-1}]$

B : leakage factor [L]

t : time since pumping started [T]

s : drawdown in the fractal aquifer [L]

n : dimension of flow.

Here the leakage rate(q) from the aquitard into the fractal aquifer is assumed to be vertical to the flow face and purely propotional to the head difference between the water table in the source layer above the aquitard and the piezometric surface of the aquifer. With these assumptions, the water table in the source bed is assumed constant(Fig. 1).

Initial condition is :

$$s(r,0)=0 \qquad r\geq 0 \quad \cdots\cdots (3)$$

Boundary condition is :

$$s(\infty,t)=0 \qquad t\geq 0 \quad \cdots\cdots (4)$$

We assume that the fractal system obeys

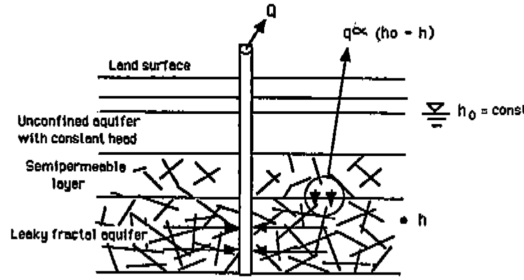

Fig. 1 Scheme of leaky fractal aquifer

Darcy's law. Consequently, the change of storage in the well can be given by :

$$W_s\frac{\partial s_w}{\partial t}=Q+K_f b^{3-n}\alpha_n r_w^{n-1}\left(\frac{\partial s}{\partial r}\right)_{r=r_w} \quad \cdots\cdots (5)$$

where Q is the discharge rate $[L^3T^{-1}]$, s_w is the drawdown at the production well [L], W_s is the storage capacity $[L^2]$ of the production well, r_w is the radius [L] of the production well, and $\alpha_n=2\pi^{n/2}/\Gamma(n/2)$. Here, $\Gamma(n/2)$ is a gamma function. In general, W_s is equivalent to πr_w^2. The drawdown in the production well will be different from the drawdown at $r=r_w$ if there exists a skin loss :

$$s_w(t)=s(r_w,t)-s_f r_w\left(\frac{\partial s}{\partial r}\right)_{r=r_w} \quad \cdots\cdots (6)$$

where s_f is the skin factor (dimensionless). From (2), (3), (4), (5) and (6), the line source solution of the drawdown in the leaky fractal aquifer (Hamm and Bidaux, 1994) is :

$$s(r,t)=\frac{Qr^{2-n}}{4\pi^{n/2}K_f b^{3-n}}G(n,u,r/B) \quad \cdots\cdots (7)$$

where

$u=S_{sf}r^2/4K_f t$

$$G(n,u,r/B)=\int_u^{\infty} y^{n/2-2}\exp\left(-y-\frac{r^2}{4B^2y}\right)dy \quad \cdots\cdots (8)$$

For example, in case of dimension 2, eq. (8) is expressed by (Hantush and Jacob, 1955) :

$$s(r,t)=\frac{Q}{4\pi K_f e}\int_u^{\infty} y^{-1}\exp\left(-y-\frac{r^2}{4B^2y}\right)dy \quad \cdots\cdots (9)$$

In case of dimension 1, eq. (8) is represented by (Vandenberg, 1977) :

$$s(r,t)=\frac{Qr}{4\sqrt{\pi}K_f we}\int_u^{\infty} y^{-3/2}\exp\left(-y-\frac{r^2}{4B^2y}\right)dy \quad \cdots\cdots (10)$$

If $1/B^2=0$, eq. (7) reduces to the generalized radial flow equation of Barker (1988).

SENSITIVITY ANALYSIS AND LEAST SQUARES FIT

To automatically determine the generalized transmissivity ($K_f b^{3-n}$) and the generalized storage coefficient ($S_{sf} b^{3-n}$) of the leaky fractal aquifer from eq. (7), we can use the sensitivity analysis which quantifies the change in calculated drawdowns caused by a small variation in the aquifer parameters.

The updated drawdown s* can be approximated by a Taylor's series where $\triangle(K_f b^{3-n})$ and $\triangle(S_{sf} b^{3-n})$ are small enough such that the second and higher powers can be ignored :

$$s^*=s+U_\alpha\triangle(K_f b^{3-n})+U_\beta\triangle(S_{sf} b^{3-n}) \quad (11)$$

where

$$U_\alpha=\frac{\partial s}{\partial(K_f b^{3-n})}$$

$$U_\beta=\frac{\partial s}{\partial(K_{sf} b^{3-n})}$$

and

s : calculated drawdown

U_α and U_β are called sensitivity coefficients of the generalized transmissivity and the generalized storage coefficient, respectively. Applying Leibnitz's rule to U_α and U_β for differentiating eq. (7) (Spiegel, 1971, 1974), one obtains :

$$U_\alpha=-\frac{s}{K_f b^{3-n}}-\frac{S_{sf} b^{3-n}}{K_f b^{3-n}}U_\beta \quad \cdots\cdots (12)$$

$$U_\beta=\frac{\Omega}{K_f b^{3-n} S_{sf} b^{3-n}}\,\omega^{n/2-1}\exp\left(-\omega-\frac{r^2}{4B^2\omega}\right) \quad \cdots\cdots (13)$$

where

$$\Omega=\frac{Qr^{2-n}}{4\pi^{n/2}}$$

$$\omega=\frac{S_{sf} b^{3-n} r^2}{4K_f b^{3-n} t}$$

Eqs. (12) and (13) can be utilized in eq. (11) to calculate new drawdown values as changing $K_f b^{3-n}$ and $S_{sf} b^{3-n}$ by small variation of $\Delta(K_f b^{3-n})$ and $\Delta(S_{sf} b^{3-n})$.

Sensitivity equations mentioned above are used to obtain least-squares fit of pumping test data to eq. (7) and thus obtain reasonable $K_f b^{3-n}$ and $S_{sf} b^{3-n}$. To apply this technique, a squared error function is defined as :

$$E(\Delta(K_f b^{3-n}), \Delta(S_{sf} b^{3-n})) = \sum(s_d(t_i) - s^*(t_i))^2 \quad \cdots\cdots(14)$$

where $s_d(t_i)$ is observed drawdown at time t_i. The values of $\Delta(K_f b^{3-n})$ and $\Delta(S_{sf} b^{3-n})$ are used to update $K_f b^{3-n}$ and $S_{sf} b^{3-n}$. For minimizing the least squares error, one can utilize :

$$\frac{\partial E}{\partial \Delta(K_f b^{3-n})} = \frac{\partial E}{\partial \Delta(S_{sf} b^{3-n})} = 0 \quad \cdots(15)$$

From eqs. (11), (14) and (15), $\Delta(K_f b^{3-n})$ et $\Delta(S_{sf} b^{3-n})$ can be solved as follows :

$$\Delta(K_f b^{3-n}) = [(SUTDIF) - (SUTUS)\ \Delta(S_{sf} b^{3-n})]/(SSUT) \quad \cdots\cdots(16)$$

$$\Delta(S_{sf} b^{3-n}) = \frac{(SSUT)(SUSDIF) - (SUTUS)(SUTDIF)}{(SSUS)/(SSUT) - (SUTUS)^2} \quad \cdots\cdots(17)$$

where

$$SSUT = \sum_i U_\alpha^2(t_i)$$

$$SSUS = \sum_i U_\beta^2(t_i)$$

$$SSTUS = \sum_i U_\alpha(t_i)\ U_\beta(t_i)$$

$$SUTDIF = \sum_i U_\alpha(t_i)[s(t_i) - s^*(t_i)]$$

$$SUSDIF = \sum_i U_\beta(t_i)[s(t_i) - s^*(t_i)]$$

Thus the updated $K_f b^{3-n}$ and $S_{sf} b^{3-n}$ after the i^{th} iteration are given by :

$$(K_f b^{3-n})_{i+1} = (K_f b^{3-n})_i + \Delta(K_f b^{3-n})_i \quad \cdots(18)$$

$$(S_{sf} b^{3-n})_{i+1} = (S_{sf} b^{3-n})_i + \Delta(S_{sf} b^{3-n})_i \quad \cdots(19)$$

Thus the values of $\Delta(K_f b^{3-n})_i$ and $\Delta(S_{sf} b^{3-n})_i$ are used to update the values of $(K_f b^{3-n})_i$ and $(S_{sf} b^{3-n})_i$. The improved $(K_f b^{3-n})_{i+1}$ and $(S_{sf} b^{3-n})_{i+1}$ are used in the eqs. (12) and (13) and then in the eqs. (16) and (17) to obtain $\Delta(K_f b^{3-n})_{i+1}$ and $\Delta(S_{sf} b^{3-n})_{i+1}$. The iteration continues until both $|\Delta(K_f b^{3-n})_i / K_f b^{3-n}{}_{i+1}|$ and $|\Delta(S_{sf} b^{3-n})_i / S_{sf} b^{3-n}{}_{i+1}|$ come into the error limit. In this study, we used the error limit as 0.0001.

The root-mean-squared error(e_0) is used as a measure of the goodness of the least squares fit and is given by

$$e_0 = \sqrt{\frac{E[\Delta(K_f b^{3-n}), \Delta(S_{sf} b^{3-n})]}{N}} \quad \cdots(20)$$

where N is the total number of observations.

ANALYSIS OF VARIABLE PUMPAGE

Because the fractal aquifer obeys Darcy's law and hence the drawdown can be expressed by a sort of linear equation, we can apply the principle of superposition into the drawdown at an observation well caused by variable pumpage at several production wells. From eq. (7), the drawdown influenced by variable pump rate is written by :

$$s_p(t) = \frac{1}{4\pi^{n/2} K_f b^{3-n}} \sum_{j=1}^{m} r_{j\ p}^{2-n} \sum_{i=1}^{k} (Q_{j,\ i} - Q_{j,\ i-1})\ G(n, u', r/B) \quad \cdots\cdots(21)$$

where

$s_p(t)$: drawdown at time t on the observation well p [L]

$$u' = \frac{4K_f(t - t_{j,i})}{r_j^2 S_{sf}}$$

k : total number of pumping stages

m : total number of pumping wells

$r_{j,p}$: distance from the observation well p to the pumping well j [L]

$Q_{j,i}$: discharge at the pumping well j during the stage i [L^3T^{-1}] with $Q_{j,0}=0$

$t_{j,i}$: start of pumping stage i at the pumping well j [T].

COMPUTER PROGRAM OF PUMP TEST ANALYSIS USING SENSITIVITY ALGORITHM

A computer program of the sensitivity analysis was developed for determining the generalized transmissivity and the generalized storage coefficient. The simplified program description is as follows :

1. Input default hydraulic parameters and drawdown data.
2. Compute chosen G(n, u, r/B) values between the maximum u and the minimum u. The function G(n, u, r/B) is calculated by Romberg integration method.
3. Compute u corresponding to the time of water level measurements, using interpolation methods.
4. Execute the sensitivity analysis and then the least squares fit. The procedures 3 and 4 are repeated until obtaining best fit between measured drawdowns and theoretical ones.
5. Verify the degree of accordance of the theoretical curve with observation data on the screen, and if necessary, change the dimension of flow and then leakage factor to get a better accordance and best-fitted values of the generalized transmissivity and the generalized storage coefficient.

CONCLUSION

The leaky fractal aquifer model is available to understand groundwater flow in fractured aquifer (Hamm and Bidaux, 1994; Hamm and Lim, 1994). Automatic calculation of the hydaulic parameters using the sensitivity analysis will be very useful to analyze constant and variable rate pump test data from leaky fractured aquifer. On the contrary, type-curve matching needs too many curves for different flow dimensions and different values of the leakage factor and so it needs time-consuming work.

REFERENCES

Bangoy, L. M. et al.. 1992. A new method of characterizing fissured media by pumping tests with observation wells. Jour. Hydr., v. 138, pp. 77−88.

Barenblatt, G. E., Zheltov, I. P. and Kochina, I. N. 1960. Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks. Jour. Appl. Math. Mech. Engl. Trans., 24(5), pp. 1286−1303.

Barker, J. A. 1988. A generalized radial flow model for hydraulic tests in fractured rock. Water Resour. Res., 24 (10), pp. 1796−1804.

Boulton, N. S. and Streltsova, T. D. 1977. Unsteady flow to a pumped well in a fissured water-bearing fomation. Jour. Hydr., v. 35, pp. 257−269.

Butt, M. A. and McElwee, C. D. 1985. Aquifer-parameter evaluation from variable-rate pumping tests using convolution and sensitivity analysis. Groung Water, 23(2), pp. 212−219.

Cinco-Ley, H. and Samaniego-V., F. 1981. Transient pressure analysis for fractured wells. J. Pet. Tech., 33(Sept.), pp. 1749−1766.

Cobb, P. M., McElwee, C. D. and Butt, M. A., 1982. Analysis of leaky aquifer pumping test data : an automated numerical solution using sensitivity analysis. Ground Water, 20(3), pp. 325−333.

Das Gupta, A. and Joshi, S. C., 1984. Algorithm for Theis solution. Ground Water, 22(2), pp. 199−206.

Duguid, J. O., and Lee, P. C. Y., 1977. Flow in fractured porous media. Water Resour. Res., 13(3), pp. 558−566.

Ferris, J. G., Knowles, D. B., Brown, R. H., and Stallman, R. W., 1962. Theory of aquifer tests. U. S. Geol. Survey Water-Supply Paper 1536−E, pp. 60−174.

Grimestad, G., 1981. Inverse solutions of the Theis equation determined with programmable calculators-II. Determination of aquifer hydraulic constants from numerical test data. Ground Water, 19(4), pp. 387−391.

Gringarten, A. C. and Witherspoon, P. A., 1972. A method of analyzing pump test data from fractured aquifers. Symposium Percolation Through Fissured Rock, Proceedings, Stuttgart.

Gringarten, A. C. and Ramey, H. J., 1974. Unsteady state pressure distributions created by a well with a single horizontal fracture, partial penetration, or restricted entry. SPEJ, pp. 413−426.

Gringarten A. C., Ramey, H. J., and Raghavan, R., 1974. Unsteady-state pressure distributions created by a single infinite-conductivity vertical fracture. SPEJ, 14, pp. 347−360.

Gringarten, A. C., Ramey, H. J., Jr., and Raghavan, R., 1975. Applied pressure analysis for fractured wells, J. Pet. Tech., Trans., AIME, 259, pp. 887−892.

Hamm, S. Y., 1987. Pumping test analysis using a portable micro computer. Jour. of the Geol. Soc. of Korea, 23(2), pp.

109–119.

Hamm, S. Y. and Bidaux, P., 1994. Ecoulements transitoires en géométrie fractale avec drainance : théorie et application, C. R. Acad. Sci. Paris, 318, série II. n. 2, pp.227–233.

Hamm, S. Y. and Lim, J. U., 1994. Computing hydraulic parameters of fractured aquifers using fractal model of groundwater flow with leakage. The Jour. Eng. Geol., v.4, n.2., pp. 219–229.

Hantush, M. S. and Jacob, C. E., 1955. Non-steady radial flow in an infinite leaky aquifer and non-steady Green's functions for an infinite strip of leaky aquifer. Trans., AGU, 36(1), pp.95–112.

Jenkins, D. N. and Prentice, J. K., 1982. Theory for aquifer test analysis in fractured rocks under linear(nonradial) flow conditions. Ground Water, 20(1), pp. 12–21.

McElwee, C. D. and Yukler, M. A., 1978. Sensitivity of groundwater models with respect to variations in transmissivity and storage. Water Resour. Res., v.14, n.3, pp. 451–459.

McElwee, C. D., 1980a. Theis parameter evaluation from pumping test by sensitivity analysis. Ground Water, 18, pp. 56–60.

McElwee, C. D., 1980b. The Theis equation : evaluation, sensitivity to storage and transmissivity, and automated fit of pump test data. Kansas Geological Survey, Ground-Water Series 3, p. 39.

Rayer, F. A., 1980. Pumping test analysis with a handheld calculator. Ground Water, 18(6), pp. 562–568.

Saleem, Z., 1970. A computer method for pumping test analysis. Ground Water, 8 (5), 21–24.

Smith, S. M., 1987. Pumptest.bas : a program to calculate transmissivity and storativity. Ground Water, 25(5), pp. 599–602.

Spiegel, M. R., 1971. Advanced Mathematics for Engineers & Scientists. Schaum's outline series, McGraw-Hill Book Company, New York, 407p.

Spiegel, M. R., 1974. Advanced Caculus. SI (metric) edition, Schaum's outline series, McGraw-Hill Book Company, New York, 384p.

Streltsova, T. D., 1976. Hydrodynamics of ground water flow in fractured formation. Water Resour. Res., 12(3), pp. 405–414.

Warren, J. E. and Root, P. J., 1963. The behavior of naturally fractured reservoirs. SPEJ, 3(2), pp. 245–255.

Tomovic, R., 1962. Sensitivity analysis of dynamic systems. McGraw-Hill, New York, 141p.

Yeh, H. D., 1988. Theis solution by nonlinear least-squares and finite-difference Newton's method. Ground Water, 25(6), pp. 710–715.

Yukler, M. A., 1976. Analysis of error in groundwater modeling. Ph. D. dissertation, The University of Kansas, 182p.

함세영 :
한국자원연구소
305－350
대전시 유성구 가정동 30번지
TEL : (042)868－3057
FAX : (042)861－9720
Korea Institute of Geology, Mining and Materials (KIGAM)
30, Kajung-Dong, Yusong-Ku,
Taejon, Korea 305－350